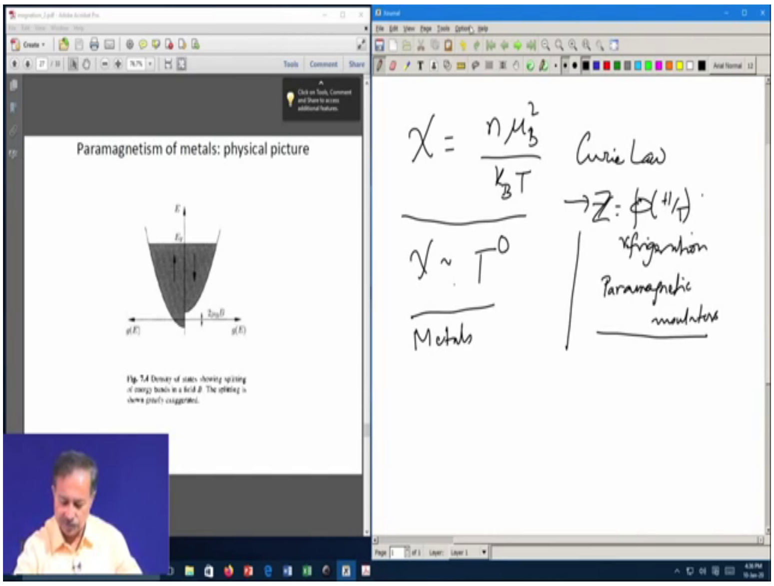
Electronic Theory of Solids

Prof. Arghya Taraphder
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture - 35
Pauli Paramagnetism

Hello we have been discussing magnetism and what we have done so far is calculated the magnetic response of free moments which are just a magnetic moment j in equilibrium at a finite temperature t. And, that then we found out the subsystems of independent magnetic moments at a finite temperature.

(Refer Slide Time: 01:01)



We found out the susceptibility of these and that gave us the famous law of Curie that which is chi equal to n mu B square by K B T ok for spin half system. So, the interesting thing here is this dependence on temperature which is 1 over T. Now of course, people in the early 20th centuries we are experimenting on several systems and then what they found that this is a good approximate good theory for insulating systems. But, when they studied metals and their metals responded differently and the susceptibility of metals was found to be independently independent of temperature. So, they behave like T to the power 0.

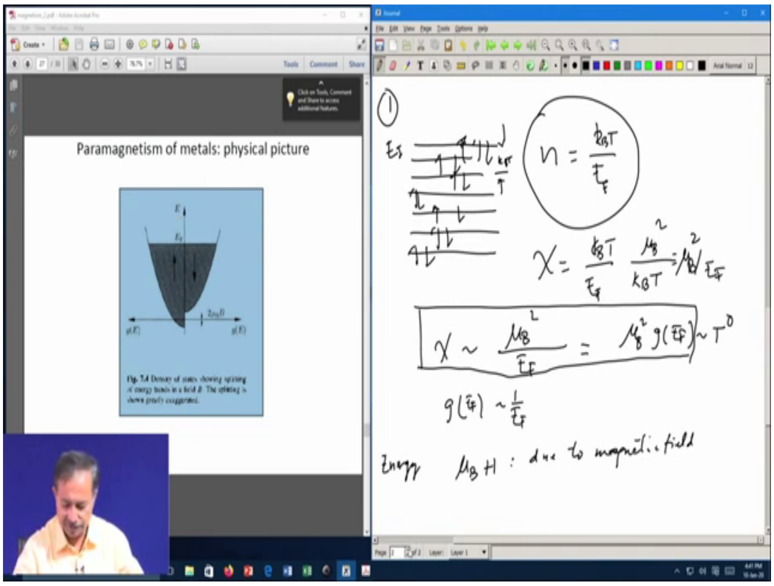
So, that 1 by T is no longer seen in metals the response of the conduction electron from the metals. Now, that immediately points out the fact that in metals electrons form a degenerate gas and that there is a Fermi level and that Fermi level severely restricts the face space of the electrons. You cannot excite an electron inside the fermi level. You can cannot take an electron from the bottom of the pile to somewhere at the up because all states are filled up.

You only can go above the Fermi level and that means, the fraction of electrons that can take the take any energy that you supply is less you severely curtailed. Similar arguments we had used for specific heat. So, when we calculated the specific heat of the unite Fermi gas and tried to find the linear temperature dependence of specific heat we actually used same concept Now, that concept was also used here and that calculation we will do.

So, the main the problem that we are trying to solve is the observation that in metals the response of electrons gives a susceptibility in presence of a magnetic field. The susceptibility which is not 1 over T not Curies law this is the celebrated Curie law, but it gives a very different behavior which is T to the power 0.You see using Curie law we actually used Curie law to get interesting physics; that the fact that Z was a function only of beta H or H by kT Sorry we used H by kT as a the functional form H by of the Z which is this H by T. This was used to find to show refrigeration to actually it was used. It was applied to refrigeration from few degrees to 100, hundreds of degrees; that this was then using paramagnetic insulating materials paramagnetic insulators right. So, this is a very different thing now that we are studying. Here we are studying metals and the response of metals is very different and as I said we will use the same concept that we used for the for a degenerate Fermi's gas in calculating specific heat.

Remember the specific heat also had a very different dependence from a constant in classical theory to a linear temperature dependence in metals. And, that is similar argument we are going to do what we will find out here is that instead of this 1 by T dependence which paramagnetic insulators indeed show we have a T to the power 0 T independent behavior of chi. So, let me just start the way we can find that its actually obtainable from a back of the envelope kind of argument.

(Refer Slide Time: 06:23)



So, let us just go back to that. So, argument so let me just go in at different levels first place a very simple argument ok. So, that is the physical way of understanding. We have these levels which are the they are all filled up. So, all these levels are filled up by two electrons. So, at any finite temperature now what is the fraction n that can energy from the temperature which is kB T by Ef right, if kB T is a very small amount its very very small compared to Ef even at room temperature it is nearly one-hundredth of the Fermi Fermi energy whereas, this is the Fermi energy.

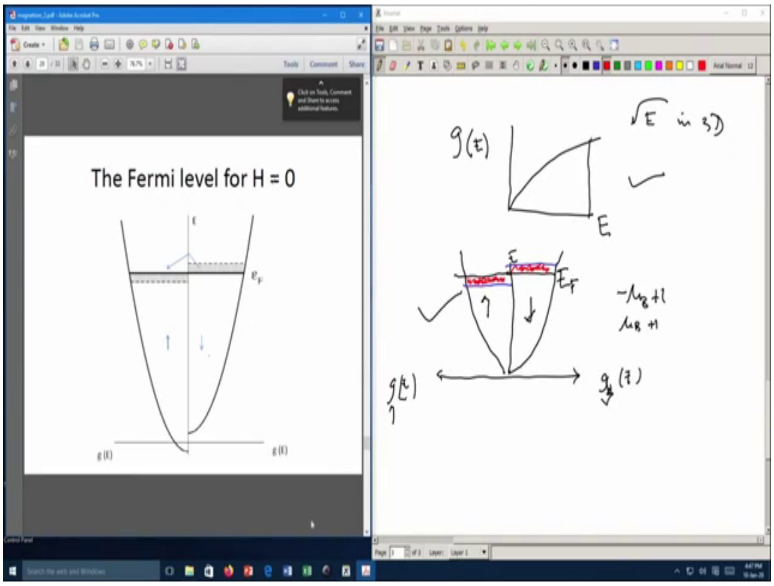
And, it is typically two, three electron volt whereas, temper at temperatures even at 100 degrees you are nearly 100 in fact 100 down from E F. So, these gives a reasonable estimate of the fraction of electrons that can take the energy. So, these are the ones which can get excited about the Fermi level. So, that that is now that is allowed. Now, again let me use the Pauli, the Curie susceptibility form the free electron susceptibilities free moments. These are the electrons which can now freely interact with the field. So, they can respond to the field and so the chi will be these this fraction n by E F into a mu B square by K B T ok.

So, this will be the response of the. So, instead of remember the n mu b square by K B T was the Curie's susceptibility right. What we n mu square by K B T and that n fraction is now not the entire set of moments. It is only a fraction which is K B T by E F and if you just multiply this mu B square by K B T by n you will get a mu B square by E F. So, chi appears to be going like mu B square by E F now in three dimension for example, the density of space is nearly flat at Fermi level. So, you can approximate g of E F the density of states by one over E F and then this becomes mu B square into g of E F. So, that is a formula that of course, its derived not derived, but argued physically and it gives me a temperature independent. So, this is temperature independent.

So, that is the argument that even Pauli who solved this problem who present he presented and this is the argument that can be sharpened further. One can do a more sophisticated calculation and one can find out that you will get a very similar formula as its just obtained here and that is what let me show you a one step. But before that let me show you what is actually happening in the density of states. So, the way the magnetic field interacts with the moment is basically mu B times H right. So, this is the energy change energy due to magnetic field due to magnetic field for an electron with in a field H.

So, the up electrons which are parallel to the to the magnetic field are they gain an energy. So, their density of spirits comes down. They are just lowered the energies are lowered the. The down electrons which are misaligned with the field they are energies will go up and so that is what is shown here. If you should remember this figure this is basically density of states versus E that but rotated 90 degree. We are familiar with this kind of a figure in 3d for example.

(Refer Slide Time: 11:27)



g of E versus E has a behavior which is like this. Square root of E remember in 3D. So, that is the behavior that we are using here and you just rotate it by 90 degree then you will find this on the picture on the left. The other thing that has been done here is that the normally we do not distinguish between up and down spins. So, the in this picture up and down spins are have the same density of states.

Whereas, here up and down spins separated out and so the way the picture is drawn is that these x-axis is now on this side E axis and we have separated out g of E in two different or two different g of E for up and g of E for down. The response of the screen is not good but, anyway let us just rotate it. So, this will be my one sector which is up. This is my other sector which is down. And in presence of a field this up sector will come down in energy by mu B H by amount. So, this will be their change will be minus mu B H for the up and mu B H for the down ok.

And, that is exactly what is shown here. So, this is now the energy axis. There is no nothing negative remember that the density of stress cannot be negative both sides are positive. This is also positive that side is also positive ok. So, what is shown here is exactly this thing except that now you have a magnetic field. So, the magnetic field pushes once with the up sector or down and the down sector up that is it.

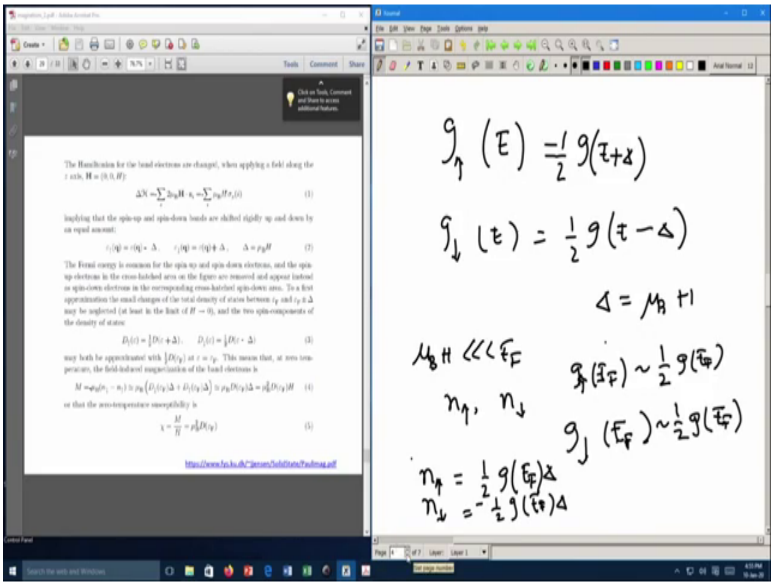
The other interesting issue is what happens then. So, remember that you have a Fermi level. So, the Fermi level is somewhere here which here translates to somewhere here ok. So, this is my Fermi level. Now if one sector comes down the other sector goes up then what happens to Fermi level. Well, its interesting . This picture on the left shows you the Fermi level.

If you have if you if you had the Fermi level which is now different for example, if up spin Fermi level, down spin Fermi level gets pushed up here and the up spin Fermi level push down here then there are there is excess electron here and the excess states here and excess x number of states here and so on.

These electrons the once which are here have a higher energy than these electrons and then immediately what will happen is that the electrons with higher energy will just come to the to the other side and becomes then. So, they if the Fermi surface Fermi levels had a mismatch then basically electrons will get transferred from here to here and then the Fermi level will again become the same. Otherwise there will always be this transfer of electrons. So, Fermi level remains at the same the system always keeps his Fermi level at the same place for up and down.

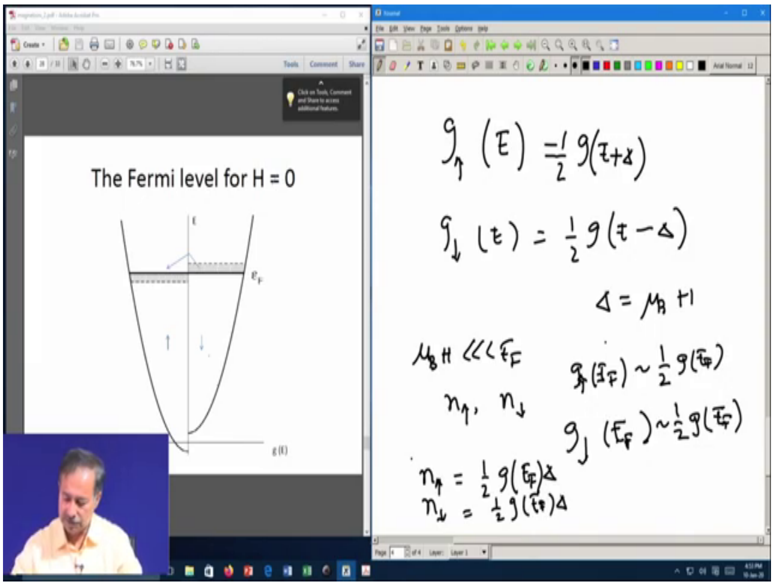
So in that case, that means there are number of electrons here. There are more number of states here states that have that accumulated here and there is less number of states on the right hand side ok. And, that basically is the number difference of electrons between up and down.

(Refer Slide Time: 16:49)



So, that is what one calculates by you can easily calculate this number. What you can do is that you just remember g not changing color. Now, it will show let me wipe it and change the color. g of up at epsilon has now become the original g at epsilon plus delta half of that. Because the you are using half of the half the densities are shared by up electrons, half down electrons and they are equal when we had no field. Now there is a field. So, this if you look at the picture.

(Refer Slide Time: 17:59)



Then see this. Since the left hand part the ups up spin density of states has come down. What was at originally at E plus delta E has now come down and so that has become. So, you choose let me just show you here. Suppose you had this density of states without field and with field your density of states has become like this then what was originally here originally here. Suppose you are looking for at E.

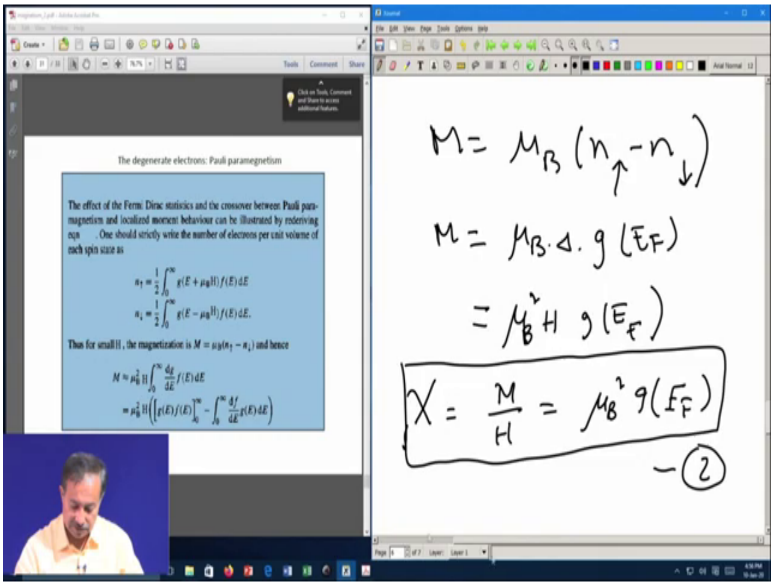
So, at a energy E. What was originally at E plus delta E delta has now come down to E. So, that is exactly. So, the original density of states which is at E plus delta is now the density of states at E for up spin and similarly it will E minus delta E for the down spin. So, this we can just forget this. This argument is very simple that you are done since your density of states. Energies have come down what was originally at E plus delta is now at E and the original density of states is just g of E and so it will be g of E plus delta that is what we will use for g up.

Similarly, g down of E will be half of g E minus delta. Here delta is mu B H because that is a shift that has happened ok. So, once you have that then the rest of the calculation is straightforward. What you do is that you again. Now you again realize that the amount of the energy much is much much smaller than E F ok. So, this is a few degrees as I said even for tesla field less than a degree and E F is some 5 electron volts 3 electron volt whatever. So, this the value that can be this value can be approximated by the see the change what you calculate is basically the change right.

So, that is exactly what is done here. So, look at the change. You need to calculate n up and n down and their changes now that. So, mu n up and mu n down after you put the field. So, what will that be right. At the Fermi level see the changes are happening at the Fermi level; now your extra states that came in at the Fermi level. So, what you can is that what you do is that at Fermi level you just approximate this g of up for example, at E F you can still approximate by g of E F original ok. And g of down E F is approximated as by g of original g of E F the right hand side is without the field.

So, this approximation is fairly can a good and just n up and n down is just multiply that by. So, n up is now g of half g of E F times this energy will be H delta. So, that will give you the extra states that you have. So, the density of states times the energy window of energy that is that extra window of energy that you have here this extra window. So, that is the number and similarly n down is the same. The same number of states have crossed over to up from down into delta. Now what you need is basically n up minus n down times mu B.

(Refer Slide Time: 22:57)



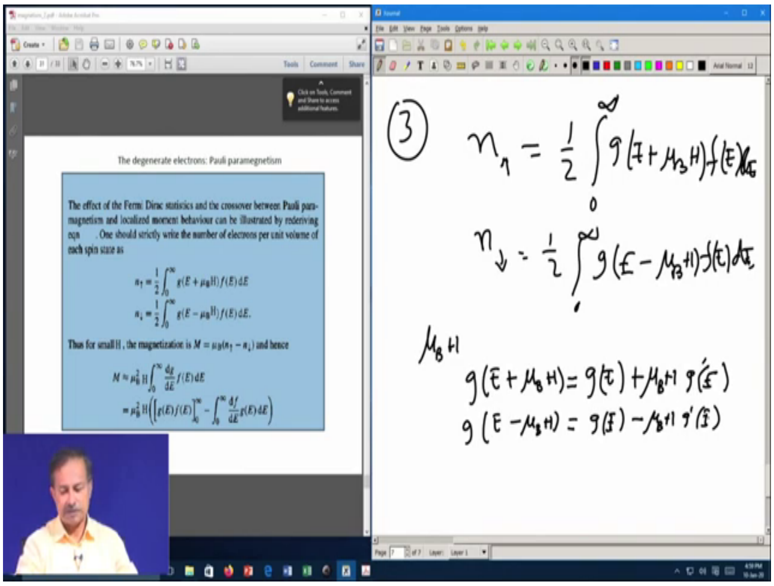
So, M is mu B n up minus n down and this number is as you see is just the. So, m is a mu B into delta into g of E F. The half and half will give you g of E F. So, that. See remember the g this n down is a negative number because n down has n has gone down. So, I could as well put a negative sign here. So, n down it this is change changes change in a number right. So, I can. So, mu n up is the original n plus this number mu n down is original n plus this number.

So, that is. So, that is the you can think of n up and n down as the increments in the number of particles in up and down. So, then their subtraction will give me a positive sign and that is what I am I am getting here and that gives me this mu B square into H into g of E F ok. So, this is what you get and then chi is M by H equal to mu B square g of E F. The same thing that I got from a very basic physical argument of the counting the fraction of electrons which can take the energy at the top of the Fermi level of the Fermi level the top of the pile.

So, it gives me back the same relation and it again shows that this is temperature independent. You can actually go further forward and you can calculate more sophisticated way there is there are ways to do it. So, this is one for example, which is more formal where it just instead of just approximating the number by half of g E F into delta. Instead we can do this integrals on the left hand side and still I will show you that you can still get the same result.

So, this was our calculation to this is the approximate calculation number two. So, we can go farther as I show on the left that we have this.

(Refer Slide Time: 26:21)



Third which is treatment which is more sophisticated. Here again the same thing, but except that we are now doing the doing in a more formal way. Again this half g of E plus mu B H times the Fermi function into d E 0 to infinity. So, this is n up and similarly n down is half 0 to infinity integral g of E minus mu B H Fermi function d E which is what is done here. Now again will do the same thing that mu B H is a very small number. So, you can make a Taylor of expansion.

So, how do you just write E plus mu B H equal to remember these half is since n up density of state is half of g of original density of state without the field that is what because we have now separated into up and down. So, that half is still this half this is no other origin that we are just writing the density of states for the up and density of states for the down separately as we did before. So, this is g of E plus mu B H into g prime of E and that is exactly what it has what has been done here.

Now, similarly g of E minus mu B H is equal to g of E minus mu B H into g prime of E ok. So, this is where we will start again and we will continue here from here and to the integrals and show that you get the same result.