Electronic Theory of Solids
Prof. Arghya Taraphder
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture – 50
Cooper problem

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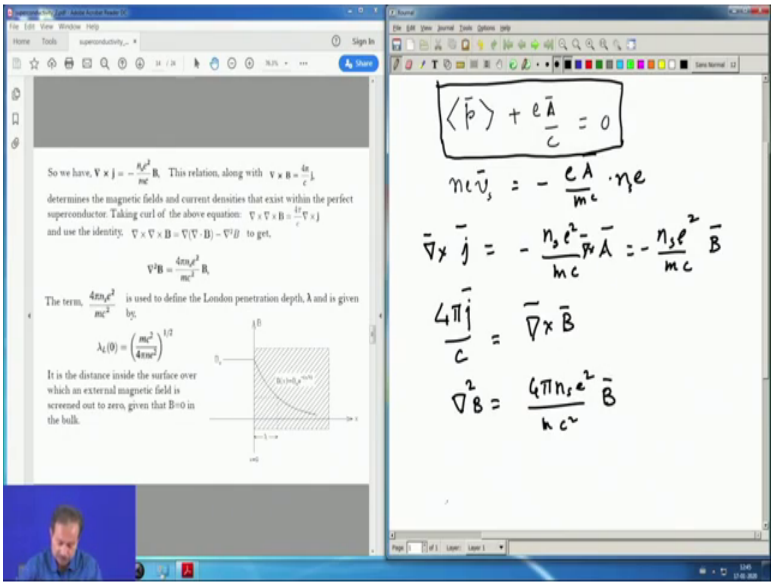

Hello and welcome, we have discussing superconductivity and the in the last lecture we discussed how London equation leads to magnetic field decaying inside a superconductor within a few angstroms of the surface and. So, that is a, that is one basically phenomenological theory which we showed and that was working fairly well to explain certain things for example, this decay of magnetic field which means Meissner effect.

So, let just to recap what we showed was that the London equation can also be written as p plus e A by c equal to 0 basically, net momentum being 0 inside which was Bloch's assertion and from here one can ne v s equal to or equal to minus e A by m c, so e A by. So, p by so, this is basically p by m velocity is p by m. So, I divide both sides by m and this is what I will get ok.

So, and then times n e and that is this equation that j equal to minus n. So, this s subscript basically means that we are dealing with the superconducting component n s e square by m c times A. So, this is this equation that is written on the left if you just take a curl of this equation you will land up with curl of A here which is B. So, minus n s e square by m c B. This was the London equation. So, in essence from this almost the entire London equation and its consequences follow and the this equation as we showed in combination with the equation that 4 pi j by c equal to curl of B will lead to this equation del square B equal to 4 pi n s e square by m c square into B.

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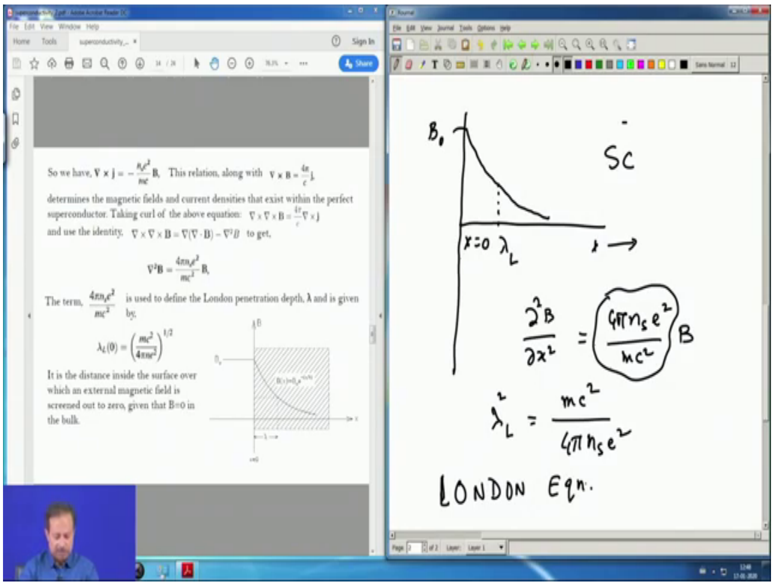

And then we showed that if you take a semi infinite super conducting system. So, this side is superconductor all the way totally this is totally superconducting that side all the way and this is the value of B at x equal to 0 and then this side is superconductor entire right hand side, x greater than 0 is superconductor.

In that case, one obtains an equation which is del square B, del x square equal to 4 pi n s e square by mc square into B and then immediately we identified that this has to have a dimension of one by length square so that, defines the so called London penetration depth is equal to mc square by 4 pi n s e square. So, that is about it and that is the decay rate. Ok decay by which one third of these is already decayed.

And we can we from experimental values if you put it in here you will find that this is a few within a few angstrom less than 100 angstrom or so. It depends of course, on the system between 100 and 1000 angstrom this thing decays in many superconductors in some it can be much it can be large larger, but that is depend that depends on what kind of superconductor you are using.

So, this was the consequences of London equation and we will leave London equation here at saw at one point in history London equation was the only equation that experimentalist had to fit their data and this was quite successful in although it is quite phenomenological there is no microscopic way of getting at it, but this is what it is and it served its purpose. One goes try goes beyond all these my phenomenological theories one is attempting to get to a microscopic theory of superconductivity, because one needs to know the mechanism of superconductivity.

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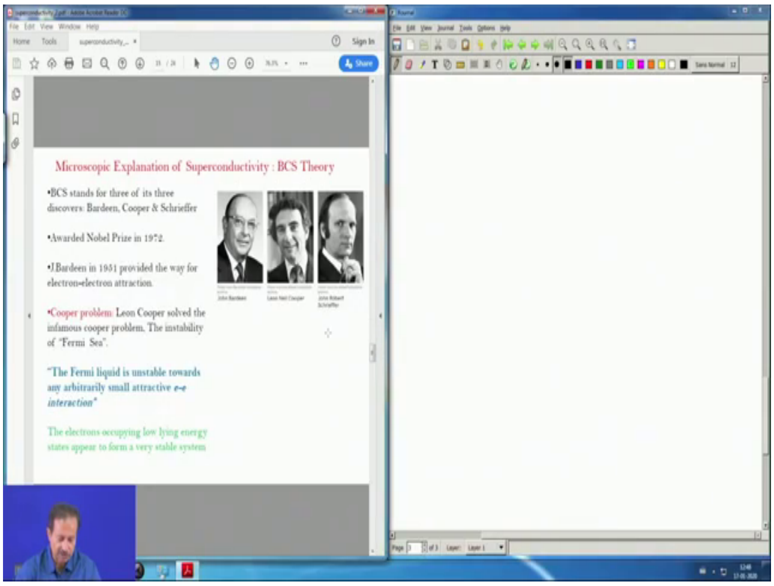

And that came from three physicists at university of Illinois at Urbana the professor was John Bardeen who is famous for co-discovering semiconductor with Brattain and so on. And so, he and his postdoc Leon Cooper and his graduate student; that means, PhD student J Robert Schrieffer these three people found out the correct theory for superconductivity.

So, before we go there it is not that in one go they when they reach that milestone there was a lot of understanding that was developing and one of the major understanding that had to come was the idea of interactions that produce the superconductor.

So, this was realized that superconductivity requires attractive interaction and it was also realized by then that superconductivity is caused by at least there is a major role played by lattice distortions which means phonons ok. So, all these things were already understood also the thermal conductivity measurements were out at that time and it was also realized that from these experiments and understandings that there has to be a gap in this spectrum.

And that really and that is not like semiconductor. So, this gap in spectrum was something that these people were looking for apart from the fact that they were also trying to understand if there is a binding. And in that, Leon Cooper was the first person who showed that indeed electrons two electrons if you put them above Fermi surface and if there is a; that means, an attractive interaction then they can form a bound state and that is a major discovery. And that is why these pairs are now called Cooper pairs because it was he who predicted their existence.

So, we will go to all that and finally, do the BCS theory, but this is how it planned out in early 50's. So, there is a bit of history they were awarded the Nobel prize in 1972. So, Bardeen also realized that there is electron-electron attraction which was also contributed by several other people Froehlich for example, electron-electron interaction through phonons for example, Cooper solved the famous says infamous, but it is a should be famous it is in famous in the sense that it showed that the Fermi sea Fermi surface is unstable.

So, that is called Cooper problem and Cooper was the first to show that in presence of an attractive interaction, Fermi surface becomes unstable in a degenerate metallic system. So, this is what it says an arbitrarily small attractive interact attractive interaction can cause the Fermi liquid state to become unstable. So, the Fermi surface does not exist anymore ok.

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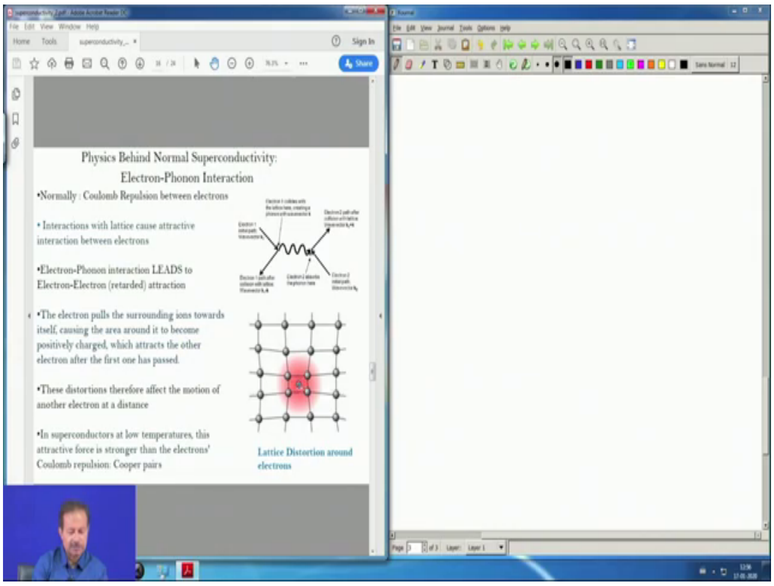

So, the physics behind the usual superconductors see there are there is now a distinction between the old superconductors and the new superconductors. We will not discuss the new superconductors or their theories at this stage maybe at the end we can make some comments about them, but our entire focus is on the old superconductors which means superconductors that existed before 1986 and also not this barium bismuth oxide. So, barium potassium bismuth oxide or so, those are not being discussed here.

So, this by normal superconductor one means the ones which are which were known for a long time and whose theory was given by Bardeen Cooper and Schrieffer that is why the BCS theory stands carries their name Bardeen Cooper and Schrieffer. So, where does it come from? So, this is how it is originated? See idea is that; the electrons are at repulsive really interacting with each other because they are both negatively charged. So, if you take a bunch of electrons and allow them to be freely interacting they will just repel from each other and move away.

So, that is something we know its coulomb interaction. So, you have to call find an attractive interaction and that was a non trivial work and that was eventually found out. So, electron phonon interaction leads to electron-electron attractive interaction there is a within parenthesis there is something called retarded. Now, retarded means that it is not instantaneous normally in most interactions that you see a coulomb interaction and all that the interaction is instantaneous ok.

The if the two particles are close by then it is instantaneous of course, at if they are further apart then there is this be I mean the way we present it in a Hamiltonian is like action at a distance because the time it takes for photon to propagate from one charge to the other to the other is extremely fast.

So, for all practical purposes these are like instantaneous interaction, but here the interaction is retarded in time. So, the interaction happens as I will show separated in time and the electron pulls the surrounding. So, the physical understanding that goes to explain this is that suppose an electron is somewhere here it pulls the surrounding ions towards itself causing the area around it to become positively charged because the ions are all positively charged because see electrons have come out of the ions and become free in a metal.

So, this distortion is something like a positively charged lattice distortion around this electron. Now, this electron effect this electron by distorting this lattice here effects the motion of other electrons. So, another electron which is passing by this will see this excess positive charge around here and get attracted towards it. So, that is what is causing the attractive interaction, but of course, if the electron was still sitting here, then the repulsive interaction will be very strong and the other electron will be repelled. So, what was what one realizing is that after distorting the lattice this electron has left.

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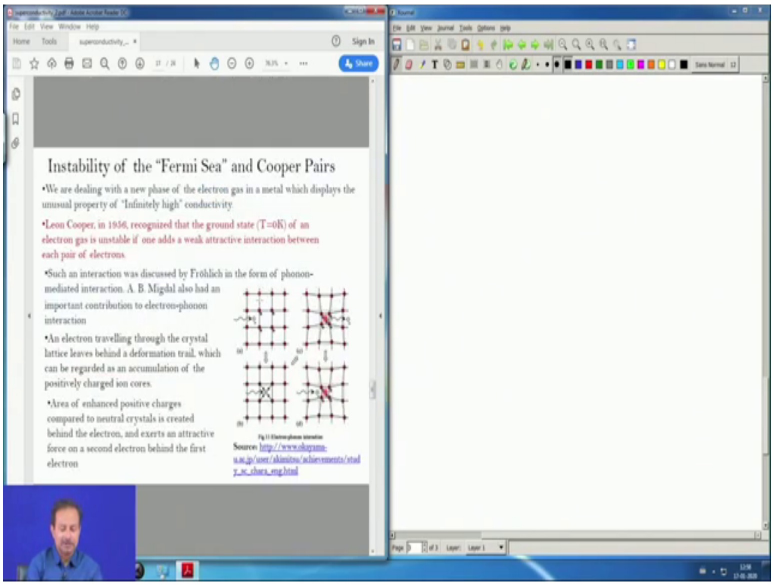

So, let us look at this picture this electron comes in distorts the lattice here. So, it attracts these other positively charged ions towards it distorts the lattice and then this electron moves away. And when then the next electron comes it sees the distorted lattice and it does not see the other electron here it sees the distorted lattice with a net slight positive charge around it and then it gets attracted towards that position.

So, it as if there is an attractive interaction between this electron and that electron, but that electron has now gone far away. So, it is the that in that kind of an interaction is called a retarded interaction, because the other electron is no longer there it is not like a contact interaction or a or an interaction which is almost instantaneous it is an interaction which happens after the first particle has left.

So, there is a time delay between the first particle passing that point and the second part, second electron coming in to experience that attraction ok. So, this was basically realized and lots of people contributed to its to the understanding of it Froehlich Migdal are some of the names that you might hear in this if you look at the literature of this. And of course so, that led to the foundation for generating an attractive interaction, but it is a strange kind it is retarded it is retarded in time.

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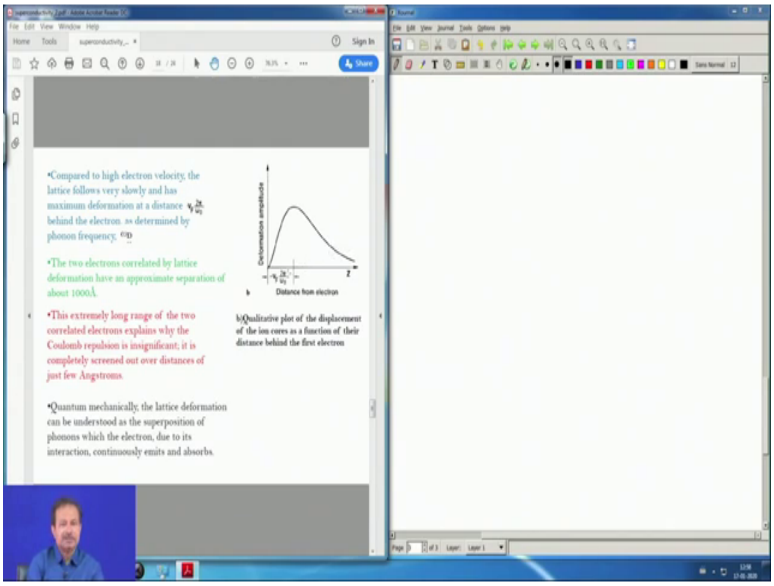

So, the reason it works is primarily, because the phonons or the lattice vibrations are much slower the lattice the ions move much much slower; at a much much slower pace than the electrons. So, the fast moving electron has gone away distorting the lattice. And now the lattice will take quite some time to relax back and that is what is shown here. So, this deformation for example, caused by the first electron peaks at a distance from the electron which is V F into 2 pi by omega D V F is the Fermi velocity of the electrons and omega D by is the phonon frequency.

See any distortion, particularly local distortion can be written down as a sum of combination of many many phonons and so these phonons have to relax these lattice distortion relaxes; that means, the phonons are generated. And then the first electron generates these phonons and the second electron when it comes it does not see the first electrons, but it sees these phonons these lattice vibrations and it absorbs these vibrations and then the net effect is a an attractive interaction which peaks around this kind of a distance.

So, the attraction is felt around this distance. And this distance being fairly large you can imagine that the screening which is operative in a metal which screens out the coulomb interaction the repulsive coulomb interaction has already screened out the interaction a repulsive interaction the bare coulomb, coulomb interaction between electrons the repulsive coulomb interaction has been screened out within a distance much less than this distance.

So; that means, the electrons do not feel the repulsive interactions because they are already gone far away. And the second electron basically feels the attractive interaction. So, the repulsion is screened away screened out and the attractive interaction is still operating at a much longer time and space scales. So, that is how the attraction is generated in a very interesting manner that is very unique in this case and it is the conspiracy of these speeds of electron the speed of the lattice relaxation the screening all of it work in such a works in such a way that somehow. And finally, there is a region of energies and frequencies or a timescales where you have a small, weak, net attractive interaction.

So, that is what generates this attractive interaction that is required to form pairs and eventually superconductor. So, quantum mechanically you can think of this as exchange of phonons and that is how it is done. So, an electron with k wave vector k one comes it goes away with emitting phonons which means it creates a local distortion, lots of phonons are emitted. And this so, it goes away releasing a phonon or momentum ksa and then another electron absorbs that phonon and goes away with the wave vector k plus k 2 plus k. And you have to then sum over all was all ks to get this local distortion and that is the way the mathematics works in quantum mechanics. And this is not what we are going to do, but this is this kind of processes led to this attractive interaction and that is found to be retarded in time only in a certain range of frequency it works.

And see since the range of frequency is small if you Fourier transform the range of time or simply time and energy uncertainty will tell you that if something is narrow in frequency that will be very large in time. So, that is exactly what happens that these interactions are retarded they operate over a large timescale and; that means, large distance also ok.

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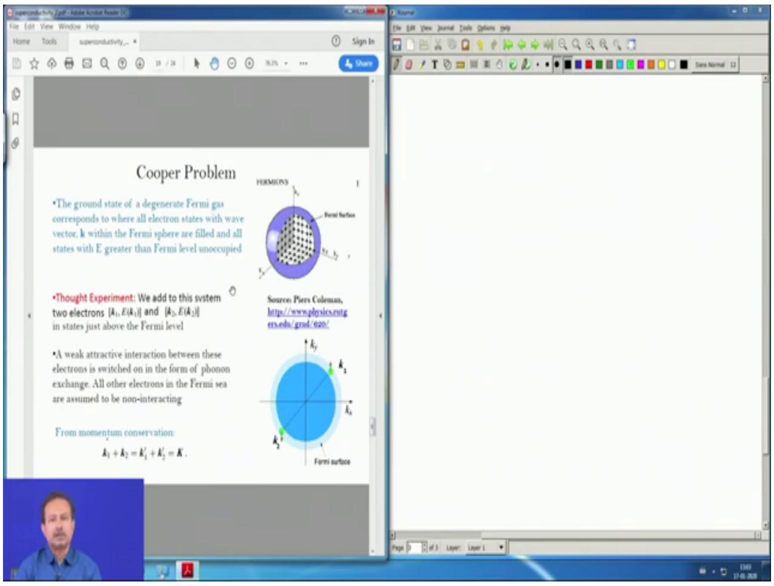

So, this is what now sets us up to do the famous Cooper problem and that is what I will motivate now, how cooper solved this problem. So, what cooper did is very simple in that in the sense that it is not simple the idea was fantastic, but the calculation was fairly simple, in the sense that. See this is a many body problem there are 10 to the power 23 electrons they are interacting with each there is an attractive interaction and so on.

Now, the question that Cooper set himself up to answer was it supposing there is this Fermi sea I mean of course, there is a Fermi sea in the metal and suppose I will take 2 electrons just above this Fermi sea which is where I can take it take them out because inside the Fermi sea all the states are filled up. So, we can take 2 electrons outside the Fermi sea above Fermi level and then allow them to interact with this attractive interaction and remember the states below the Fermi surface Fermi level are all occupied, so that is what is shown here.

So; that means, again the same idea comes in that the presence of a Fermi surface has a dramatic effect in the sense that it restricts the scattering phase space to a very minimal region. And that it from the enormously large phase space that you have in a free electron gas in this situation in a metal you do not have that. So, the there is a huge restriction in the phase space available for electrons to scatter into. So, that is what we will discuss and so, the problem that cooper took up was as let me repeat is just two electrons above the Fermi level interacting via a retarded attractive interaction and there is a Fermi surface below to restrict the phase space.

So, this is almost a cartoon problem in the sense that there are these enormous number of other electrons. So, this all of them are attractive are interacting where these attractive interactions and so on and so forth, but coopers problem is just this that is a Fermi surface plus 2 electrons above it attracting via a retarded interaction.

Now, let us just look at it k 1 and k 2 are the 2 electrons we have added to the system. So, there is a Fermi sea and 2 electrons are added just above it and these 2; these 2 electrons are interacting via this attractive interaction their momentum moment or k 1 and k 2 and energies are E k 1 and E k 2; E k 1 and k 2 are pretty close to the Fermi energies just of the Fermi level. And similarly, k 1 k 2 are also pretty close to be Fermi surface Fermi momentum ok.

So, that is what this picture shows. So, the k 1 plus k 2 is. So, after they scattering after they interact via this interaction they scatter in 2 states k 1 prime and k 2 prime and by momentum conservation the k 1 plus k 2 and k 1 prime plus k 2 prime is equal to k which is conserved.

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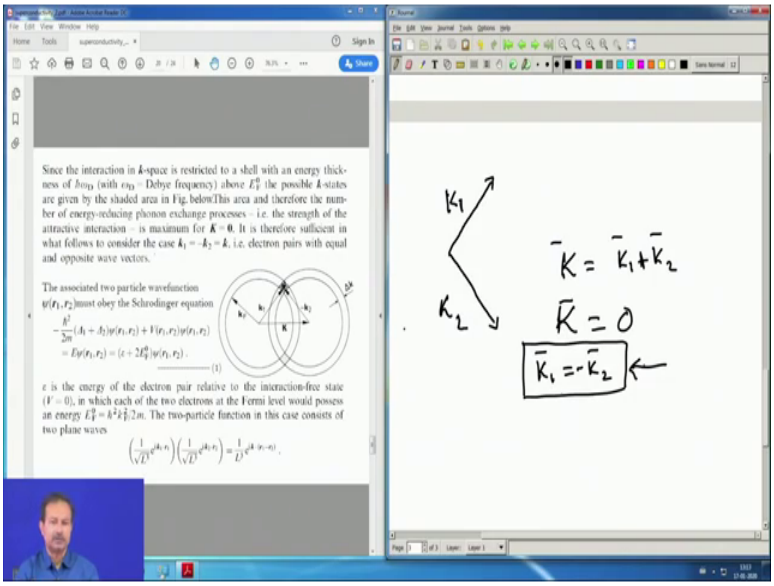

Now, let us look at the k space which is a restricted to a shell with an energy thickness of h cross omega Debye, because the attractive interaction works as we showed in the previous discussion, that it works in a very narrow range of space and the corresponding energy range over which it operates is basically the phonon frequencies right.

So, the phonon frequency is omega divide and so, the energy range over which this interaction works is extremely narrow. And that is this h cross omega Debye and so the problem now is that so, the k states are the k states that are that we are interested in this k 1 and k 2 see look at k 1 k 1 is this and k 2 is minus of k 2 is this. So, k 2 is around this direction. So, let me just draw k 1 and k 2, so k 1 is in this direction and k 2 is in this direction ok and k 2 this is k 1 ok.

So, that is what we are, we have and these two interact where there are attractive interaction which operates only in an energy range h cross omega Debye about the Fermi surface Fermi level ok. So, we can so, then let us look at this so, what has been done here in this picture is where as we drawn 2 different Fermi surface corresponding to the two different electrons for example, suppose they are centered here and here and these are the corresponding Fermi surfaces and two and their separation is this k.

So, k is how much? k is so, this is the k is k 1 minus k 2. So, k equal to k 1 minus k 2. So, the two centers are separated out now by a distance k and now you see the reason we are doing all these things is one wants to find out, what is the value of k for which the scattering is maximum. The phase space available for scattering is maximum, so that is what we are trying to find out. So, this so the region over which the interaction these two spheres interact; these two circles interact when in three dimensional of course, this is a sphere is this narrow shaded region here and shaded region here.

So, these two regions where these two rings in intersect one is here and another is here. So, these are the two regions, so where the electrons will interact and one wants to find out when is that area of that region maximum. And you can immediately see that area is maximum when these two spheres basically coincide with each other so; that means; that means, this entire annular region then becomes the region of phase space where the scattering can happen.

So, there were the number of energy reducing phonon exchange processes the scattering means that there are phonon exchanges that are happening which reduce the energy this area where this is happening is maximized when these two spheres, these two circles basically merge with each other; that means, K equal to 0. So, that is the upshot of this discussion that there is this region of phase space where if k 1 is equal to k 2 sorry this is k 1 plus k 2 because, this is the distance, so this is just the opposite of minus k 2 right.

So, this is k 1 and this is minus minus k 2. So, this is k 1 plus k 2 and so there then k 1 plus k 2 equal to 0 means k 1 equal to minus k 2. So, that is the region that is the restriction on which around which the scattering will be maximum. So, if you can set up your k case in such a way that k 1 and k 2 are just the opposite of each other that will give you the maximum scattering and that is what and that will of course, reduce the energy the maximum because that these are the processes which reduce energy when they exchange these phonons.

And so that so, we will stick to this that the two electrons have their momenta opposite of each other and the rest of the regions I mean for other momenta the amplitude of this process is much less compared to this one.

So, this is the one we concentrate, this is one Cooper also concentrated and for physical reasons as you can see that this is what gives you the maximum reduction in energy. So, that will be the region we should be concerned with. So, then let us follow the argument and the calculations of Cooper as he did.