Electronic Theory of Solids
Prof. Arghya Taraphder
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture - 47
Superconductivity

Hello and welcome. We are more or less done with Magnetism. The essential topics, we covered in magnetism are basically the atomic responses to magnetic field, which is done in the first part. And then, we have worked on the long range magnetic order of permanent magnets like the insulating magnetic materials which are you know ferromagnet para ferromagnet, antiferromagnet and so on. And then what we did was to write down the model from which these long range orders can be derived and can be understood microscopically and thereafter, we showed in certain cases, that these models are exactly solvable in very fortunate circumstances.

Then, after that we calculated the excitation spectrum of a one particular case, which is in a Heisenberg ferromagnet and showed that there are at any finite temperature, there are large numbers of these excitations, which reduce the magnetization from its saturation value. And then, this was in 3 dimensions. But then, we also mentioned that there is a very strong theorem which is extremely important called Mermin-Wigner theorem which tells us that with models which have continuous symmetry. Like for example, x y model or x y model is a model, where only x and y component of spins are kept in ah. It is one more step from the Ising model and Heisenberg model is where all three components are kept.

Now, in x y and Heisenberg models for example, where the spin can have values. It is the order of parameter which defines the ordered state has continuous symmetry. In such cases these magnetic excitations, these excitations proliferate in dimensions equal to or less than 2 and the proliferate in such a way that at any finite temperature, they kill all the order. So, that is one thing you should remember that the examples, we choose are very specific 1D Ising model and Heisenberg model in 3D, where we had ordered states at finite at 1D Ising model does not have; but 2D Ising model has order in at finite temperature. And 3D Heisenberg model of course, also has ordered state at finite temperature.

So, that is the message that we got from study of magnetism. And it is a historically one of the main most important branches of condense matter of physics and it teaches a lot about how quantum mechanics placed out at the microscopic level at the level of atoms and where it is suppose to play. And it leads to a states which are long range order; although the interaction may be short range, the order that it produces can become a long range and give rise to microscopic quantum states.

So, in that line of studies, we will study one of the most exotic microscopic quantum state, which is superconductivity. That is what our next topic is and it was discovered in 1911 in Leiden by a Dutch physicist called Heike Kamerlingh Onnes and what he did was that he was a basically trying to generate low temperatures in his lab and he was at that time, he could go down to the lowest temperature in the world and.

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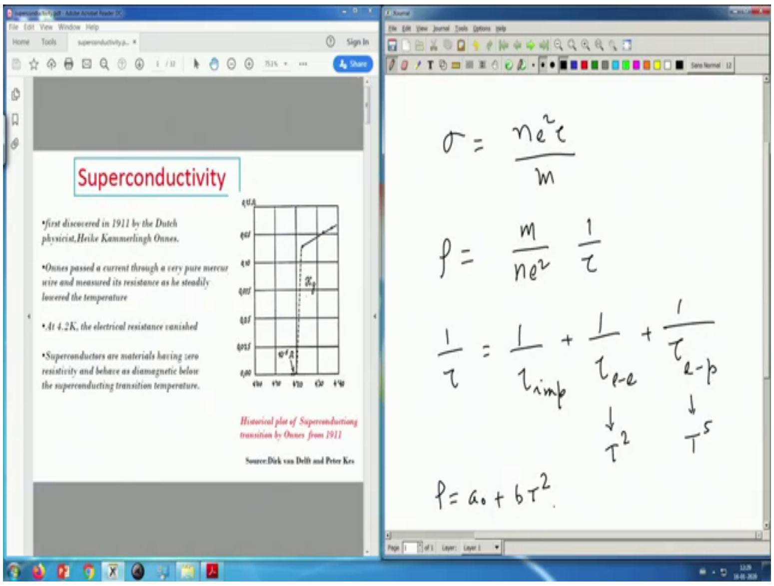

Superconductivity

- first discovered in 1911 by the Dutch physicist,Heike Kammerlingh Onnes.
- Onnes passed a current through a very pure mercur wire and measured its resistance as he steadily lowered the temperature
- At 4.2K, the electrical resistance vanished
- Superconductors are materials having zero resistivity and behave as diamagnetic below the superconducting transition temperature.

Historical plot of Superconducting transition by Onnes from 1911

Source:Dirk van Delft and Peter Kes

$$\sigma = \frac{ne^2\tau}{m}$$

$$\rho = \frac{m}{ne^2}\frac{1}{\tau}$$

$$\frac{1}{\tau} = \frac{1}{\tau_{imp}} + \underset{\downarrow \atop T^2}{\frac{1}{\tau_{e-e}}} + \underset{\downarrow \atop T^5}{\frac{1}{\tau_{e-p}}}$$

$$\rho = a_0 + bT^2$$

So, he was studying one of the other metals to see what happens. And the reason is that that for metals for example, we know that sigma is n e square tau by m was the dude formula is the dude formula and so, rho is m by n e square into 1 by tau. Now, this 1 by tau has contributions from many things; 1 by tau impurity plus 1 by tau electron-electron interaction plus 1 by tau electron phonon interaction ok. So, 1 by tau imp the tau impurity is basically constant. It does not depend on temperature too much. At low temperature, certainly does not depend on temperature at all and it. So, 1 by tau e this depends on temperature as T square at a low temperatures and this depends on temperature as T to the power of 5, well below the theta Debye temperatures.

So, essentially rho goes as some constant plus some b T square at low temperatures and this is what Dr. Onnes was trying to look at and check whether the metals really do behave this way. Indeed of course, nowadays we know that many metals do not behave this way, but good metals more or less is give similar behavior and that is what. In those days this was the formula, they had and this is what he was trying to check. And so, then what he found was that of course, in the gold and platinum, for example, he got a temperature resistivity versus temperature which was going down smoothly.

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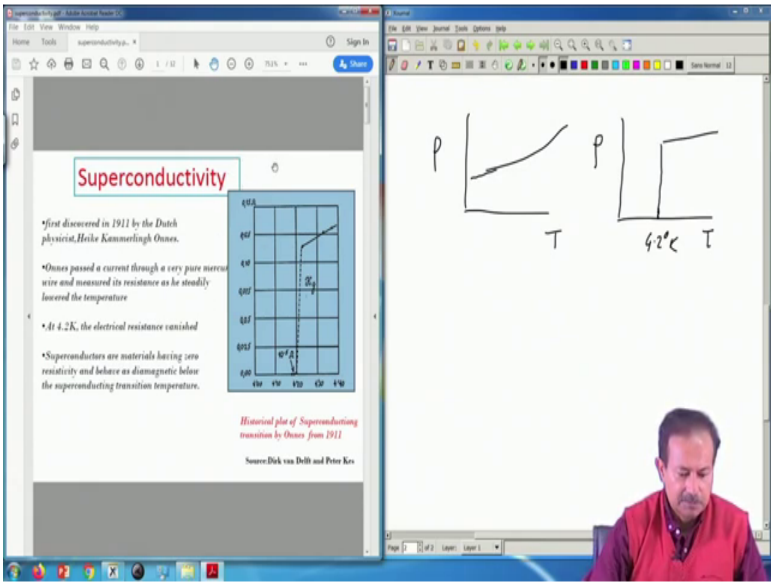

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And so, this was there was no surprise here. The temperature up to which he could go down it was behaving like this ok. But, but then in mercury, he choose mercury because mercury is one of the most pure metals you can get and then, there was a big surprise. And what he found was that mercury, the resistivity just dropped immeasurably below his instrumental resolution at four point around 4.2 degree Kelvin. So, this was real surprise and so, this picture is shown in a here on the left. This was the original picture of Kamerlingh Onnes.

So, this was on a graph paper which is what students use these days also to draw a plots. So, this was the reproduction from his paper and that shows that he could be measure up to 10 minus 5 ohms and his resistivity went below that. So, this he was a person, who was he realized that this is a new state of matter and he actually reported, it saying that this is probably a new state of matter. So, this is the history. The Onnes passed a current through a very pure mercury wire and measured its resistance and he steadily as he steadily lowered the temperature and at 4.2 degree the electrical resistance vanished.

By vanishing one basically means below the instrumental resolution goes below the instrumental resolution. So, superconductors are materials having zero resistivity and they behave as diamagnetic below the transition temperature. So, these are the two characteristics features and that define a superconductor.

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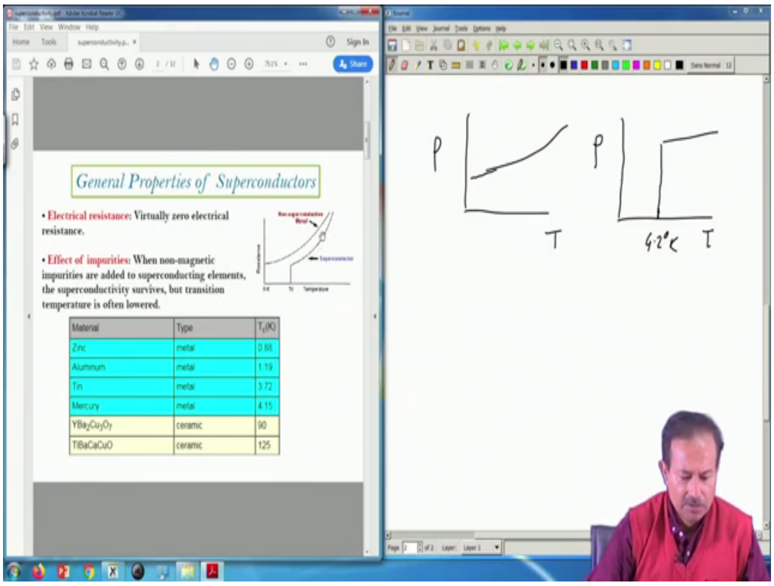

General Properties of Superconductors

- Electrical resistance: Virtually zero electrical resistance.
- Effect of impurities: When non-magnetic impurities are added to superconducting elements, the superconductivity survives, but transition temperature is often lowered.

Material	Type	T_c(K)
Zinc	metal	0.88
Aluminum	metal	1.19
Tin	metal	3.72
Mercury	metal	4.15
$YBa_2Cu_3O_7$	ceramic	90
TlBaCaCuO	ceramic	125

So, general properties of superconductor that the virtually zero resistance. By virtually again I mean that ideally it should be zero resistance its, but of course, you measure you measure resistance in some instrument and their instrumental resolution determines what is the minimum you can measure. And there are of course, effects of impurity; the purer the metal the better the transition is. But unless the impurity is magnetic and there is not much effect on the superconductivity as such, the transition temperature sometimes goes down if you have strong disorder.

But for nonmagnetic disorder in most superconductors that we discussed; we do not see much change in superconductor. Superconductivity basically survives. So, there are some materials that are listed here zinc, aluminum, tin. Mercury. There are others which are even higher superconducting temperature. But then, you can see that for example, lead has 7 degree nearly 7.2 or so degree; niobium is 9.2, 9.3 degree that is the highest element of the superconductor with highest T c.

Then, suddenly there is there are these two materials that are listed YBa2 Cu3 O7 for example, it is a ceramic material, undoped material is a is actually a actually a an insulator a magnetic insulator and it becomes superconductor at around 90 degree Kelvin when doped. So, that is a remarkable jump from 4 to 9 to 10 degrees centi 10 degrees Kelvin to 90 degrees ok. I will come back to it at a later stage. This is another example where this is superconductor which is a ceramic material which has T c about 125 degree Kelvin. So, this is the typical feature that we will expect for; for a non superconducting material this will go like this, there is a residual resistivity always because of impurities and no matter what you do in a material, you will always have impurities in it like defects and so on.

So, it will tend towards a finite value. One assumes and asserts that at zero temperature, everything will become a super. Every metal will become a superconductor, but there are materials where superconductivity has not been found so far like gold for example. But any way, it is it is just that maybe we have not gone down to sufficiently low temperatures. But that is a different matter, we will not discuss it. Superconductor has this typical feature that it follows this way a constant plus T square kind of behavior and then, suddenly drops ok. At least, this raises with a temperature as I said and then suddenly at certain temperature finite temperature, it drops to 0 immeasurably to 0 ok.

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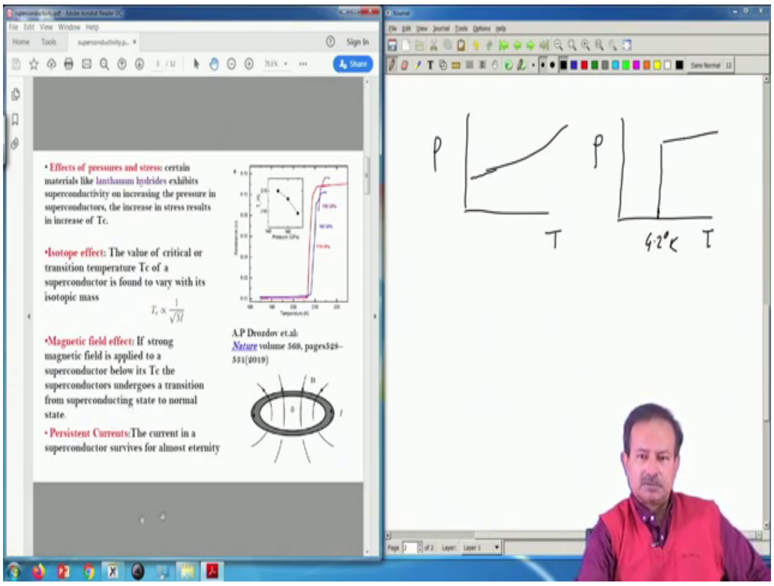

- **Effects of pressures and stress:** certain materials like lanthanum hydrides exhibits superconductivity on increasing the pressure in superconductors, the increase in stress results in increase of Tc.
- **Isotope effect:** The value of critical or transition temperature Tc of a superconductor is found to vary with its isotopic mass

$$T_c \propto \frac{1}{\sqrt{M}}$$

- **Magnetic field effect:** If strong magnetic field is applied to a superconductor below its Tc the superconductors undergoes a transition from superconducting state to normal state.
- **Persistent Currents:** The current in a superconductor survives for almost eternity

A.P Drozdov et.al: *Nature* volume 569, pages528–531(2019)

So, there is a there are several things that one actually studies this, I will come later. First let me just discuss this conventional things, which is the value of critical or transition temperature T c of a superconductor is found to vary with its isotopic mass. So, supposing a material is superconductor and you are replacing the atom by its isotope and then, you will find that the superconducting. T c is proportional or if you can do it for several isotopes, we will find its proportional to 1 by root M. And this is called the Isotope effect. It is very measure land mark in the understanding of the old superconductors and the mechanisms, because it is tells you that there is an involvement of the square root of mass appears in the frequency of a vibration of lattice right.

And that means, that this lattice is somehow connected to superconducting mechanism; whatever is responsible for superconductivity. So, this is strong indicator that superconductivity of the type that conventionally, we discussed is has a long, a strong dependence from lattice or phonons. To be precise from phonons that is vibrations. Then, there is a magnetic field effect which is a very important as for as applications are concerned, the thing is that every superconductor has a critical magnetic field above which it becomes non superconducting. So, that is called a critical magnetic field. So, below T c this happens even for a for a very low temperatures and way below T c, if you apply a strong magnetic field at some point, the super conduct will become non superconductor..

Then of course, there is this famous notion that there is a persistent current and its measured and if you were take a superconductor below its T c of course, is this is the superconducting state and you allow and you let you current to flow through it which is which is what superconductor does. It allows currents to flow without dissipation and end one can actually check how long does the current survive and it is found that it would be the from the measurement that has been done on a laboratory time scale for example, one can extrapolate that this will be more than the age of the universe.

So, in very clean materials. So, that the really says if is an amazing discovery that the current set of in a superconductor will last almost forever, till eternity. This is a very interesting new direction in super research of super conductivity effect of pressure and stress. This was predicted long time back actually. The there are many materials which are insulating, but under a high pressure, they become a metallic and the notion was that that once they become metallic, they may become superconductor. And one of the holy grails of such research was metallic hydrogen.

So, it is known or it was predicted and now it is known that metallic that that hydrogen under extreme high pressure becomes a metallic and if the if that happens that will become a superconductor with a T c which is nearly 300 degree Kelvin or even more. So, that was the estimate; theoretical estimate and. So, people actually went ahead and finally, finally, found that metallic state, but the superconducting conductivity has not yet been observed there. On the other hands, sulfur, oxygen for example, they become superconductor under very high pressure.

So, so, these are interesting observations. For example, even iron which is a magnetic material and magnetic materials are supposed to be non superconducting. We will explain the reasons later on. And even soon even iron becomes non magnetic has a nonmagnetic phase at a very high pressure and that that phase has become superconducting. So, there are many examples where this thing, these things happen.

The high pressure leads to super conductivity and the latest in that saga is the class of materials which are hydride materials, it started with H2S, from a group in Germany Drozdov and company and he, they went to very high pressure nearly 200 giga Pascal and that is an enormous pressure of course. And at that pressure, they found H2S became a superconductor at fairly high temperature.

In that saga, the latest is the lanthanum of hydrides LiH10 super, these are called super hydrides. And look at the T c for example, as a function as you increase, the pressure ah, so at about to 170 giga Pascal, this has a superconducting T c more than 205 degree Kelvin and latest one that they found in this hydride series has a T c at are about 245 to 250 degree Kelvin So, that is really remarkable. These papers have come out in a nature and. So, they these group is trying to push the idea of having superconductivity and very high temperature and of course, the holy grail is again to go to room temperature, but then if the pressure is so high, one does not know what applications will come out of it.

But never the less, the fact that something can be super conducting at that high temperature is still amazing. And for a for any scientist that is a is a remarkable discovery and one has to understand what is happening in the system; what causes that high temperature T c, high value of T c so, ok. So, that is that is one decision. But then, there are materials like metals like silver, copper, gold which are which were not found to be superconducting and tin has been has become super conductor has been found to be superconducting at milli Kelvin's or even less.

So, So, there are metals like lead, mercury; these are fairly high by high I mean 6-7 degrees, 4 degrees and so on. Whereas, there are other metals which are better metals, then these which are non super like platinum, gold, silver, copper, they are much better metals, better metals, but not do not have higher T c than this. They are actually have very low T c and in some cases, we have not been able to make them superconductor.

So, that is a sort of mystery in a sense that one tries to understand it, based on whatever understanding we developed microscopic understanding. But not all is understood, there are other superconductors which are a more so called exotic. These are for example, heavy fermion superconductors, then superconductors with different class of order parameters which are not the conventional s-wave. I will explain isotropic superconductors.

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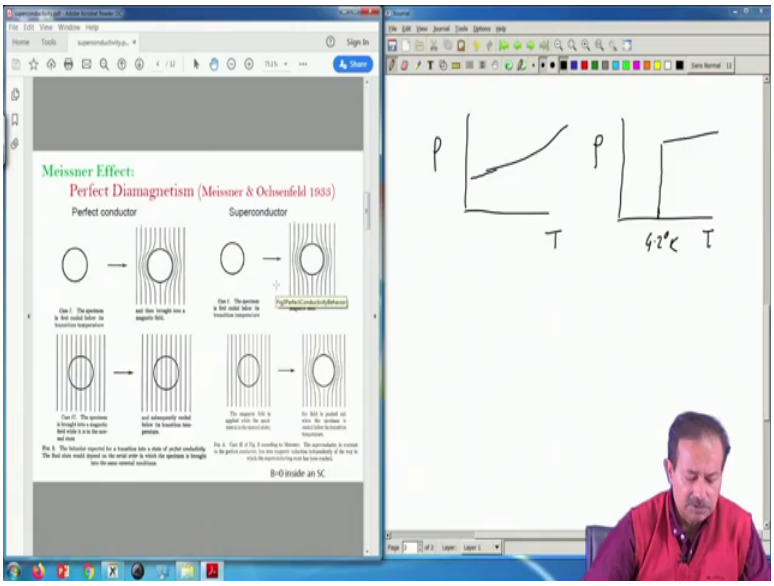

Meissner Effect:

Perfect Diamagnetism (Meissner & Ochsenfeld 1933)

And there are even magnet superconductors or conductivity and magnetism co existing. The heavy fermion superconductors for example, where a big mystery and mostly containing cerium or uranium and the this is they are still being worked on and one is trying to understand, what is happening in those systems. So, in heavy fermion superconductors, in heavy fermions you remember we discussed long back that these are materials, where the electron-electron interaction is very strong; strong repulsive interaction.

And you normally do not expect superconductivity to form in such a system. Never the less they do become some of them become superconductors and that is still a mystery being worked on. More than nearly 30 years, more than 30 years after it I have gone I have passed since the discovery and we are still working on a basic understanding of those things. So, superconductivity is still an unsolved problem in that sense. There is a class of superconductor which is this as I said this mercury, lead, niobium and NB3g and all that, where this is more known.

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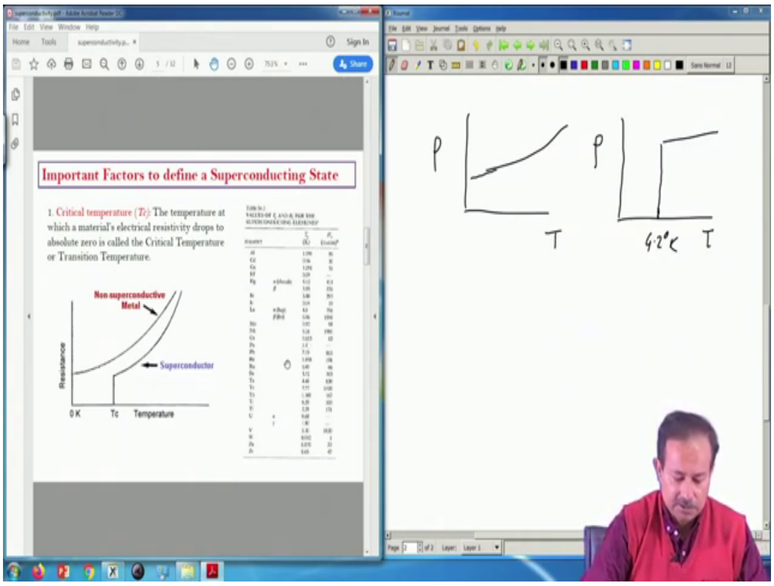
Important Factors to define a Superconducting State

1. Critical temperature (Tc): The temperature at which a material's electrical resistivity drops to absolute zero is called the Critical Temperature or Transition Temperature.

So, these are this is a list of a superconductors; large list of superconducting elements and as you can see that niobium has the highest superconducting T c amongst the element, nearly 9.3 degree Kelvin. So, these were better understood than the new superconductors that are have very high temperatures ok. So, there is a very important effect that defines the superconducting state and that effect is called the Meissner effect and what it tells us is that, it is related to the magnetic field effect on a superconductor.

So, in a nutshell, it is basically saying that a superconductor is a perfect diamagnet. So, this was discovered in 1933 by Meissner and Ochsenfeld. Ochsenfeld and Meissner found that the superconducting state is a first of all it does not depend on the history of application of the field. And secondly, in order what you do the magnetic field from inside the superconductor is always expelled. So, that is these are two examples shown here. This is a perfect conductor for example. In a perfect conductor also a field gets expelled.

So, if you had a suppose a super conductor was only a perfect conductor, then the specimen suppose you take the specimen. Then, cool it below its transition temperature, becomes super conductor and then you put a magnetic field. Then, the magnetic field will be expelled. I mean the magnetic field will not enter the perfect conductor. So, this is assuming that a super conductor is a perfect conductor, then you do the other thing. You take the super such a perfect conductor and first put a magnetic field above its T c and then.

So, its this is when it is in normal state, you bring it in a magnetic field. So, the magnetic field now penetrates its inside this were the perfect conductor and then, we cool this material. This perfect conductor below its T c and its becomes superconductor. Then, the magnetic field will remain inside the superconductor. It is basically trapped inside the superconductor. So, this is a this is the scenario, if a superconductor is just a perfect conductor ok. Now, what happens in the real superconductor? A superconductor does not bother about the history of how you applied the magnetic field. So, that is what is shown here.

The specimen is first cooled below its transition temperature which is the similar situation here and then, brought into the magnetic field. And the magnetic field does not enter the superconductor. Now, you do the other thing, it is called field cooled; this is called zero field cooled and. So, you take the material in its normal state above Tc put a magnetic field. So, the magnetic field enters, just like here; it is a normal. So, these and these are the same. The first picture and this picture are the same.

But now, you below bring the super actual super conductor to below its transition temperature, while the field was inside and then, at T c you will find that once it becomes superconductor, the field is expelled. So, the final outcome for a real superconductor is the same, the field is expelled from inside the material. Whereas, here it depends on the history whether you are you have done zero field cooling or field cooling and they have different final states for a perfect conductor. This is very easy to show and a every book shows this, many books shows show this.

So, this is you can also work it out yourself that this is what is going to happened in a perfect conductor; whereas, in a superconductor the final state is always the same. Since it is a thermodynamic state, it is a new state of matter, where the superconductivity expels magnetic field. This is called the Meissner Ochsenfeld effect and this is this means that it is a perfect diamagnet.

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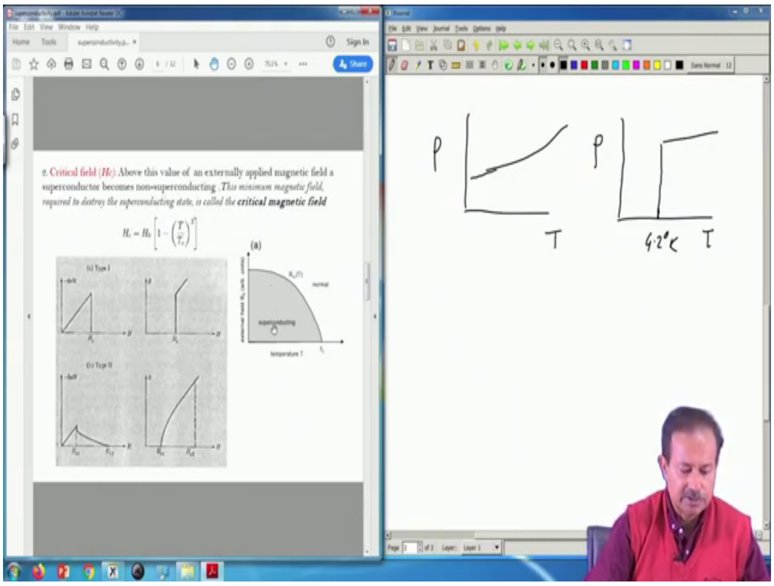

2. Critical field (Hc): Above this value of an externally applied magnetic field a superconductor becomes non-superconducting. *This minimum magnetic field, required to destroy the superconducting state, is called the* ***critical magnetic field***

$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

So, important facts that defines superconductivity are this that there is a critical field above this value and externally applied magnetic field will kill the superconductivity and this is how it goes typically. This is called the critical magnetic field. As a function of temperature at T c of course, you do not need any field to kill superconductivity.

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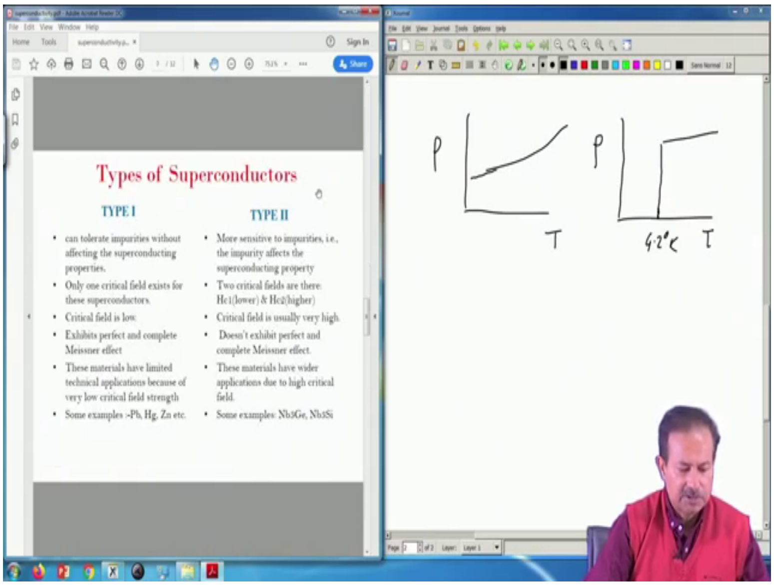

Types of Superconductors

TYPE I	TYPE II
can tolerate impurities without affecting the superconducting properties.	More sensitive to impurities, i.e., the impurity affects the superconducting property
Only one critical field exists for these superconductors.	Two critical fields are there: Hc1(lower) & Hc2(higher)
Critical field is low	Critical field is usually very high.
Exhibits perfect and complete Meissner effect	Doesn't exhibit perfect and complete Meissner effect.
These materials have limited technical applications because of very low critical field strength	These materials have wider applications due to high critical field.
Some examples :-Pb, Hg, Zn etc.	Some examples: Nb3Ge, Nb3Si

So, H c goes to 0 and so, the typical behavior is 1 minus T by T c square. There are two types of superconductors; type I and type II, which I will discuss later on.

And then of course, so, this is these two types are basically outlined here. This I will discuss again later and the type II one has two critical fields; after one critical field magnetic field can penetrate the superconductor, but only in vortex certain vertex tubes ah. So, that is it is a very different state and that we will discuss when we discuss electrodynamics of a super conductor. So that means, it is not a perfectly perfect Meissner state, but it does not show complete diamagnetism so.

But that is in that is an interesting state and that was one example where theory predicted, predicted it first and then came the experiment ok. So some examples are given. These are the materials the type II materials which have a applications, which are widely applied and so, there because of the high critical field called Hc2. So, I will come back to all these and start discussing the theory of superconductivity; what leads to superconductivity from the from the next few graph.