Electronic Theory of Solids
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Lecture – 03
Drude Model continued: Hall Effect

Hello. We are not done with Drude model yet. The model has been around for more than 100 years and it has its share of success. So, let us just try to understand a few more phenomena using Drude model which actually helped us in understanding several experiments in the past. Let me introduce to a very important experimental tool which is called the Hall Effect.

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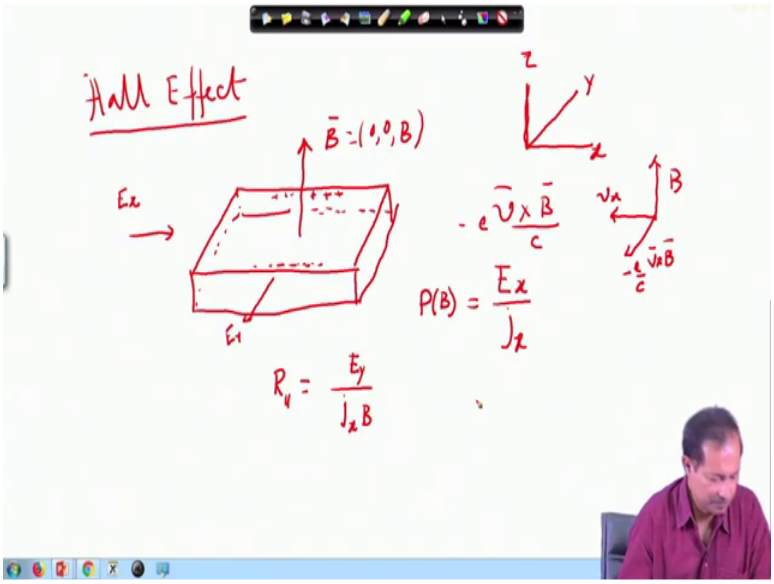

Now, Hall effect is a way one measures the charge of the carrier in a particular solid and so, it has been around for a long time and it also tells us many more things which is why we will come back to it at a later stage again. What we do now is that we start from the equation of motion that we wrote down and consider a geometry that is different. So, what does Hall effect do?

The original idea of Hall was that if a current flows in a conductor and if you put a perpendicular magnetic field across that conductor then some of the electrons will get deviated because of the Lorentz force coming from the magnetic field and this will deplete the number of electrons arriving at the other end.

So, that means, effectively the resistance has increased and he was looking for the increment in resistance due to magnetic field. The geometry is simple, let me draw a slab of conductor. In this situation let us draw the axes also: x, y and z and the electric field is along x direction.

There is a magnetic field perpendicular to this which is along the z direction equal to (0, 0, B). Its magnitude is B and let me show you what will happen. There is this Lorentz force v cross B divided by c into minus e. This Lorentz force component coming from the magnetic field will turn the electrons away and what will happen is that there will be a concentration of positive charge on one side and negative charge on the opposite side as shown.

So, that means, an electric field gets generated in the transverse direction, y direction here and that field actually opposes further movement of electrons. This field basically cancels the force that comes from the Lorentz force and that is where the steady state is reached.

Let us calculate; first let me show you the directions. This is my B along z direction and this is the direction of v along x and this is minus e v cross B by c. Remember the currents are flowing in this direction; that means, the electrons are flowing in that direction and that is why I have drawn the v vs x in this direction.

There are quantities that one needs to define. The two quantities that one defines here are this quantity which is a standard E_x by j_x called magneto-resistance simply because there is a magnetic field. Had there been no magnetic field this is just the resistivity that it would calculate, but because there is a magnetic field this is called a magneto resistance.

And, then there is another quantity that is very important which is R sub H (R_H) called Hall constant whose definition is E_y by j_x into B. Why does one chose this kind of a constant? See for example, this E_y as I said is the field that is generated due to the movement of electrons across, accumulation of electrons on this surface and that surface.

So, that means, that this E $_y$ should be proportional to the current. It has to be proportional to the current because of the movement of the electrons is due to current and in addition there is a bending caused by the magnetic field. So, it should be proportional to B also.

So, E $_y$ is a constant proportional to j $_x$ and B. And, so that is why this constant was chosen and it is called the Hall constant. Our aim is to determine this constant. So, let us go ahead and do the calculation.

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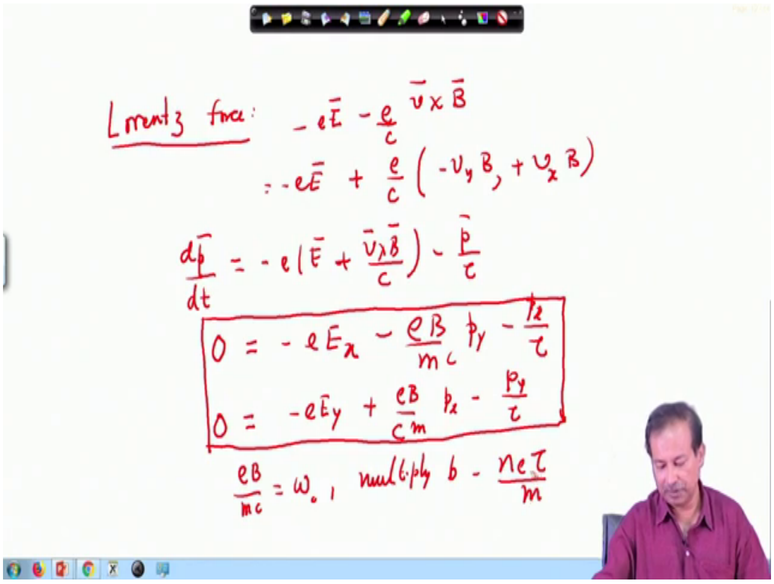

See the Lorentz force is minus e by minus eE electric field plus minus e v cross B by c in Gaussian units. Now, if you do that then this part gives me minus eE that this is the electric field minus e by c. So, let me just show you what this simple cross product, remember B = (0, 0, B).

I will have no z component of velocity involved here. I will have minus v y into B plus v x. So, these are the two components v y into v x into B. So, the x component is minus v y into B and the y component is v x into B. Under this force let us calculate the equation that we already had and try to find out what happens to the numbers that we wrote down, the Hall coefficient and the magneto resistance.

Now dp/dt equals to minus eE plus v cross B by c and this term that we derived, the relaxation term, the drag term or whatever it is called (usually called drag term or just the the relaxation term).

In the steady state of course, as we did the other day, we will put this dp/dt to 0. Therefore, I have two equations: one is for the x component which is minus eE x; remember the field I started with was in along x direction, but another field has now generated which is in the y direction because of this term. So, I have to keep both the fields minus e B by m c into p y minus p x by tau.

What I have done is a I have written as p by m which will do for the y component also minus eE y plus eB by c into m into p x minus p y by tau. So, these are the two equations that I have to simultaneously solve. So, this eB by mc is usually written as omega sub c, will come back to it when we do electrons in a transverse magnetic field again and also what we do is that multiply both the equations by minus n e n e tau by m. See, you can see the logic for doing this because if we use this multiplication these terms will become n e square tau by m and there is a e B by m c. So, I will have a omega c times tau.

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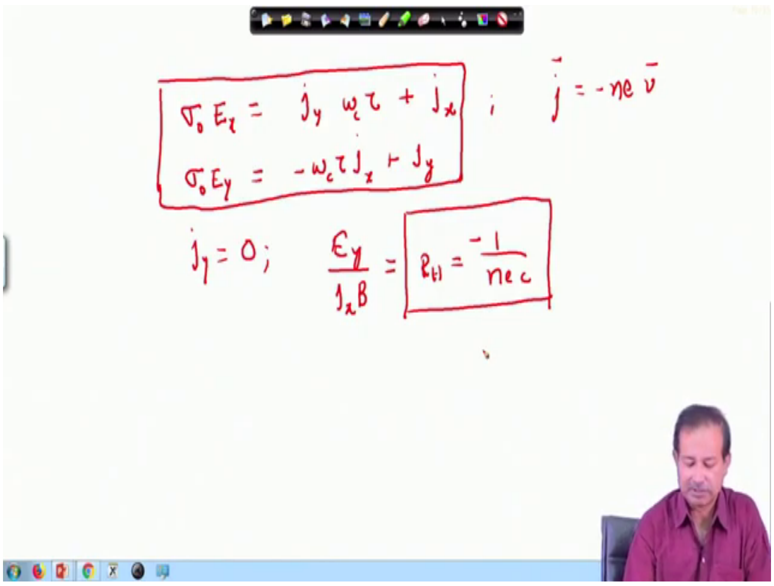

That gives me the equation that sigma$_0$ E x equal to j y into omega c tau plus j x. See that I am using the fact that j equals to minus n e v. So, the other one is sigma $_0$ E y equal to minus omega c tau j x plus j y. So, these are the two equations that I have. They need to be solved. Now, the what I want is a the this Hall resistance and for that I have to put the current in the y direction to be 0, because that is when the equilibrium has reached in the steady state a. So, no more current is flowing in the y direction.

So, that gives me j y equal to 0 and if I do that then E y from this equation is equal to. Let me write the full thing. E y by j x into B, you can calculate these, equal to R H will turn out to be 1 by n e c with a minus sign. This minus sign is very important because now this is the quantity I defined as R_H.

So, R_H turns out to be minus n e c. This minus sign tells me that the carriers here are electrons. If there is a positive sign what I can assume then is that the carriers in the system are of opposite sign to the electrons. For example, when holes carry the current which happens in metals or semiconductors sometimes.

This relation is a celebrated relation, this is the Hall constant which tells us the sign of the carrier. I will ask you to remember this because this is such a simple relation it only depends on the density and then two fundamental constants electric charge and c with a minus sign. So, that is what one is really after in this experiment.

And, once you do experiments what you will find is that this relation is more or less valid in many metals. However, there are systems where the sign is different which I actually preempted and I said that there are different kinds of carriers. That is indicative of the fact that there are different kinds of carrier in some systems which are positively charged and which brought in the idea of holes. The other problem with this is that this relation is not always valid, such a simple relation does not work always.

This n is of course, temperature dependent, in a metal it is not so strongly dependent, but this simple relation fails in many materials, but it is still a celebrated relation because for many systems it works out and it is one of the very few ways in which you can determine the sign of the charge of the carriers.

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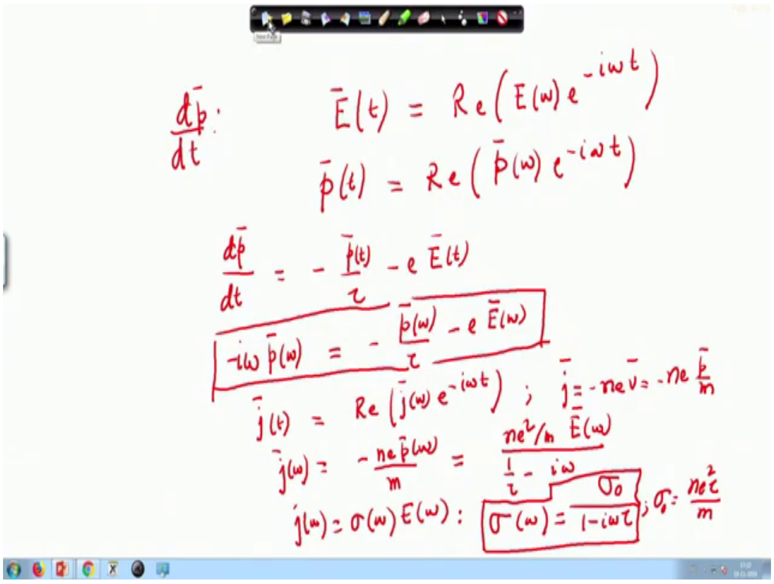

Now let me wrap up this discussion by showing you one more simple application of this equations that we got and then I will be done with it which is what I have been mentioning from the beginning that the this dp/dt, the dynamics, has so far not played much role in this deductions. We have always used the steady state condition and said dp/dt=0, but there are situations where it is not and one such situation obviously, is where there is a time-varying electric field.

So, when there is an electric field that is time-varying then in that case this will not be 0. It will follow the time dependence of the electric field and we can work out the equations for conductivity in such a situation. I will show you that the conductivity sigma becomes a complex number in this case and it becomes frequency-dependent.

Let me show you: it is a very short calculation and anybody can do it easily. This is what I will work out now. Suppose there is an electric field which is varying in time with a single frequency and this I denote by real part of E of omega, e to the power minus i omega t .

Then, I will use this equation dp/dt equal to minus p by tau plus the force coming from here and I remember that since E is varying with a single frequency the electron momentum will also have the same frequency. I can write the momentum of the electron as the real part of p of omega e to the power minus i omega t. Now, of course, what I can do is that I simply substitute these two complex representations of the field and the momentum in this equation and I get this equation for example, the one that I have been writing by p by tau minus eE.

This equation now becomes minus i omega p of omega simple time derivative will give you that is equal to minus p omega by tau minus eE omega because E to the power minus i omega t gets canceled from both sides. So, that is the equation. Now, this equation is very easily solved. I mean you can write down the current from this equation.

Current is again I and we can assume the current to be following this frequency. So, this current is again real part of j of omega e to the power minus i omega t and j equal to n e v for electrons, equal to minus n e p by m ok.

If you combine these two then your j omega is j omega turns out to be you just put it back here in this equation in these equation, this equation. So, then j omega is equal to minus n e p omega by m equal to n e square by m into e of omega by 1 by tau minus i omega. Now, this is a very important relation. It comes by replacing the p of omega from here you can replace p by j times m by n e into minus i and that will give you this relation.

That it is the relation j omega equal to sigma omega, now everything is frequency dependent. Remember E omega, we can easily find out that sigma of omega equals to sigma_0 divided by 1 minus i omega tau where sigma $_0$ is n e square tau by m, the good old DC conductivity.

That is what happens when you have a time varying electric field applied to the system. The conductivity is now a complex number and this complex behavior is dictated by sigma naught divided by 1 minus i omega tau. In most cases i omega tau is a very large number and then in that case you will recover your sigma known i sigma 0 by omega tau.

If omega tau is a small number then of course that means, your frequencies are very low and you go back to the DC limit which is a sigma 0. But, typically generally these frequencies are fairly high, particularly for electromagnetic radiations these are in the terahertz region: 10 to the power 15 Hertz or so. There are you have to keep it. From this relation you can go one step up and you can use Maxwell's equation.

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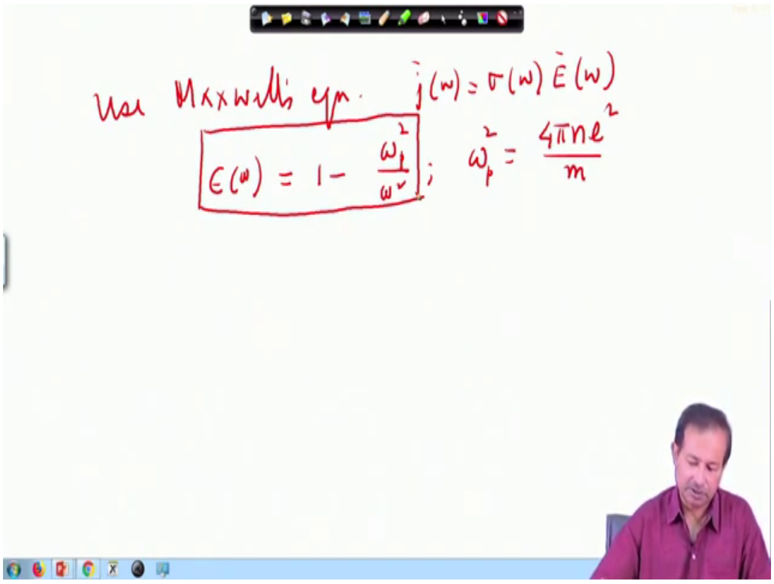

You can use Maxwell's equation and combine with these result that j omega equal to sigma omega E of omega then you will find a very useful identity which is the dielectric constant which is our good old dielectric constant which is now 1 minus omega p square by omega square, where omega p square is 4 pi n e square by m.

Now, if the dielectric constant has to remain positive, then this omega has to be greater than omega p. So, that means, this quantity has to be less than 1; whereas, if omega is less than omega p then there is a trouble. Because then it becomes a negative number, this goes into the frequency and in the E to the power i omega t and then it becomes a complex number and that means, there is a decaying part.

Which means that your frequency, if it is not more than omega p which is called the plasma frequency, then electromagnetic radiation will not penetrate into the system, it will decay down whereas if it is greater than this then it will pass through. So, oscillatory radiation will propagate and the metal will become transparent if your omega is greater than omega p.

This relation is actually very useful and the value of plasma of frequency is typically around 10 to the power 15 Hertz range for metals. So, a conclusion from this simple classical model with relaxation time approximation is that a metal can be even transparent to an oscillating electromagnetic radiation or the radiation can decay.

A lot of useful physics can be obtained from a simple model like this and that is why Drude model has been studied for so long. It has run its course. Many simple minded pictures can be obtained from this extremely simple and classical model of Drude.

What we have done today is we are concluding our discussions on Drude model. We will come back to it later, to discuss some of the successes and failures at some point in the future, but basically it shows that a model of such simplicity treating electrons as a classical gas with a relaxation time approximation gives us a lot.

Many useful and interesting results came out while the discussions are all classical. There is no quantum mechanics involved in this and still we get very useful results which tell us the story of a metal both in static condition as well as under oscillating electromagnetic field.

Now, apart from that there are situations of course, I must admit where Drude model fails quite miserably. For example, specific heat is a great example. You just saw that specific heat in Drude model is still assumed classical, 3/2 n k which is not what happens in a metal, it goes linearly with temperature.

The success of Wiedemann Franz law from Drude model is also a bit qualified in the sense that it works out at low temperatures and at high temperatures, but there is a large range of temperature where this constancy, the Lorentz number being a constant, does not really work.

Nevertheless it is a very useful model and it has run it is course for nearly 100 years now. For many simple metals where electron-electron correlatio is not very strong or other anomalous effects exist and if you leave aside those, then for many metals it works it gives you very nice results.

In the next lecture onwards I leave Drude model and start using quantum mechanics which is what finally dictates the world of elections and one has to calculate things starting from quantum mechanics of many electrons. So, the interactions might become important. We will discuss all of that at a later stage.

At the moment we will start with simply a system of non-interacting electrons, with quantum mechanics and see what comes out of it. So, we are in for completely new world.

Thank you so much.