Electronic Theory of Solids
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Lecture - 28
Magnetism

(Refer Slide Time: 00:24)

Hello, we have been discussing certain interesting properties of magnetic systems for example, where the spin of these of the carriers are used to transport information; transfer information from one point to the other rather than using the electrons themselves in the charge which cause a lot of coulomb energy and of course it also slows down the slows down the process.

So, the idea here is that you encode your information with the magnetic state of a system. So, the one example we gave was this famous giant magnetoresistance systems which is which has earned actually Nobel Prize and this system is like this that the magnetoresistance; the resistance of the carriers depend on the spin state of the material.

So, the spin alignment in the material and for example, in this case for example, remember this is not a case where charge transport is completely quenched; it is still using charge transport, but it charge transport depends on the spin out the charge. So, it is not fully a spintronic device, but it is a step towards spintronic device.

For example, here we said that you have a ferromagnetic material tool these red regions are ferromagnetic and this similar region is paramagnetic. And the for example, if the two ferromagnets have opposite alignment, then this is high resistance state because. For example, if you want to transfer an electron from the bottom to the top, then the up electron will enter this ferromagnetic region the all the electrons here more or less carry up spin.

So, you are transporting that into this region which is which contains down spins. So, then they spin has to turn back into down maybe the if the Fermi energies are not compatible and if the such transport; so at this spin is in different direction is not an easy transport.

So, once there is there could be scattering there could be Fermi surface Fermi level mismatches. So, primarily the spin scattering is going to prevent it from entering this region; Now that on the other hand suppose I now turn the spin on the top to up; then of course, an electron can easily move into the up region.

So, this kind of situation where you can magnetically control the transport is called the magnetoresistance the because resistance is high in this state in the right hand side and low in the left hand side. And that can be used to read and write information to send information; for example, here at the current is one; suppose there are this magnetic recording medium has this moments N S S N and so on.

And this read write this kind of GMR read write head is moving on it then depending on the magnetic field that this produces; this will change and so, this will either allow an electron to come in or not allow an electron to go. So, this for example, the bottom magnetic recording head has this N S kind of arrangements where magnetic states are our data.

And so that can be read off by this that will align this regions which is where there are there magnetics magnetic materials; these are magnetic materials; so the spins are aligned and align. And then depending on whether these two are parallel or perpendicular aligned or misaligned; then current will flow or not and that is how you can encode data or information into a this kind of a magnetic arrangement.

So, this is an example where magnetism is used to transfer data to record and transfer data. And then this is one spintronic application that is already in use, where as this is not what you actually want we do not want to transport any charge anywhere and so that kind of spin spintronic applications has not come in the market yet and the still in the drawing stage; theoretical analysis goes on.

And finally, one day one should be able to get such a material where you do not need to transport electrons at all; only spins get transported. Spin state gets carries the information and that state gets transported spin arrangement gets transported and that is how this is going to be done in the future.

The interesting thing about this is that all these requires you to understand the magnetic interaction between materials between spins. And the moments that reside in an atom interact can interact with a nearby moment and that can lead to magnetic interaction these are called exchange interactions. And that is what we have to study to understand and work on magneto, resistive materials and use magnetism how to use magnetism to do any applications.

So, that is what I will go to and besides a point there is a fundamentally interesting. So, fundamentally interesting subject; it is actually one of the first known macroscopic quantum state which I will come to.

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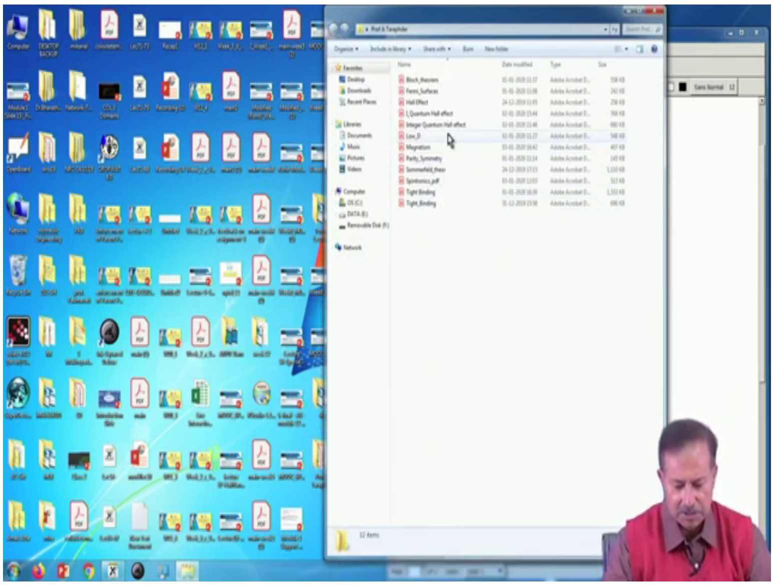

So, let us begin by discussing something called just an electron; take an electron into a magnetic field which is travelling in a magnetic field.

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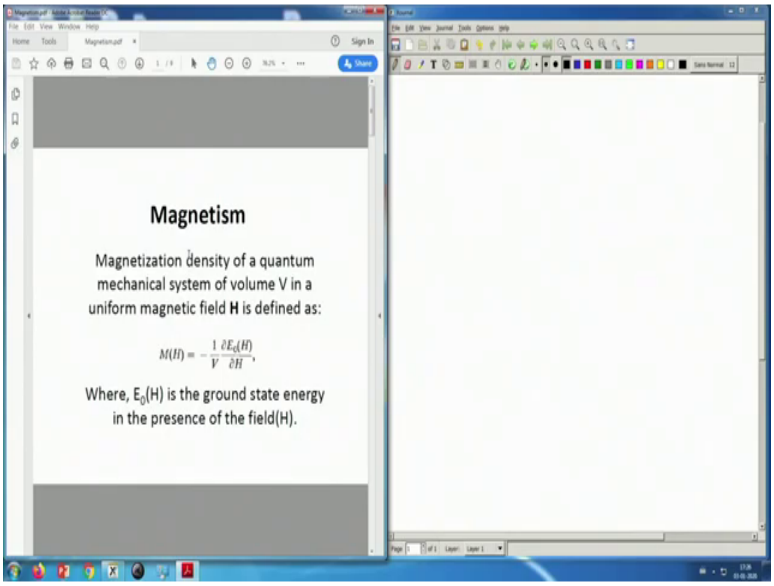

So, that is the subject of magnetism; an electron in a magnetic field basically that is what one wants to study and of course, later on we will study the interaction between moments in an electron where moments are not moving, but they are interacting with each other where something called an exchange.

So, first let us start with the usual magnetism that we have done this.; There are many ways you can study magnetism; one branch studies magnetism starting from the magnetic materials. So, that is important those are materials, magnetic materials are very important because those are the ones for applications are done with the bulk applications of magnetism.

Whereas there are fundamental microscopic theories and this is what I am going to concentrate on. Because to understand magnetism you have to basically understand how a magnet with its moment behaves in presence of a magnetic field. And then of course, how do such magnetic moments in a solid interact with each other and produce so called long range order ok.

So, that is a microscopic theory of magnetism, whereas the bulk magnetism which is where applications are done involves the magnetic bulk magnetic properties and these properties are more or less described by classical theory where as what I am going to do is a quantum theory of magnetism.

So, let us start with in; so there are certain definitions you need to know one is the magnetization density; in a volume V in an uniform field H is defined as the at 0 temperature is the ground state energy derivative with respect to H divided by volume this is a density; so divided by volume and with a minus sign so that is the definition of magnetization; magnetization density. So, that is one has to remember that, one has to calculate it from any description of microscopic magnetism.

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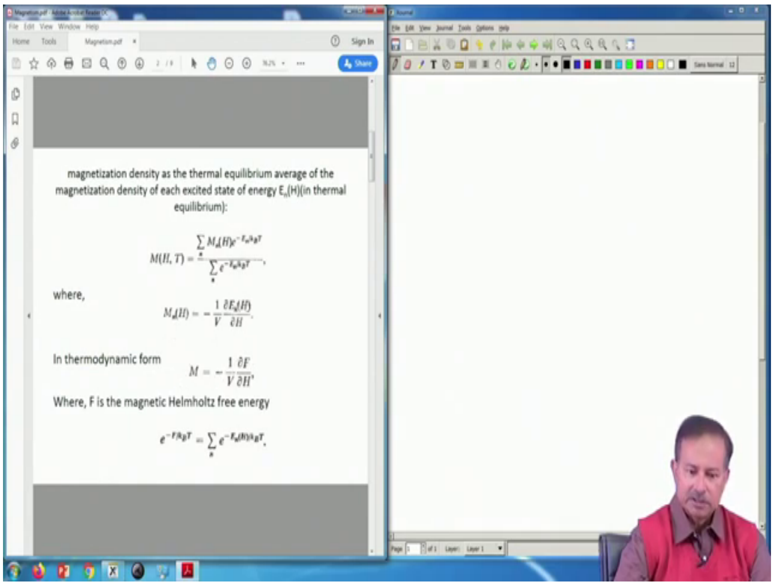

Now, how does one calculate it? You calculate it by at any finite temperature for example, you calculate it from a not from the ground state energy because now you will have entropic corrections, entropic contributions to your energy.

So, all that can be subsumed in this fact that if you know; if you know your eigenstates of the Hamiltonian; then the if the energy of these states is our e sub N, then you have set of states e sub N then of course, you can calculate the if you know the magnetization of each of these states; then just multiply that magnetization of each state by the probability of having that state and some overall the probability.

Therefore, you get a form a which is the in finite temperature of course, this e ground state energy is then replaced by the free energy ok; the Helmholtz free energy which is defined here as the sum of this quantity ok. So, that is the; that is the standard definition at infinite temperature. So, all you have to do is to find out the free energy and then take a derivative with respect to magnetic field and divide by the volume with a minus sign and you get the magnetization density.

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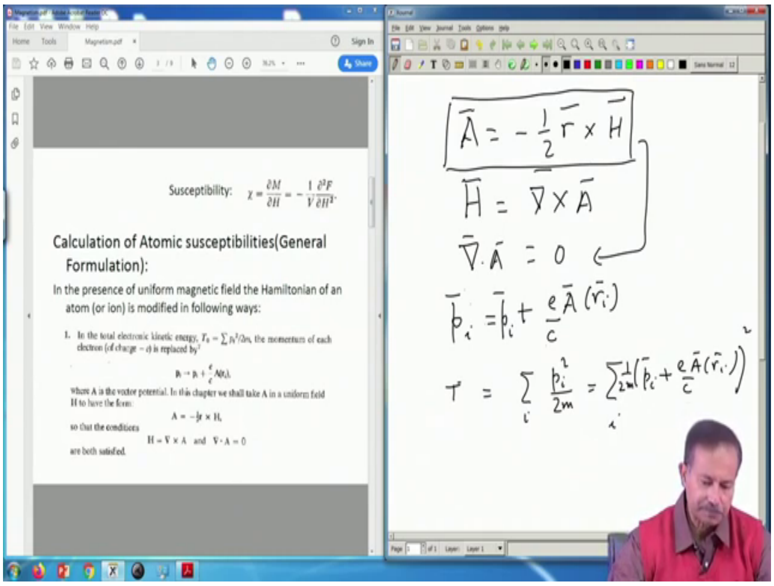

Susceptibility is a quantity that of course, we need and we it is basically a response of the system to a magnetic field and that definition is how M changes with field. So, this is just a derivative of del M del H. So, that will become two derivatives with respect to F with respect to H of the free energy; so that is just use the definition of M which was minus del F del H into 1 by V and that gives you the second derivative with respect to magnetic field.

So, first one has to start from atomic susceptibilities because after all susceptibility of an atom has to be calculated and then it has to be summed over all atoms. If these atoms are not talking to each other in terms of magnetic interactions, then that will become the magnetic property of the system that is the; suppose I know the magnetization or magnetic moment of an atom and if all the atoms are summed over, there magnetic moments of summed over then will get the magnetic moment of the entire system that is how one does it.

Now, to do that of course, you have to do a standard quantum mechanical calculation. So, I mean we can do classically also this can be this is there is nothing quantum mechanical here this is a standard canonical replacement of momentum as we have done in the case of quantum Hall effect remember that p has to be replaced by e by p plus e by c into A; A is the vector potential. So, curl of A will give you H and A is the way one writes A is choose a gauge which is minus half r cross H.

So, that is use that to calculate your A; in this gauge this is the value the formula for A. And then of course, the celebrated relations that H equal to curl A and divergence A is taken to be 0; so often called Coulomb gauge. So, this choice actually; you can easily see that this choice will give you this.

So, under this and then of course, p i which is the momentum at the of the ith atom the electron at the ith atom is of the ith electron is sitting in ith atom probably and. So, the ith electron momentum is p goes to p i plus this plus comes from the minus e otherwise is p minus e A by c e at the point i; r sub i by c.

So, it is ith electrons momentum is now shifted by an amount which is e by c into a of r i; so that is the canonical momentum. So, my kinetic energy is T equal to sum over i; p i square by twice m and this is basically p i plus e A at r i by c square by twice m; that is the standard expression for kinetic energy now in presence of a magnetic field ok.

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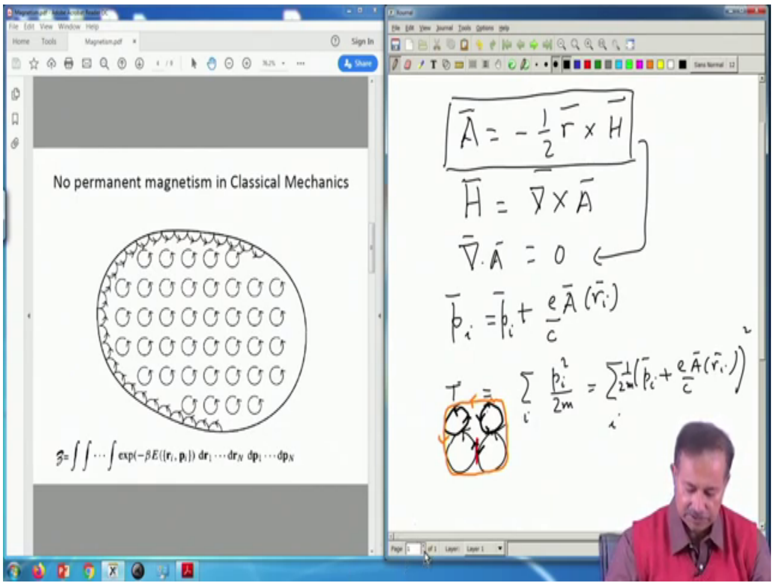

Before I go further, I have not introduced quantum mechanics or classical mechanics anything so far; this is still in this can will be a classical description. But let us just ponder a bit and go back to some beautiful ideas that came from Bohr and his student Miss Van Leeuwen and they had a beautiful argument to show that you cannot have permanent magnetism starting from classical mechanics.

So, that is kind of interesting because that show that tells us that whenever you look at a permanent magnet; how big it is the underlying mechanism has to be quantum mechanical. And that is why I said in the beginning that this is the in the first known and used macroscopic quantum object quantum phenomenon, the without knowing of course, because magnetism was discovered in some 600 BC.

And so this relation actually tells us that this assertion that this proof by Bohr and Van Leeuwen tells us that you cannot have permanent magnet starting from classical mechanic. So, it has to be a quantum phenomenon; so that makes the oldest it probably the oldest quantum phenomenon at a macroscopic scale.

Nowadays of course, superconductor you can see a superconductor right in front of your eyes at liquid nitrogen temperature. Those all well known exotic quantum phenomena, but this is a quantum phenomenon that we use everyday and for last 3000 years also or 2500 years.

So, let me just explain what this picture tells us. If you look at the if you think still classically then look at the; these electrons all the in electrons are rotating about a about the magnetic field. So, the magnetic field is out of the plane and the electrons are rotating about the; about that magnetic field.

So, each electron forms an orbit and then you can easily see and the classical way of looking at reason of orbiting an electron circulating in an orbit produces a magnetic moment proportional to its current to the current that it generates. So, but now let us look at these all the orbits that are inside; these closed orbits. If you look at them the orbits are they each of this suppose in two neighboring orbits a like this and both rotating the electrons in both rotating along this direction.

So, along this boundary for example, this boundary you can see the motion of the left electron is up where as the motion of the right electron is down. So, they actually cancel; these contributions from these orbits at the boundary at the point where their come close is cancelling. So, then similarly another electron which is here it will cancel the moment coming from this one.

So, the orbits we will get basically cancelled internally all internal orbits are cancelled which orbit remains? So, suppose I put another one here; the orbit that remains is the one which is the total current that will be remaining still is the one which is coming from here. And it has a sense which is this; these are not out of this four, if you take this four then this is the one that will remain ok.

So, this will cancel this one this will cancel this one and the net current comes from this large orbit ok. So, that is the classical way of thinking of a magnetic; large magnetic moment forming out off when you put a magnetic field. Now, if you look at this picture on the left then you can see that.

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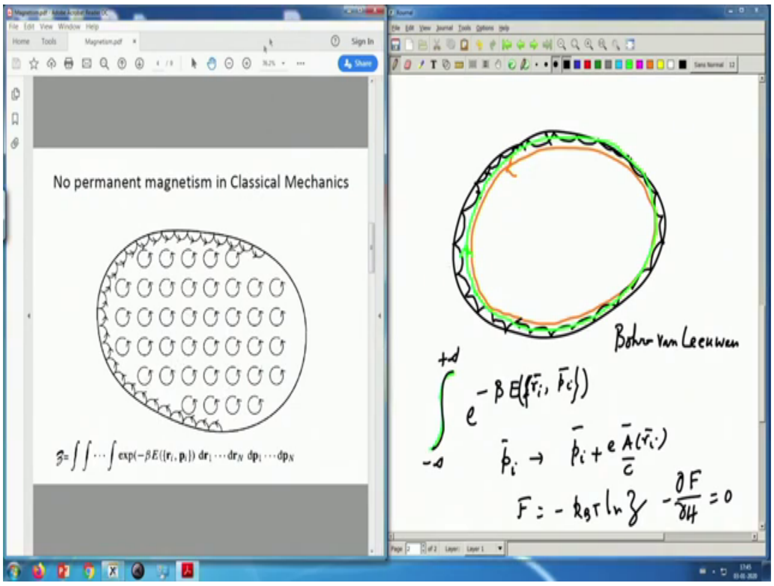

So; that means, in this geometry in this left hand side in the; in this whole system currents are moving only in this direction, there is a giant loop and this is the net movement of current due to the internal orbits.

But then you can see that there at this other orbits at a drawn at the boundary which is not a complete orbit these orbits basically their orbits size of the orbit is such that they cannot complete the hit the boundary, the electron hits the boundary and then reflects back; so these are called skipping orbits. So, these skipping orbits which are at the boundary are like this. So, they are like this and so on.

Now, the each of them represents direction of rotation like this, but since they cannot complete the orbit they have to reflect back; the total sense of the orbit is in this direction; so that orbit is in this direction. Now, you see that these two loops big loops they cancel each other. So, that is the; that is one way to sort of warn you that it is possible that the total magnetization may not survive ok.

So, now let us do a mathematical theory to just show you that this is indeed the case if you are thinking classically which is what we have just done. Now, look at this object this is basically the partition function right exponential minus beta E which is the function of r i and momentum of each of the electrons and then and that that is basically the Hamiltonian that we wrote the energy corresponding to that.

So, this integral is giving us; this assumes that there continuous distribution on energy this is classical this is a continuous distribution on energy and if you look at it the integrals are all from minus infinity to plus infinity. So, that is the; that is the way we calculate partition function right because in classical mechanics moment and position can take all possible values continuously and we integrate over all of that to get the classical partition function.

And then from partition function we can calculate the free energy and two derivatives of free energy is susceptibility; one derivative will give you magnetization and so on with respect to field. Now, since this integrals are from minus infinity to plus infinity and the only place where magnetic field enters in this e to the power minus beta E{ r i, p i}.

So, this; so curly bracket means is all of all sets of entire set r i's and p i's ok. So, the only place it enters; magnet field enters is through p i; so p i changes to p i plus e A i; A r i by c ok. Now, since this integral goes from minus infinity to plus infinity; it really does not matter if you add another term to p i because you just shift the origin of your p i and absorbed this term and because is a minus infinity to plus infinity integral; this integral will be will remain the same.

So; that means, the integral with A; this vector potential and without A, they are the same. And if they are the same; that means, the partition function has finally, no dependence on H. So, if you now calculate the free energy which is log minus k B T; log z. So, free energy is minus k B T log of z and this has no dependence on H and then if you take any derivative with respect to H which is m, which will give you 0 and that is exactly what you do in classical mechanics.

You find that the magnetization vanishes; then there is no question of magnetic moment susceptibility or anything for a classical system described by classical mechanics. So, this was actually done by worked out by Bohr and student of him Miss Van Leeuwen; this spelling is bit complicated . I hope I have done it correctly, but you can find it in many textbooks mentioned; this they mention this theorem.

And this is a very very easy proof in that there is almost no proof you just do one line and you get the answer and that is surprising that it was overlook by most people. But nevertheless, the question then is how quantum mechanics restores; the after all in quantum mechanics also you will calculate the partition function.

And the resolution of that is that in quantum mechanics you do not have continuous distribution of momentum and energy and therefore, the you have to sum over the eigenstates and then you of course. So, this integral will be replaced by sum and then it will be a completely different ball game and that sum turns out to be non zero and gives you a magnetic moment.

So, we will start right from here in our next class to start again from the previous slide which is the Hamiltonian that I have written down. So, what I will do now is just to put this Hamiltonian.

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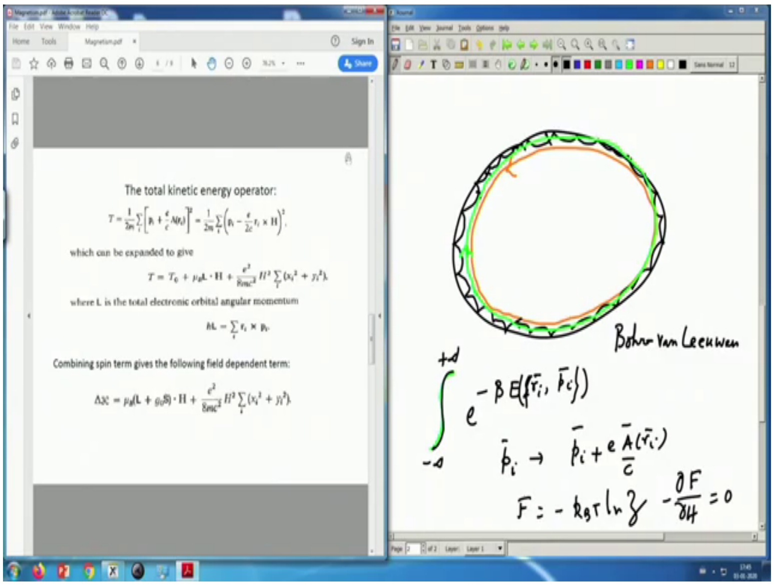

I will just go straight to a quantum description and just up to this far is nothing quantum, but after this I will start a quantum. So, this is the new; this is the Hamiltonian and this is what you get out of it; all this I will work out and we will continue from here.