**Electronic Theory of Solids**
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**Lecture – 49**
**Meisner Effect from London Equation**

So, we have been discussing the London Equation which is the one of the first major developments in the theory of superconductivity trying to explain the phenomena that happen in a superconductor, and one of the most important phenomenon that happens is Meisner effect ok.

So, London and London, this F and H London - two brothers, they understood that the superconductivity can be broken down into two parts which is one part of the conduction electrons are superconducting and the other part is normal. And the normal part and superconducting part they are in parallel, and therefore, the resistivity is shunted by the superconducting component, and therefore, you will still see zero resistivity although the other part remains dissipative.

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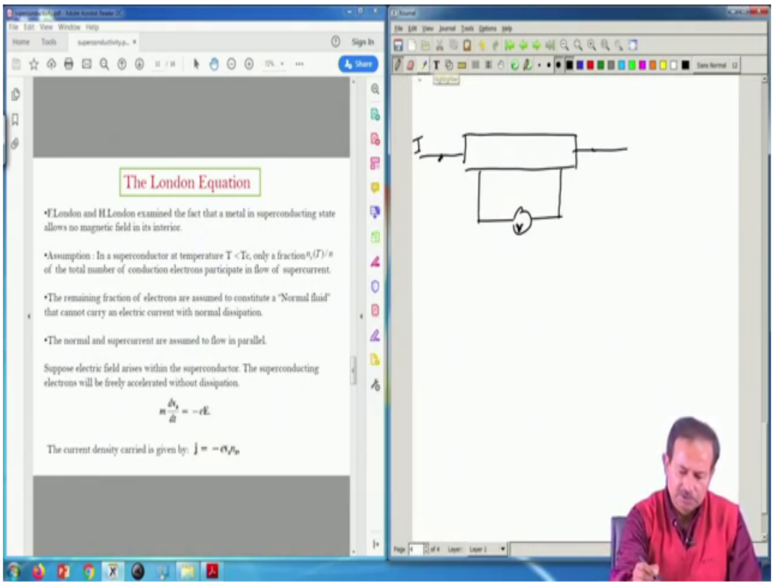


Typically, these measurements are done in four probe geometry. So, it is just as an aside that if you have a superconductor, you would suppose you do this kind of a measurement two probe measurement, and then of course, there are this the you measure the voltage here, and you send a current through this, and then you are measuring the voltage the voltage includes the contact voltages, contact resistance and all that, so that needs to be avoided. So, these contacts are they have to be avoided.

And ideally one does four probe measurement which is just this instead of measuring the current and voltage at the same lead, one measures current and voltage at two different place and leads ok. So, and you find out the voltage drop across these two leads, and then you know the current that is flowing, and so you know the resistance between the two leads, so that is the standard method of calculating of measuring low resistances and low very low resistances and so on, so that the contact resistance does not come into picture ok.

So, this is just an aside we are starting to do this again I let me just show you what to we did this is London equation, and this is not London equation, this is the equation from Drude a right for transport.

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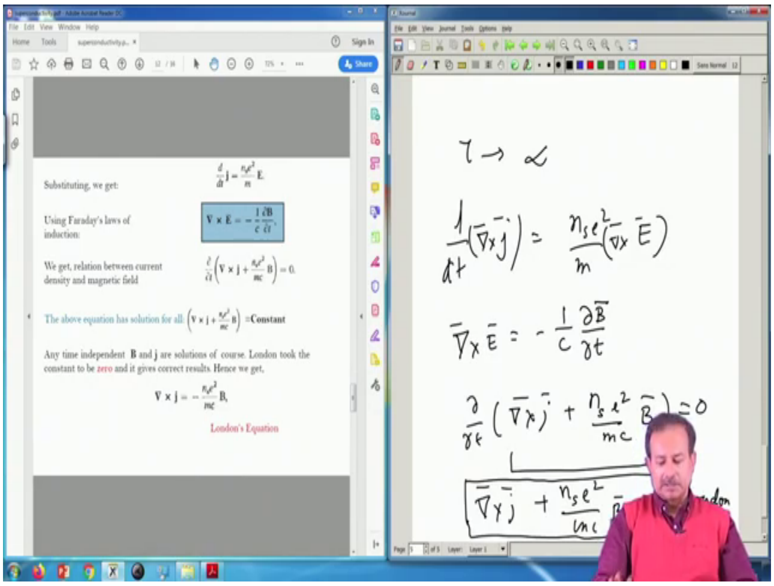


When we caveat that tau goes to infinity ok. So, this 1 by tau term which was there Drude equation can be safely dropped, and you can have a just like a Newton's this is a Newton's law basically. So, for the super currents this is the equation of motion. And then you write j equal to minus n e v, n s e v, v sub s also, this s means super superconducting part component.

And then of course, what we did is that we used we just took a multiplied by this by this equation by e v n e v, n e, and that is what we got. So, n e square by m on the right, so this is what we have just done.

Then one uses the Faraday's law which is this and combines it with this. So, what one does is that one takes a curl. So, this equation for example, dg d dt of j equal to n s e square by m into E, you take a curl on both sides. And then curl of E from Faraday's law is equal to 1 by c del B del t and put it back in, so that is so you will get time derivative on both sides. So, you can combine the two one on one side, and write del del t of curl of j plus n s e square by m c into B equal to 0.

And this means that this, whatever is inside the bracket is time independent, it is a constant in time. Now, out of all the possible constants that you can choose you can choose any combination of constants where large number of constants you can choose I mean in finite in principle. So, you choose the one which is just 0. So, this object is 0 is what London chose, and this is called the London equation. So, this is London equation.

So, it is actually two London. So, Londons' equation you could write as well, but anyway this is where historically called London's equation, London equation. So, that is about it that we did.

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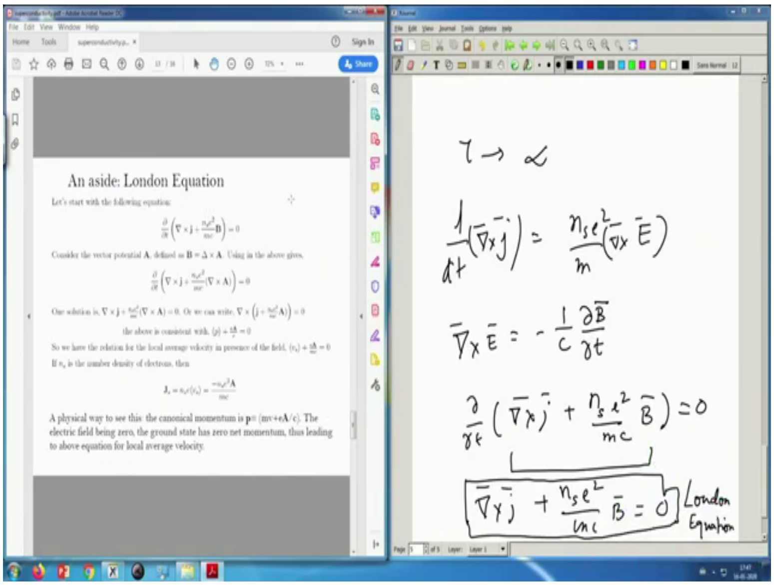

And then we sort of connected that London equation with the theorem by block which said that the overall momentum average momentum in a superconductor must in the ground state must be 0.

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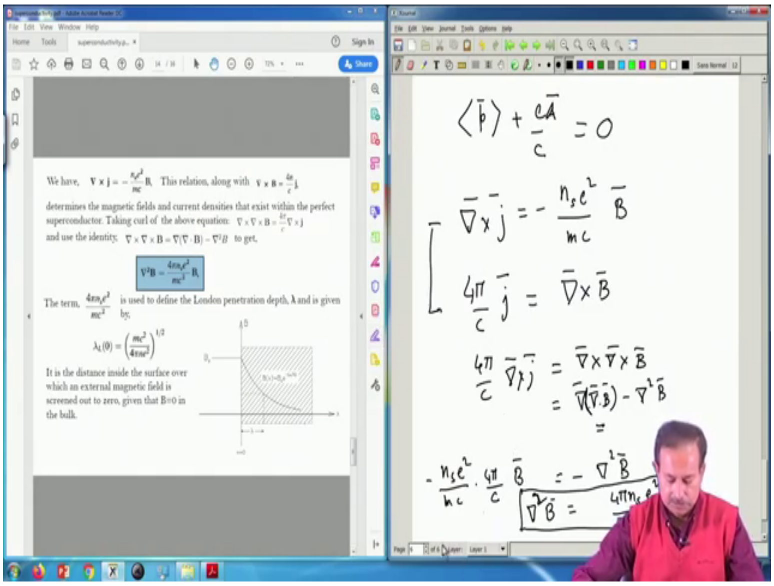

So, p plus e A by c equal to 0. So, this was Bloch's assertion that in the ground state this is what will happen and that is so the ground state will carry no net momentum and that is what this is and this is actually consistent with what London had written down, so that is what we showed the consequences of London equation of course, are dramatic. In the sense that it is that equation which showed that there indeed is a Meisner effect in a system.

So, how does one do that? Let me just show it. So, from London equation, we get a curl of j equal to minus n s e square by m c into B right ok. Now, of course, there is another equation that 4 pi by c into j which is a Maxwell equation is equal to curl of B. It is Ampere's law ok. So, these two can be combined easily. So, I take a curl here 4 pi by c curl of j equal to curl of B ok. So, we have come up to this far.

Now, the left right hand side is we know that this is gradient and of divergence B grad B minus del square B ok. So, this is a, this is of course something we know from our vector algebra. And from Maxwell equation, we also know that divergence B is 0. So, we can just write this as minus del square B.

Now, we know that curl of j is minus n s square n e n s e square by m c. So, you just multiply both side by 4 pi by c, and see what happens minus n s e square by m c into 4 pi by c right, into B is then this. So, that means, del square B equal to 4 pi n s e square by m c square into B, and that is this equation this one, so that is the equation that we have to now understand.

So, this is an equation that follows trivially from, but just by doing two more steps and using Maxwell equation starting from London equation ok. So, what does this say suppose you are in one dimension ok, so then this equation tells you that ok. So, let me just draw this geometry.

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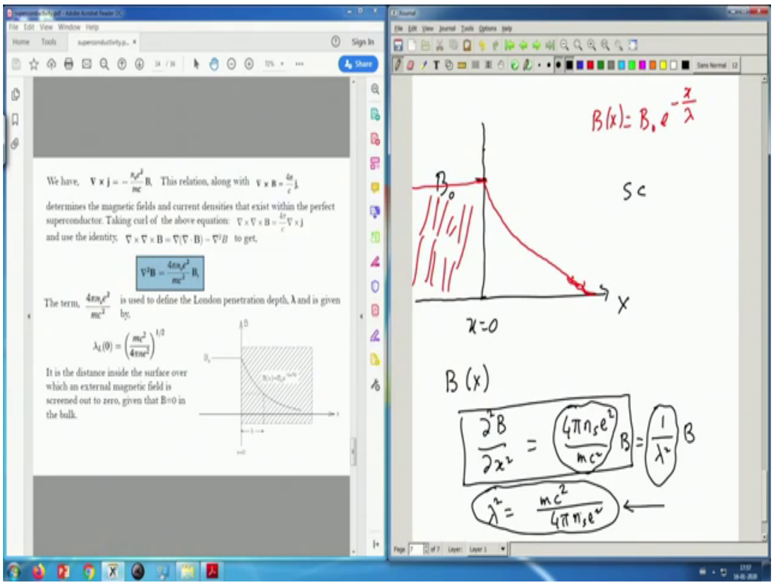


So, suppose you have a is this is called a semi-infinite superconductor on this side. So, the x equal to 0 onwards, x equal to 0 onwards right hand side is superconductor, and left hand side is vacuum – I mean there is not no superconductor here, it is vacuum side. And then you are putting a magnetic field here B 0, which is at x equal to 0, and left of it you have applied we have applied a magnetic field.

Then, what one would like to see is how B behaves with x, and beyond x equal to 0 of course it enters the superconductor, so it is supposed to enter the superconductor. And one can write down this equation along only the B in that direction, so that is del 2 B del x 2 equal to 4 pi n s e square by m c square into B which is conventionally defined as 1 by lambda square B.

Why is it defined as this? You can immediately see from two sides of this equation, this part of the equation that this quantity has the dimension of 1 over length square, so that means, I can define a length lambda which is such that 1 by lambda square is this quantity.

So, lambda square equal to m c square divided by 4 pi n s e square, so that is a length scale that I obtain from this calculation. So, this length scale appears naturally from this equation, a combinational London equation and Maxwell equation. So, this solution to this is trivial. We know what the solution is. The solution is B x suppose B starts from here B naught is here then let me just draw it in a different ink. So, B naught has a constant value on this side.

So, this is B naught. And then this side is non superconducting vacuum, and this side is superconducting. So, B naught has a value which is this, and it so let me wipe this out. So, this side is completely superconducting and that is my B naught value. And so B naught has a solution which has to decay. So, what is the solution B naught B of x equal to B naught e to the power minus x by lambda right, so that has to be the solution of this equation, and that is it starts from B naught and then decays.

So, within a length scale of lambda it becomes nearly one-third of the value. And this lambda is typically depends on the system of course within some hundreds and thousands or thousands of angstrom it decays down to almost 0. So, that means, if you go inside the bulk of the superconductor, then there is absolutely no field. You will not see any field no matter how you cool, what you do, it does not matter. This is the equation and this tells you that there is no choice that the magnetic field has to fall down to 0 within in a inside the superconductor ok. It is almost vanishes by a certain distance.

So, this basically explains the Meisner effect. So, this is in a way Meisner effect except that at this it also tells us that at the surface of course you will have some region typically of length lambda over which you will have a magnetic penetration only at the surface, but beyond that there is no magnetic field inside the superconductor.

So, this was the way magnetic field going to 0, perfect diamagnetism was explained by London's equation ok. And it also predicted this length scale that there is a length scale over which this will happen, and it came out naturally from, look at this only thing that depends on the system here is this ns. So, it just in this equation at least is the dependence is only through the conduction density of the super curve superconducting fraction, so that is what a.

So, and for example, at an extremely low temperature where the n s is almost equal to n, you can figure out what your lambda is, and compare with experiments to check the validity of it, and indeed this has been done and checked that this is what happens in a superconductor. The other interesting thing that I was mentioning was that the existence of a super current.

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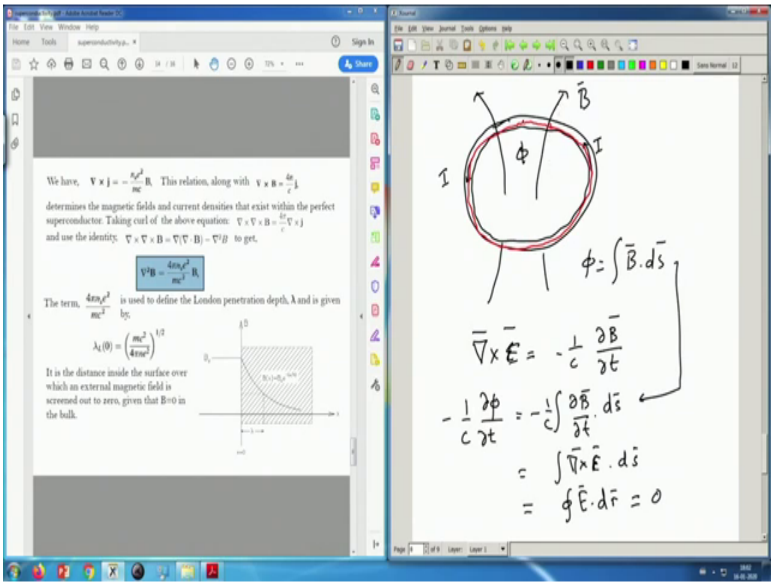

Now, that is actually very easily shown, if you for example, take a ring ok, and a thread a magnetic field through the ring. So, suppose I have a ring like this which is superconducting, and a thread a magnetic field perpendicular to the ring through the ring. And there is a there supposed to be a current I ok. So, how do I find that? So, this, for example, suppose this flux is phi, so phi equal to B dot d s.

And so this is to be integrated over this surface, and d s is a surface normal to B and so on and so forth, this you know already, it is a surface element normal to the plane of this ring for example, ok. Now, you also use this equation curl of E, is equal to minus 1 by c del B del t ok, which we have been using consistently. So, then curl of E dot, so let us just put it here. So, let us just start from this equation, and write del phi del t ok.

And so 1 by c del phi d minus 1 by c del phi del t is minus del v del t 1 by c into d s ok. Now, 1 by c del v del t is this. So, this will become curl E, curl E dot d s ok. Now, this using stokes theorem we can write as E dot dr, where r is a line element that a over a contour which is enclosing the surface S ok.

So, what we can do is to choose this line element the line over which we are integrating, just choose it inside here, you take the line inside the superconductor. So, that is the contour over which you will do the integration. And we know that inside the superconductor E is 0.

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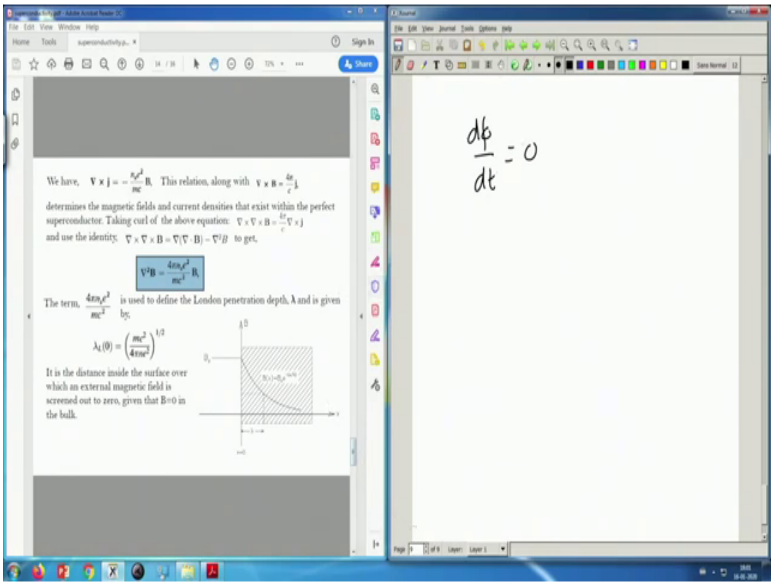


So, this will lead to 0, that means, d phi dt if change of flux is 0. So, that means what, that means that suppose now I switch off the magnetic field, so magnetic field is externally applied, I can switch it off from outside, so we do that. So, this magnetic field is switched off now. But then this equation tells us that you the flux remains constant. So, flux has to be whatever it was before the B before B was switched off.

So, this area times B is the flux and that has to remain constant, so that means, that the this there has to be a current that keeps on flowing to generate this flux to remain to keep the flux at what it is when I put the magnetic field right, so this is kind of interesting.

This means that supposing you have a superconducting ring, and you are you have put a magnetic field, and so this is the geometry in a ring you put a magnetic field you have a flux.

So, flux is created by the B field that is applied from outside. Now, from simple Maxwell equation and the fact that the electric field inside the superconductor is 0, I can show easily, and what I did was to show that the flux has to remain constant.

It is also the other way around. So, suppose I started with a flux which is 0, and I wanted to put a magnetic field, then the flux will keep remaining 0, because there is this. So, there will be a current generated that will try to keep the flux as it is. So, what how does one do this experiment, you can just take the ring – superconducting ring, take it above its magnetic you take it above its transition temperature put the field, so that there is a field inside. So, there is a flux passing through the through this ring ok.

So, once that is setup, then start cooling, and you cooled it below superconducting transition temperature, so the ring becomes a superconductor. And now what you do is that you simply try to withdraw the field. And you can of course, you can switch off the source of the field from outside. And what it will do the superconductor is to have a super current which will keep on flowing through this, so that the field remains constant whatever it was the same value, and that is called a persistent current or super current, and that since the field flux has to be constant the current has to also remain constant.

So, it is a dissipation less current and that flows in the superconductor. And it just keeps on flowing, because no change in flux is allowed and that is exactly what happens. And that is what I mentioned in the earlier in the lecture that the one of the ways to see that there is you are in a superconducting state is also to see that there is a super current. And you one can actually so the how do you know there is a super current you can actually measure the flux and see that the flux is not decaying.

So, therefore, one can actually do it, do this experiment. And one finds that this flux is not only constant, and it is a constant over an enormously long time. People have done this experiments for days, for months, for years and kept it on and tried to find any decay in the super current and so far, and the estimates are that the super current will not decay, and decay time is probably is indeed more than the life of this the age of the universe itself so far which is about 13.7 billion years, and it is certainly more than that.

If you if the material has little very little defects and say pure good material a superconductor is almost absolutely no decay of the current in a very pure superconductor without any perturbation of course. As I said it has to be pure materials for conducting material where you set up this super current and measure the super current or the flux year after year, you will find absolutely no decay in the super conducting current, super current in it, so that is another way of seeing that something is a superconductor, but of course, much easier is to see that it is a diamagnet, but these are all evidences that you are in a very very different new state and that state is something that one has to understand.

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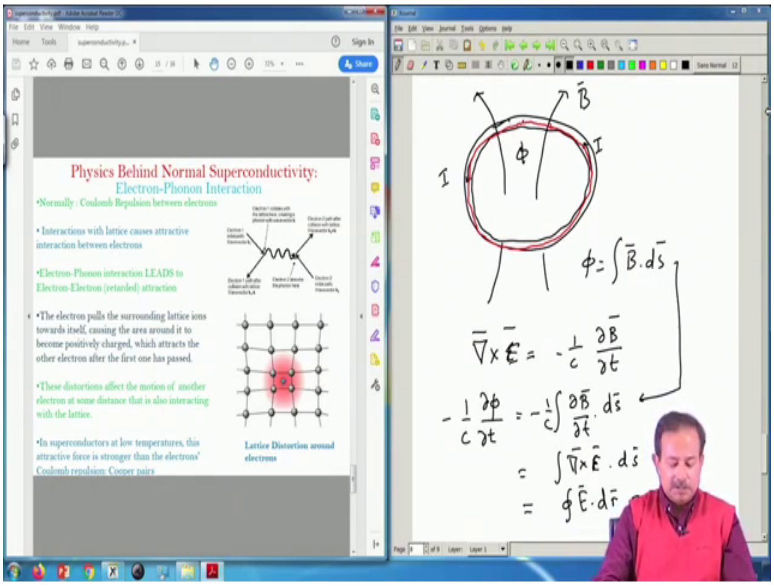


And so that understanding came from a remarkable theory. And this theory was proposed first by Bardeen, Cooper and Schrieffer, and they later on got Nobel Prize for it. But it has a long history almost every big physicist every famous physicist that you know of starting even from Einstein, Neil's Bohr, Heisenberg, many of them tried their hand on a theory of superconductivity.

And although it was realized that superconducting mechanism involves the lattice vibrations because remembered that isotope effect T c goes as 1 over root m that told us already that the lattice vibrations are the frequency of the lattice the vibration goes as 1 over root m, and that is involved in that has to be involved in the mechanism that gives rise to superconductivity.

The main difficulties of course that electrons are both all the electrons are negatively charged. So, they will repel each other right. And it was realized that at least people thought about it that the superconductivity is a state where you have a condensate is like a Bose condensate, and so therefore, you need to pair up electrons and you need an attractive interaction for that to happen. And now of course, we know that indeed there are these so called cooper pairs two electrons form a pair a bound state which we call cooper pair which is responsible for superconductivity in a otherwise repulsive Fermi gas right, Fermi system.

So, that is a theory that is where the theory of superconductivity starts from, and that is something that we will carry on doing from now on. So, the phenomenology is now established that you have a Meisner effect, you have 0 resistivity, and both of them are required for you to have to have to define something as superconductor.

And there is this London equation which is a phenomenological equations, but that shows how magnetic field decays inside a superconductor leading to Meisner effect. And then of course, there are these experimental evidences that tell us that there is a and of course, there is this persistent current which I just showed.

And then there are remember all these are phenomenological theories. So, far we have not done any microscopic theory not started from underlying basic interactions in the superconductor to produce a superconducting state and that is the task that as I said many people tried. And finally, Bardeen, Cooper and Schrieffer achieved in 1950s.

Now, the that theory requires to understand how the electrons attract each other and that attraction has to lead to form a bound state of two electrons, and that will lead to a sort of condensate state which is where the resistivity vanishes. All the phenomenology that I described has to come out of such a theory and that is exactly what BCS theory does and that is what we will start in our next class.