Electronic Theory of Solids
Prof. Arghya Taraphder
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture – 48
London Equation

Hello. Welcome back, we have started discussing superconductivity and we gave a brief outline of what superconductors are which are the materials that are superconductor of course, not the entire list that is not even possible because every other day there is a new superconductor. Now, however; we described what superconductivity is and we made a distinction between the two concepts that it is not a perfect conductor.

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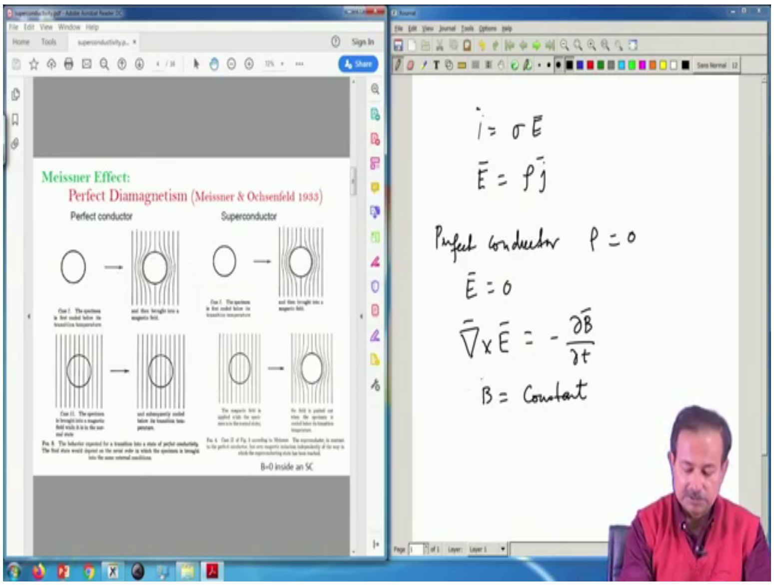

And you should not define superconductivity as just resistivity going vanishingly small. You should also include and that is more important Meissner effect which is the perfect diamagnetism to define a superconductor. Now, the reason for that is actually clear is that a Meissner state should Meissner effect is part of the equilibrium property of the system whereas, a transport is always a non equilibrium property. So, superconductor defined as an equilibrium state a new state in equilibrium requires this Meissner effect perfect diamagnetism to define it.

Now I also asked you to check whether the scenario that is depicted on the left that a perfect conductor and a superconductor are different and because this is as you can see in perfect conductor it is history dependent. How you cool whether cool 0 field or with field it will be different. Whereas, the outcome in a superconductor; no matter how you do it, whether a field cool or 0 field cool it is the same by I do not know if you can tried doing it, but let us see I will just give you an outline of how to do it and its as I said its extremely simple.

See for example, j equal to sigma E is a standard equation that we use for superconductor for any transport, this is the current density proportional to electric field. So, I can invert this to write equal to rho j ok. Now, as I said a perfect conductor is a rho equal to 0. Now in superconductor of course, we know that the current is finite, we can have a super current and then; that means, that E has to be equal to 0 inside a superconductor.

So, electric field cannot be sustained inside a superconductor. Now if that is the case, then of course, we also know that curl of E from actual equation is equal to minus del B del t. So, in that case you have, so C equal to 1 and all that I have chosen. So, in that case if E is 0, then, B has to be constant. B cannot be time dependent right.

So, whatever B you start with has to remain in a super in a perfect conductor and let us see why that is what explains this diagram for example. See in this diagram you had B equal to 0 inside when it was in the normal state then you cooled below its superconducting transition temperature and bring it inside by superconductor I mean perfect conductor. Suppose, there is a perfect conductivity transition temperature. So, that below that one brings it inside a magnetic field and it will keep its magnetic field inside whatever it was so, that is 0.

So, that remains constant and since it was 0 it is 0. So, that is what this is, now in here it is just the same. See I have we had filled when we had the system in a normal state. This perfect conductor was in a normal state at t greater than Tc and it then it is in the same situation the T is reduced and 1 is coming below Tc it is brought down below Tc and became a perfect conductor and it cannot change its magnetic field.

So, the magnetically whatever it is it remains trapped. So, that is what a the reason for this scenario in a perfect conductor. Whereas, in superconductor; experimentally one found out in 1933, that this is the scenario. No matter how you cool, you have to be expelling the magnetic field from inside the superconductor. So, that is the way one justifies there is not just a perfect conductor it is more than that, it is something different, it is actually a new state.

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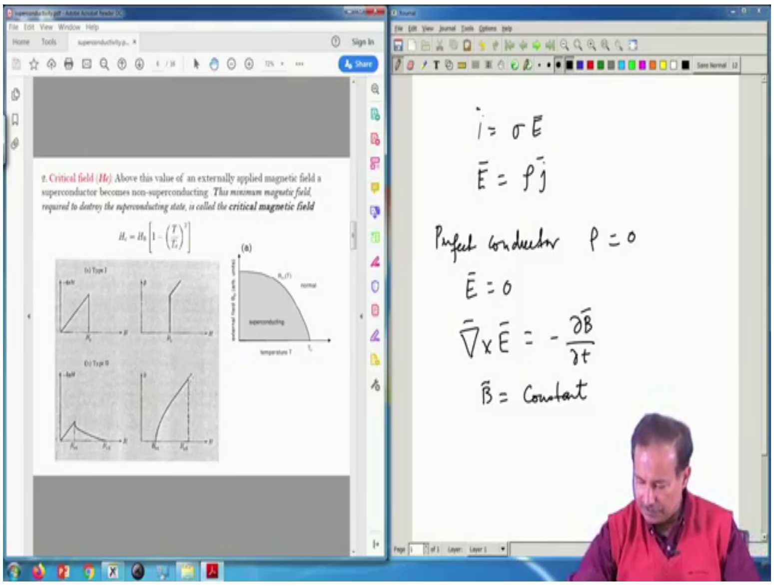

Then we discussed this upper critical field for example, the critical field and then of course, the lower and upper critical field in type two superconductor, we just mentioned that and. So, the critical field goes like this it vanishes of course, at T c because after that it is a normal state and at 0 temperature; it is fairly high this value is the critical field value critical magnetic field. So, this typically goes like H 0 into 1 minus T by T c square and that is the behavior shown here.

This for example, is type two, type two has a behavior in which there is an Hc 1 and then the field starts decreasing and there is an Hc 2 after that the field is completely expelled and that is interesting because, in this state beyond Hc 1, the system is in a mixed state where magnetic fields do penetrate the superconducting material, but in a sort of vortex lattice which will like tubes magnetic field passes through in tubes those are called vertices then those vertices form a lattice. So, that is an extremely interesting physics and we will discuss it towards the end.

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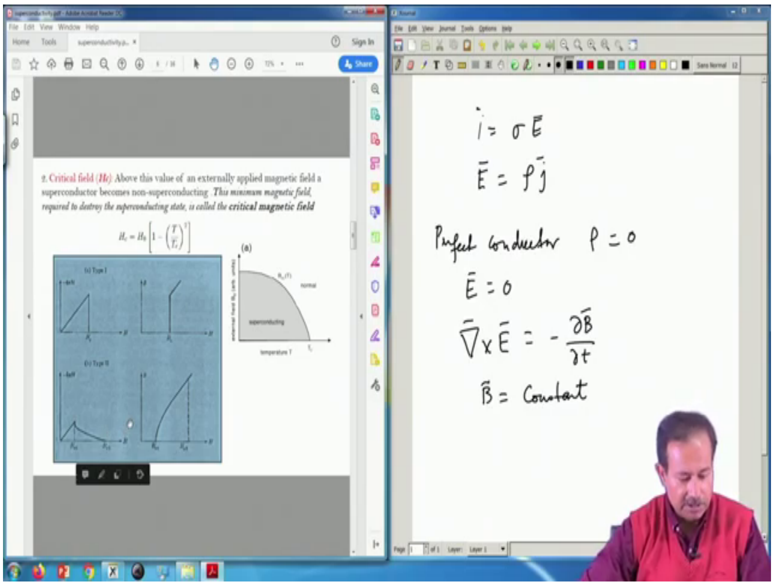

Then we just as a phenomenology, we discussed there are these two types of superconductors and their properties. So, as we said that this type two superconductor is the one which is mostly in use. The materials that are used as superconductor are type two superconductors ok.

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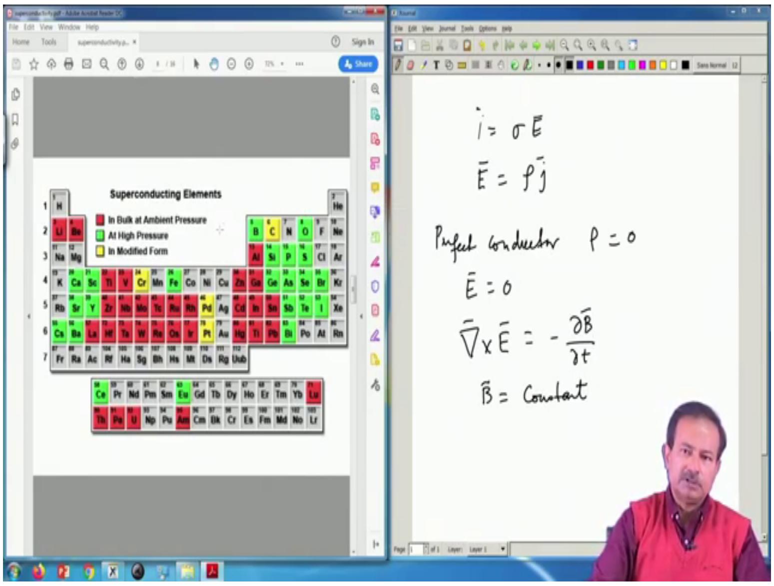

So, superconducting elements for example, in the periodic table, they abound I mean; there are lots of elements which are superconductor the red ones are in bulk at ambient pressure, the green ones for example, at high pressure and this is somewhat older. There are many there are probably there are I mean these things changes a very often.

So, this just gives you a list of things that was a few years back and for example, calcium Sc scandium, strontium, it all these are under high pressure superconductor at high pressure. The other thing is this yellow one; these yellow ones are interesting in the sense that these are in modified forms.

So, carbon for example, carbon is given in yellow which means that there is a form of carbon which is superconducting and we know what this is called the buckminsterfullerene if you dope it C 60, basically C 60, C 60 is when you dope it with potassium for example, alkali metals they become it becomes superconductor.

And C 60 on the other hand pure carbon graphite for example, is not superconductor. So, diamond is not superconductor, at least not normally. So, I mean. So, diamond is a different story; we will not discuss it here, but carbon normally as for example, as a suit is not superconductor, just typical carbon is not superconductor. Whereas, that is an insulator and whereas, doped C 60 molecule for example, easy superconductor.

So, but diamond also under doping become superconductor, but that is still controversial what is super conductivity, superconducting or not, so I will not get into that ah. There the cerium and europium are also normal superconductors and they are also superconducting when they form this heavy fermion when they are part of a heavy fermion compound.

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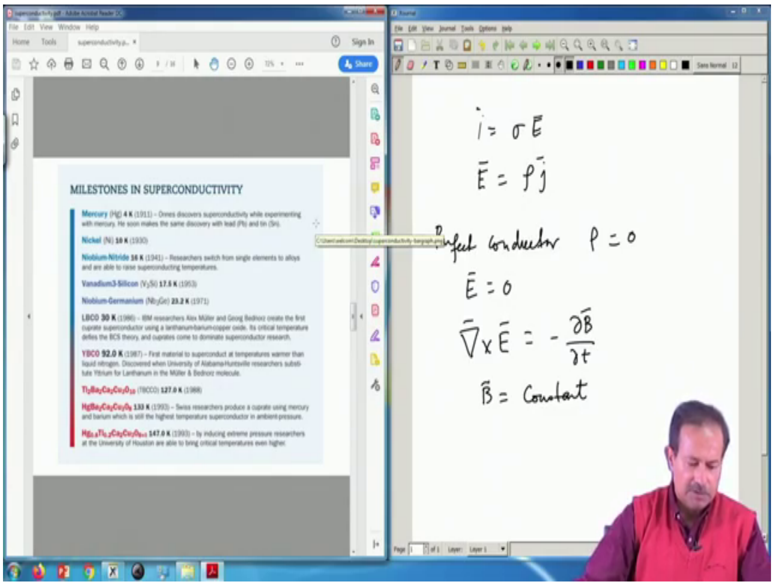

This is a part of history and it should be; it should be told at least once and this is the discovery nearly at 4 point just above 4 point 4 degree Kelvin in 1911, then in nickel, 10 Kelvin, niobium nitride is fairly high Tc for example, 1941, so these are alloys.

So, this is the first this is the alloy which has a high fairly high Tc compared to other systems elemental systems like say mercury nickel alloyed and so on. Then of course, there are the series this is another landmark LBCO which is Lanthanum Barium Copper Oxide, in 1986 Muller and Bednorz, they found out that the ceramic compound is a superconductor under doping. So, then so, LA 2 C O 4 is a antiferromagnetic insulator; whereas, if you dope with barium or lead they become superconductivity, the superconductors and these superconducting transition temperature is fairly high 30 degree Kelvin.

So, and that is called a high temperature super conductor. That was the first high temperature superconductor that was discovered and they got their Nobel Prize within a year of their discovery. And then of course, within a very short time came this there is a superconductor called YBCO, the same year even just of a year later that first broke the barrier of liquid nitrogen temperature. You see the liquid nitrogen is fairly easily available, it is not very costly and for industrial applications; if something requires liquid nitrogen it is not difficult to obtain, it is also not very costly.

So, once this material came, the barrier of liquid nitrogen temperature was broken; this is 92 degree Kelvin, this yttrium barium copper oxide and it is a discovered by Paul Chu and his group. And then came this the series of high temperature superconductors came as I said, the mercury one mercury doped compounds are mercury compounds are fairly high temperature have fairly high Tc and they are fairly high temperature superconductor and these are, these were the high temperature superconductor still some years back.

But then (Refer Time: 13:05) as I said a group from Germany Drozdov and company, they found superconductivity at high pressure in Hydrides and that is now come up to about 250 degree Kelvin or near about.

But that is an extremely high pressure; 140, 150, 170, 180 GPa giga Pascal. YBCO for example, is an ambient pressure, this 92 degree Kelvin superconductor. So, thallium dope superconductors were at ambient pressure. So, for application purposes, these are more useful in the sense that if you can bring it down to using these superconductors, then the cost is much less all you require is liquid nitrogen.

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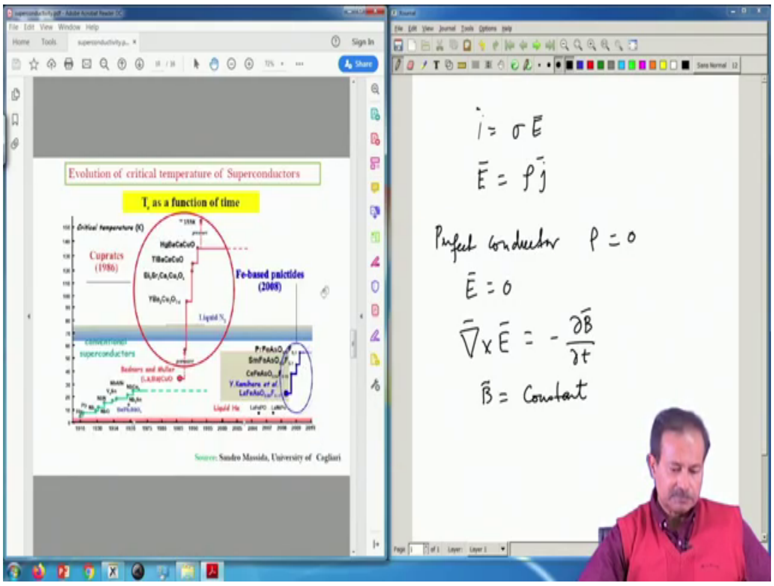

So, this is a picture you will see in many places and this is basically how the so called Cuprates which is Bednorz and Muller's discovery. They change the way, we look at superconductivity and it also dramatically changed our understanding of the theory of superconductivity. So, what it entails is that look at the way this curve progresses between 2000, between 1910 and 1986 superconducting transition temperature in this region increased only by only to N B 3G for example, is about 23 degree Kelvin and that was the highest Tc superconductor we had till about 1986.

And in 1986, this dramatic thing happened that Bednorz and Muller discovered lanthanum barium copper oxide superconducting at 30 degrees. Before this of course, there was a material which is BKB barium potassium. So, that let me not discuss that that is the compound which is which was high temperature superconductor, but see this one is actually mentioned here, this is BKB and BPB, barium lead bismuth oxide and barium potassium bismuth oxide these were also fairly high temperature superconductors, but not in the class of this LA BA CUO 4.

So, the reason became B A B B A PB B I O three is useful and important is that it also shows the undoped compound source charge density way. So, that is a different matter say, that is why let us not discuss it, but this high Tc class which is the basically started with cuprates; they became famous and their super conductivity has changed our understanding of superconductivity dramatically and also they have changed the way experiments are done much much more precise experiments are done these days since the discovery of high Tc superconductivity.

The other class of superconductors that came about in to around 2008 was this these are called Fe based pnictides pnictides are for example, this is a pnictide. In the periodic table; this column nitrogen, phosphorus, arsenic, antimony, bismuth, these when you form compounds of that kind they are called pnictides.

So, for example, here is a iron based ASO. So, these are oxides and these are basically pnictides they are called pnictides, they became superconductor and these are interesting because these systems were magnetic also. And these are fairly complicated superconductors and we will not discuss it. But these are also fairly high temperature, but not as high as the cuprates in this class ok.

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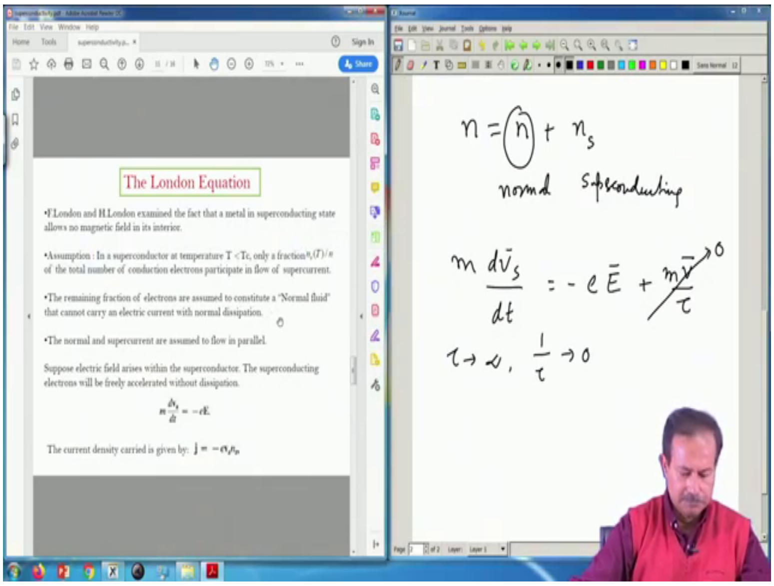

Then of course, one, I once with this phenomenology is that place. Let us discuss the earliest understanding of superconductivity as we go along and one of the earliest equations that where that was put forward by London and London two brothers that tried to explain superconductivity in a phenomenological way was this London equation. So, that is what I will come too.

So, let us just discuss what it. So, their motivation was of course, to explain the Meissner effect and the started by and then they of course, knew that it is perfect conductor is not what the story is this is a very different beast. And because perfect conductor does not expel magnetic field if you feel cool. So, this only a factory so, their assumption was that only a fraction of the electrons total number of conduction electrons become superconductor.

So, that so, they have divided the total conduction electron density n two parts. So, n normal plus n superconducting. And So the superconducting fraction is fairly less, but the advantage is that if you if both of them are carrying current and superconductor superconducting fraction has no resistance then of course, any measurement of resistance will still give you 0, because that part will short the current.

So, under this assumption they say that the remaining part is n, this remains normal and dissipative, but this is superconducting. So, this is called superconducting fraction. So, they flow in parallel and so therefore, is like two channels and then since one channel has no resistivity it just shorts it. So, you will find 0 resistance till. Now you can calculate this starting from a simple Drude equation, right.

So, that Drude equation that we wrote down is now you write it for the superconducting fraction and this is what you do. Minus e times the electric field that is applied. Now of course, you see that there was a term which was plus m v by t, in the Drude formula, but what you have done here is that since this is super conductor.

There is no current that there is a no resistance there; that means, that 1 by tau goes to 0 right. So, that means; tau goes to infinity. So, there is no resistivity at all. So, that is what one has used here and said this term to 0. So, your equation is only this one.

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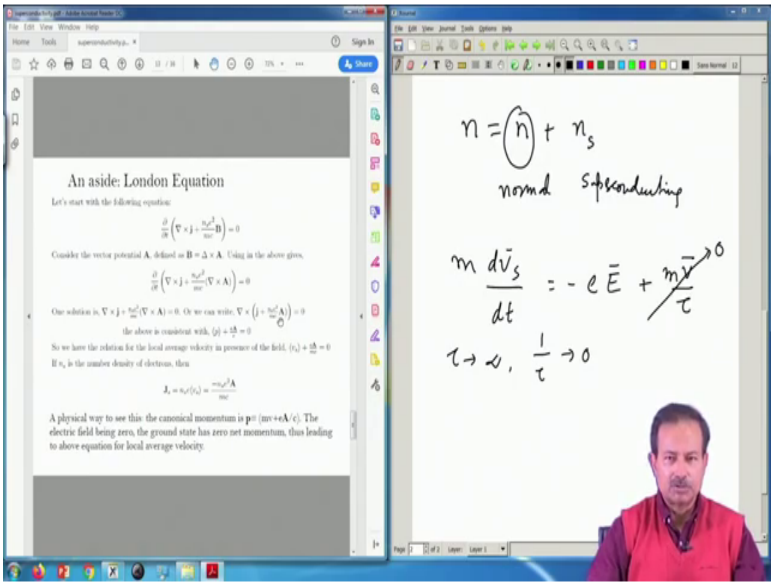

Now, that there is a super current in a super conductor is actually obvious from the theory of, from the basic idea that there the that, if you ok. So, let me just show you how you understand that there is a super current, but let us first go over and finish the calculation of London derivation of not derivation, but a argument that leads to London equation ok.

So, this is where one starts from and then of course, you can multiply v both sides by to make it minus e n s v s this is your j, n e v basically, and so multiply both sides by n e minus n e minus comes from the electric charge. Remember that at that time this e, we do not know what this value. At that time it was thought to be just e electronic charge, but let us just keep it what it is and. So, this is the charge of the carriers the kernel, so that is what is being done here.

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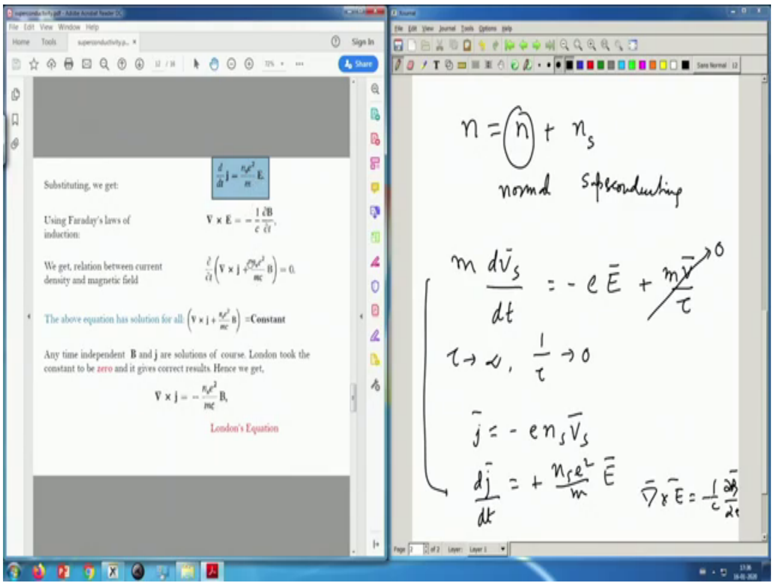

Now, you put this in and e is the charge of the conduction of the conduct those which are responsible for conduction and then you can just now take this equation then becomes d j d t equal to minus plus n s e square by m into E. Then the procedure is fairly straightforward use Faradays law which is this and so, curl E equal to 1 by c minus 1 by c del v del t which I wrote a few minutes back and that is the equation and you can go ahead and put take a curl on both sides of this equation the above equation.

So, if you take a curl, then you will get del del t of curl j plus curl of E ns e square in by m into curl E which we will give you this ns this equation right.

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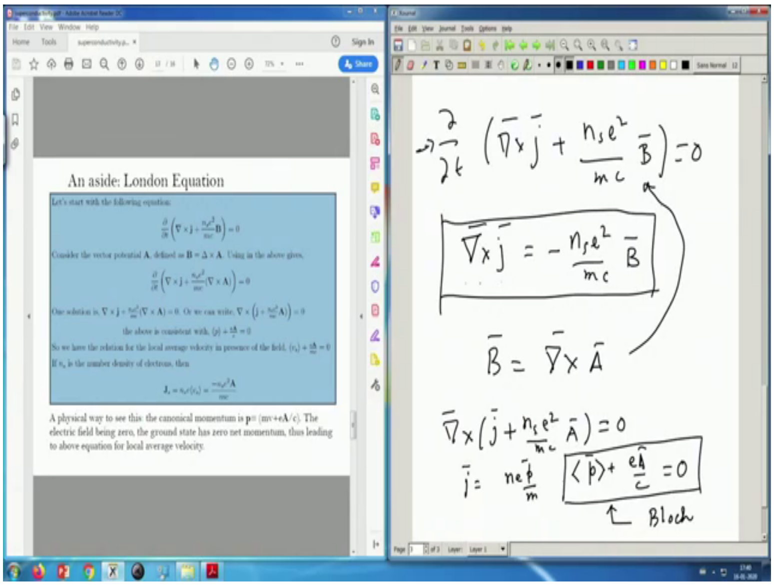

So, that leads to the fact that del del t of curl j plus n s e square by mc into B equal to 0 ok. So, the above equation clearly gives you a solution which tells you that this whatever is inside the this bracket is timely independent ok. So, that is all this equation tells you.

And so this is a constant independent of time. So, that means; any time independent B and j will give you a solution of this equation. What London did was that, he took this to be equal to 1 is 0 minus n s e square by mc into B. So, this is what London did and this is what London's equation is.

So, in effect out of all those solutions; you choose the one which is just this. So, as an aside we can just discuss this thing that you know this equation, the one that I have written here del del t of curl j plus ns e square B by m c equal to 0, you can actually put B equal to curl of A, which is a vector potential, A is the vector potential and then of course, curl j plus ns this thing into curl B that will give in to curl A here put it in here. So, you will get a curl both the both of them are curls.

So, this is a solution or for example, curl of this quantity is 0 right. So, that means; curl of j plus ns e square by mc into A equal to 0. So, this thing for example, again j is n e v and so, you can cancel n e and then therefore, this gives you. So, this is j equal to n e p by m right. So, you put that in, then what you will get is p here for example, plus eA by c equal to 0.

Now, this p is momentum and this p is the momentum corresponding to the super electrons and ideally, this is replaced by the average momentum and then this equation tells you that the total average momentum is 0.

So, this is the canonical momentum and that is that being 0 was actually men says that the ground state has 0 net momentum there is no its a and being no electric field inside this is what was originally predicted by Felix Bloch that this is how the ground state should be that it should not have it should not carry any net momentum. And so, that Bloch's idea is basically consistent with London's equation.

So, that is all this aside tells us that this Bloch never published it. So, but Bloch actually mentioned that the average momentum canonical momentum inside a superconductor should be 0 and in the ground state and that is actually consistent with the equation that we got from London's equation anyway.

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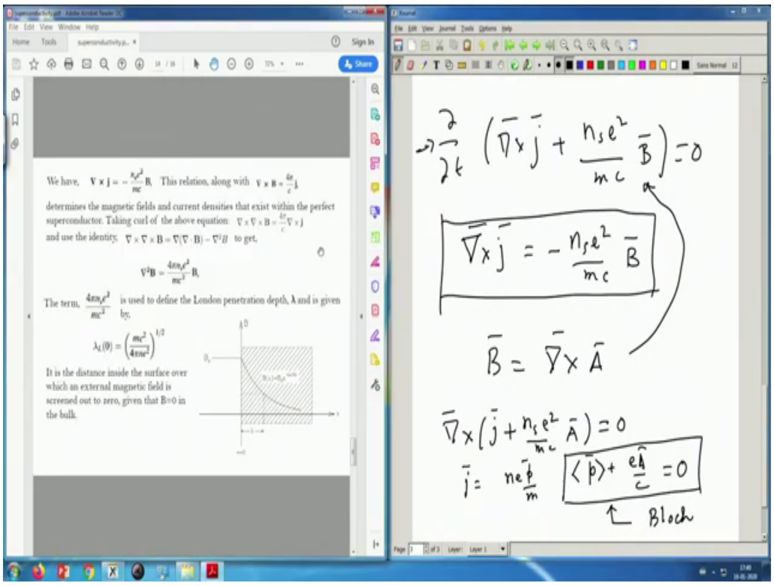

Now, we can go further and what I will do is the, I will show you how magnetic field decays inside a superconductor explaining the Meissner effect starting from London's equation.