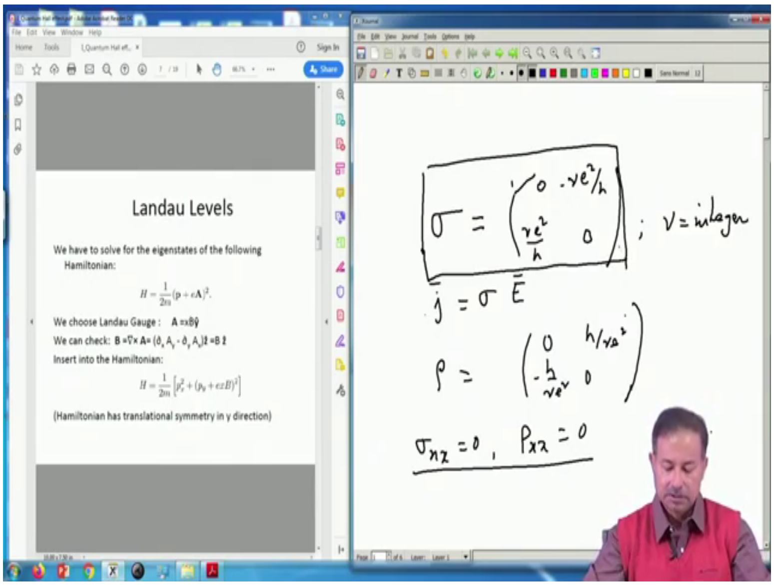
Electronic Theory of Solids
Prof. Arghya Taraphder
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture - 26
Electronic in a Strong Magnetic Field and IQHE

(Refer Slide Time: 00:28)



We have been discussing a very important effect that almost shook the physicists community, condensed matter physics community when in 1980 Von Klitzing discovered the so called quantum Hall effect. Now, in the quantum Hall effect in one word is basically observation of the conductivity tensor in a two dimensional electron gas and a strong magnetic field and at low temperatures whose form is of this type.

Sigma equal to for examples if I do not have a diagonal conductivity then this is nu e square by h and nu e square by h, where nu is an integer and your sigma is defined, you should that usual definition j equal to sigma E. So, this constitutes a quantum Hall effect measurement and observation and this is what is called quantum Hall effect.

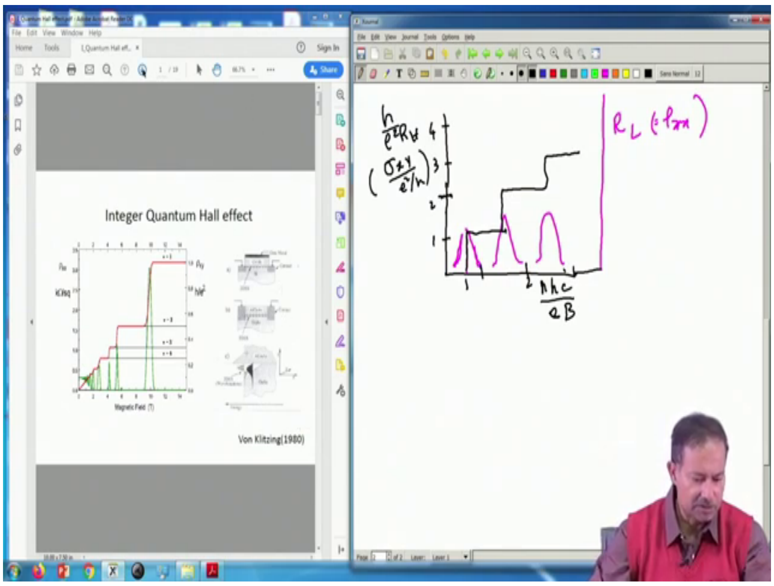
Now, why is it so different from the classical Hall Effect? In the classical Hall Effect; of course, you do not have this kind of a quantization. So, let me just show you the plot. Before that I emphasize again something that I wrote the other day as well that remember if I invert this matrix which is sigma then I will get a resistivity matrix and that will be like 0 0 h by nu e square and minus h by nu e square, ok.

Now, this, if you look at this two matrices that when rho x x is 0, then sigma x x is also 0 and similarly the sigma y y. So, the sigma x x is what is the longitudinal resistivity, the resistivity along the direction of current and see rho x x y is the resistivity along the direction of current, direction of field and current. And, sigma x x equal to 0 is along the conductivity along the direction of the field the voltage that the voltage that you apply.

So, from external sources; so, this is really remarkable, because both rho x x and sigma xx being 0 simultaneously which is what we do not expect from our class 11 12 level physics. And, the reason for that is that there is a strong magnetic field and the magnetic field changes the entire scenario of conduction and we will come to it towards the end of this lecture.

The fact that there is a strong magnetic field alters the mechanism the basically the nature of the states and therefore, the mechanism of current carrying current carrying of the underlying carriers. So, let us just go ahead and then what we said was that the, result that Von Klitzing obtained was really dramatic.

(Refer Slide Time: 04:39)



And remember this as I said this nu as well I will see in this graph, we will see in this graph for example, so, if I plot h by e square R H; e square R H, which is basically sigma x y by e square by r h, e square by h, sigma x y by e square by h.

Remember your previous viewgraph that we had sigma x y was nu e square by h, so I am just dividing that sigma x y component by e square by h while plotting and that is what then that will give me just nu, if that relation that I showed is correct and that is exactly what Von Klitzing got. And, this side is n h c is inversely proportional to B. So, the units that are used are n h c by n is the density of electrons by e B.

So, this plot is just the other way of representing this plot. This plot is in resistivity, so it is plotted against B. And, now I am plotting the conduct conductivity off diagonal conductivity and it is plotted against 1 by B inverse to B with some constants. Now, the constants are such that the units are just 1 2 3 4 on both sides. So, if you write 1 2 3 4 and so on and 1 2 3 and so on. Then this is what you will get. So, this is your, so, let me plot it in a different ink that will make it better the different color. So, let me plot it in a in a different color, ok.

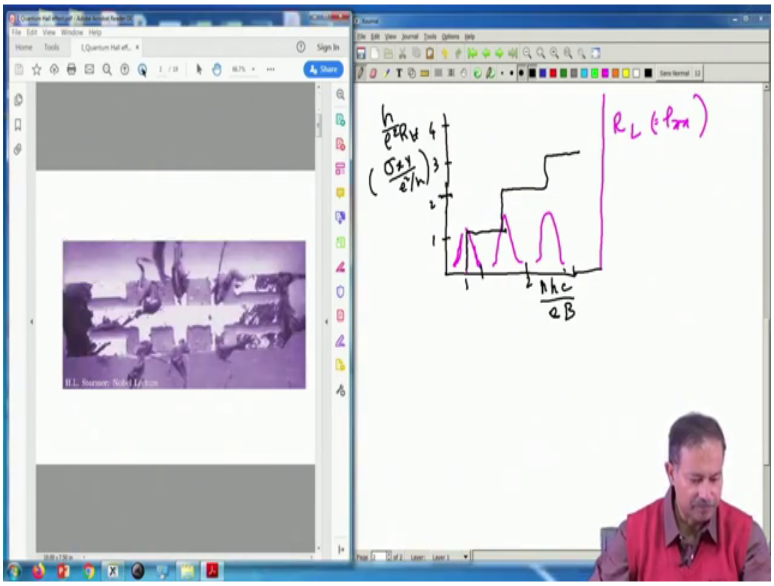
And the other one, this one will be, sorry it should be in black, it will rise up to 1 um, let me do it properly. So, this will rise up to 1 and then come straight and then it goes up becomes 2, then go straight, then three go straight so, this is your 2, for example. So, this is how the plot looks like. And, on the right hand side what one is plotting is the longitudinal resistance R L in arbitrary units which is basically l rho x x.

So, that is the, that is the graph that is actually shown on the left hand side also the black one is on this side on the left side and the purple one is on the right hand side. Now as you see the this there are very sharp jumps at certain values of these 1 2 3 and so on. So, 1 2, this is the v plot that is done on the left hand side as well.

Now, look at these numbers this 1 2 and 3 that is exactly what makes this. So, spectacular these 1 2 and 3 the plateaus where the plateaus appear in sigma x y are quantized; that means, this 1 is 1.0000 up to almost 9 decimal places it is a it is quantized up to a billion nearly a billion few parts in a billion. And this plateau 2 at 2 is where the value is 2.0000 up to nearly nine decimal places 3 is the same 3.000 to 9 decimal place.

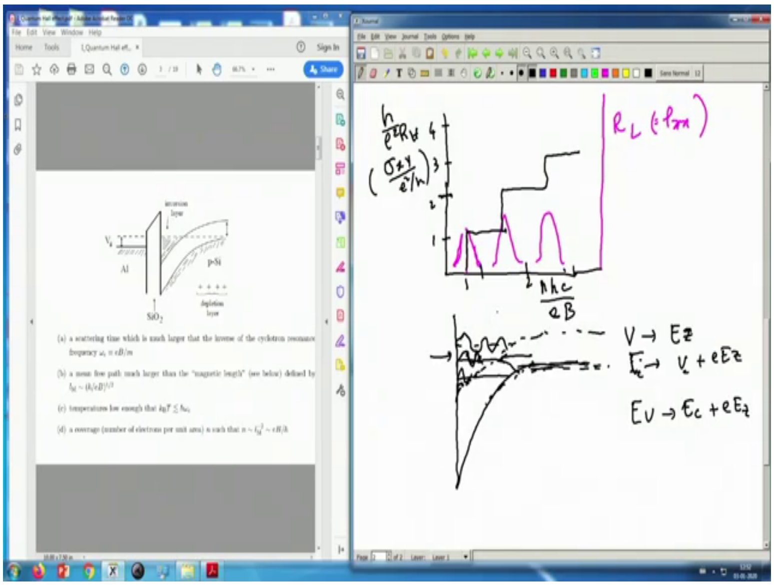
And that is really remarkable, because you are dealing with resistance, resistivities which are material properties they depend on defects and a lot of things. So, this means that this really does not depend on the material parameter whereas, in normal resistivity's resistances of course, depend on the material parameter. Now the this suggests that there is something spectacular, something which is quantized and that quantization is actually it turns out it comes from the physics of Landau levels which is what we have started doing. And so, what we did was that there was this set up by Von Klitzing and company.

(Refer Slide Time: 11:21)



And nowadays of course, you get a much cleaner and better setups, but the originals, the basic idea is this that you have to quantize your, you have to confine your electrons along one direction with a constant electric field, constant electric potential which gives you a field which is proportional to z.

(Refer Slide Time: 11:23)



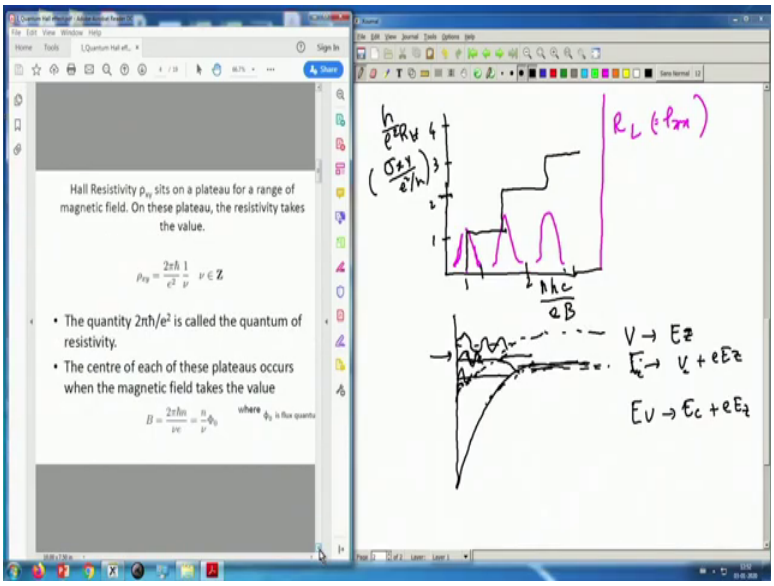
So, the constant electric field which is which is proportional to e times the z. So, the voltage will be just voltage will change to V plus. So, for example, the conduction band and I put at the E z, the conduction band and the valence band for example, there the energies of those will change to.

So, this will be E valence for example, will be given by E c plus some e E z which means there is a bending there the levels will conduction band and valence band will bend. And the conduction band bends below the conduction band bends below the valence band and then you will confine electrons within this region right.

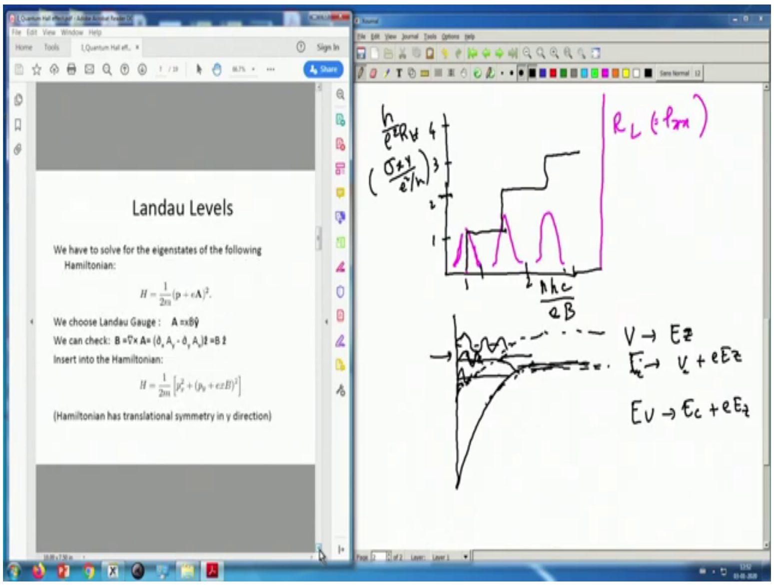
So, there will be some electrons transferred from the valence band into the conduction band. So, from here to here, because the conduction band has come down below the valence band and then you will have confinement within this region this region, ok. And that confinement means that you have this similar to what you learned for example, in particle in a box or simple harmonic motion there will be levels which are like this.

So, there will be there will be levels which are and so on, so, so, on and so forth. So, there are different kinds of they deal with many nodes. So, that is the kind of geometry we are we are in where the wave functions are more or less confined along the z direction. So, their energies are also quantized accordingly as we know from any confined confining potential, ok. So, that is what we did in the last class.

(Refer Slide Time: 14:23)

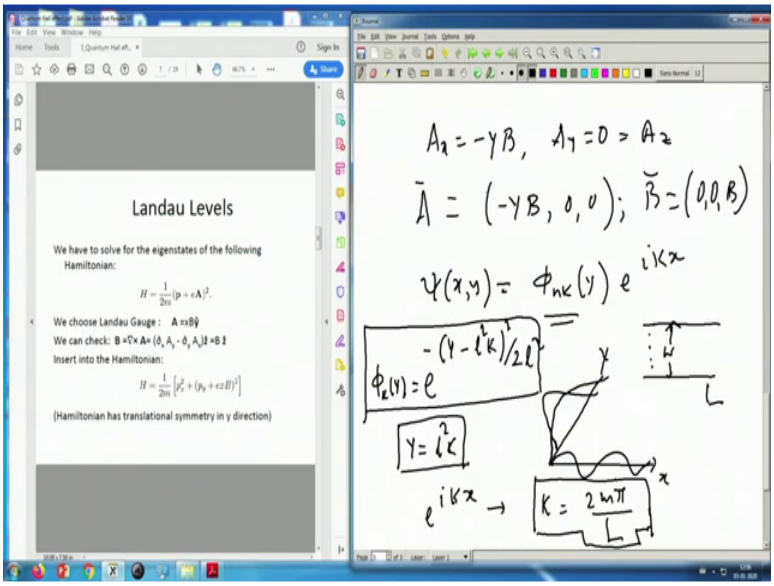


(Refer Slide Time: 14:26)



And, then we started working out the Landau problem and what we did is that we worked it in a gauge which is which was our choice of the gauge was we took the y component to be this 0 and x component.

(Refer Slide Time: 14:36)



So, we chose A x equal to minus Y B A y equal to 0 equal to A z of course, is 0. So, we do not have to bother. So, then B is along the z direction and so, the A vector was minus Y B 0 0. So, that is what remember B was 0 0, B.

And we solved the started solving the problem what we did was we solved the Schrodinger equation. And then finally, what we obtained for Schrodinger equation was something like that phi n K of y e to the power i k x and this phi n k is a Hermite polynomial whose first one is basically a Gaussian the n equal to 1 is e to the power minus y minus y minus l square k square by twice l square.

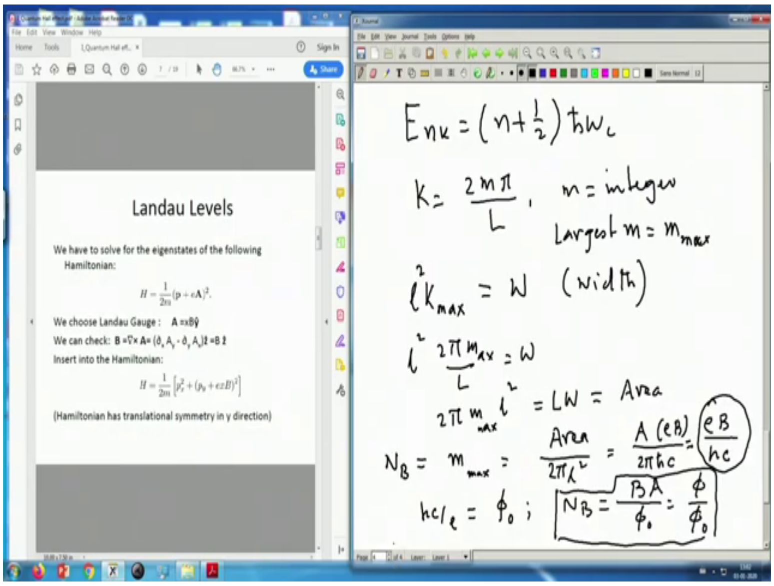
So, phi k y was this is the first one. So, this is a Gaussian basically, this is lowest Landau level and this is a basically a Gaussian wave. So, along y direction if I plot along x and y along this direction it is a plane wave whereas, along this direction we have a Gaussian which is sharply peaked somewhere.

So, this is typically a Gaussian wave function. The thing is that the center of the Gaussian is shifted from x equal to 0, y equal to 0 to a point which is y equal to l square k along the width. So, y direction, along the y direction it is shifted by an amount l square k. Now, the e to the power i k x solution e to the power i k x solution we can again use this is the plane wave.

So, you can use the born one Carmen boundary condition which will give us k equal to some 2 m pi by the width of the sample which is let us call it anyway w is the if you call it w or you can call it l, it does not matter. So, let me width it since, we use width for w let me call this l. So, if the length is l along the y direction, ok. So, this is the, this sorry this is the x direction. So, the length is l and y direction we have. So, our sample is like this that we have a length along x direction is l and along y direction we have width, width w.

And remember that y equal to l square k means that y the, core the center of this Gaussian can be placed anywhere along the y direction and we will find out how many y's we can have how many ways we can place the center and that will actually determine the degeneracy with the condition that the k has this quantization relation, right; so, along x direction, ok.

(Refer Slide Time: 18:57)



So, then let us find out what the number of degenerate states that we can have remember the degeneracy appears here n plus half h cross omega c. So, we have to find out the remember this will note that this energy does not depend on k, it only depends on n. So, for each n I have to find out how many degenerate k values exist and that is exactly what we will do now.

Now, k as I said is just 2 sum to m pi by l and m, m is an integer. So, let us assume that the largest number of largest value of m is m max. So, largest m equal to m max. So, that is exact that is my degeneracy, because I start from k equal to 1 and then I go up to m max and that is the number of degenerate states, number of states along this for number of k states that I can get. So, that means, but how, how many so, that gives a value of k max right how many such values are allowed.

The maximum k max is bounded by the fact that it has to be equal to the width, k max cannot go beyond the extent of the system, right the size of the system. So, l square k max must be equal to W it must be equal to or less than W. So, that is a maximum W is the maximum, ok. So; that means, l square 2 pi m max by l is equal to W, ok. So, I take the W on the other side. So, 2 pi m max into l square equal to L into W. Now l into w is basically the area of the system being in use.

So, m max value which is the degeneracy number of states which is the degeneracy of this for a particular magnetic field of any state of any of these energies is area divided by 2 pi l square. Remember that the area has a dimension of area l into l square this also has a dimension l square. So, this is the right hand side is a dimensionless quantity which is exactly what we should have gotten.

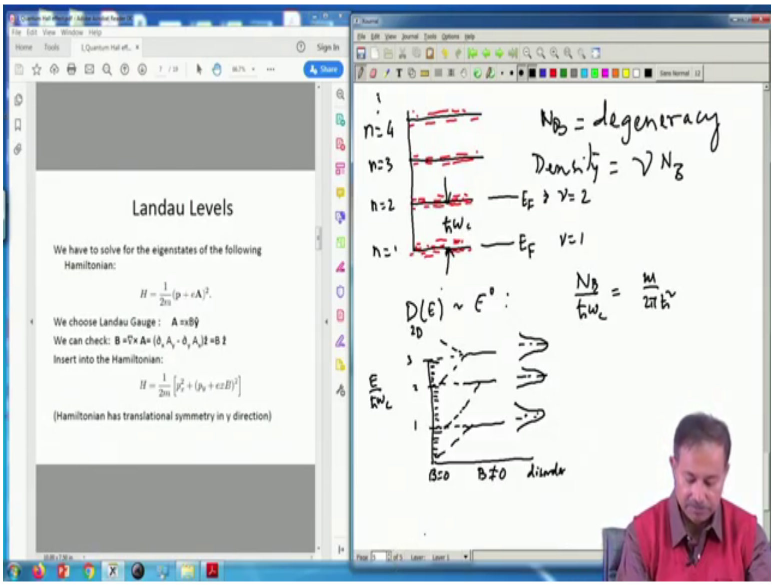
So, if you put the remember your l square. So, l square is h c by e b. So, this is area A times e b by 2 pi h c and this so, let me, so, let me just find out what was my l square l square was h h c by e b. So, this h h cross c by e b, e b by ok. So, this is basically the suppose I have a unit area then this is e b by this was h cross c so, e b by h c, ok.

So, h cross h by 2 pi, it is cancelling the 2 pi and e B by h c is my degeneracy and that is, that is actually quite interesting, because h c by e is something called a Dirac flux, this is the unit of flux magnetic flux you can have. So, this n b is then the total magnetic field you have applied from outside divided by into the area I have taken to be unity. So, in b times the area divided by phi naught which is the flux supplied from outside divided by Dirac flux.

So, it basically counts the number of Dirac fluxes that can be accommodated in that area when you put an external magnetic field. So, that is interesting, because if you increase this b then of course, you will have more and more fluxes the more and more degeneracy that you can have and so the three flux the external flux is large then larger and it you divide that by the quantum of flux which is called the Dirac flux.

And, then that number is the degeneracy of the levels this is also equal to E B by h c it is an interesting exercise that you can calculate that the if you have an uniform distribution.

(Refer Slide Time: 24:44)



So, this was like this we plot it right E's are just delta function and each of them are hugely degenerate. So, that degeneracy is there is a large number of states here. Each of them has the same number of states which is at all, all of them are at the same energy and the number of states is N B equal to degeneracy, ok.

So, this is this number is fairly a very large and so, when you put electrons in the system then you can fill up all the states and; that means, your first Landau level n equal to 1 is filled up or you then you change your chemical potential or the Fermi energy you hit here then you will fill up all these states.

So, your density will be just the so, the density is just a number times the N degeneracy. So, if your Fermi level is here if you have you have n equal to 1, nu equal to 1 Fermi level is here then your nu equal 2 implies nu equal to 2 and so on and the number of electrons then is nu times n b. So, that is the whole idea of Landau quantization Landau level quantization of the electron moving in a two dimensional plane with a cross magnetic field in a cross magnetic field.

So; that means, you are if the moment you change your gate voltage for example, or your magnetic field depending on how you want to change your magnetic field will change the degeneracy. So, suppose you have a certain number of electrons. So, and n b has to match that number of electrons if you are, if you want to fill that entire lowest Landau level.

So, you can change your b and then you just make n b equal to n and then nu equal to 1, then n b equal to 2 then the nu equal to 2, n b becomes higher then you then you have to and you change your gate voltage you come here you have to fill in two of the Landau levels. So, nu it becomes 2 and so on.

So, you can actually control this basically you Fermi level jumps from one to the other and then the degeneracy also can be controlled by changing the magnetic field. So, that is how this whole thing works. So, these, these distances are h cross omega c, right., what you can see is that if the if you had the remember the original this thing the that in density of states in 2 D, in 2 D is independent of energy, right is e to the power 0 that you can actually have here also.

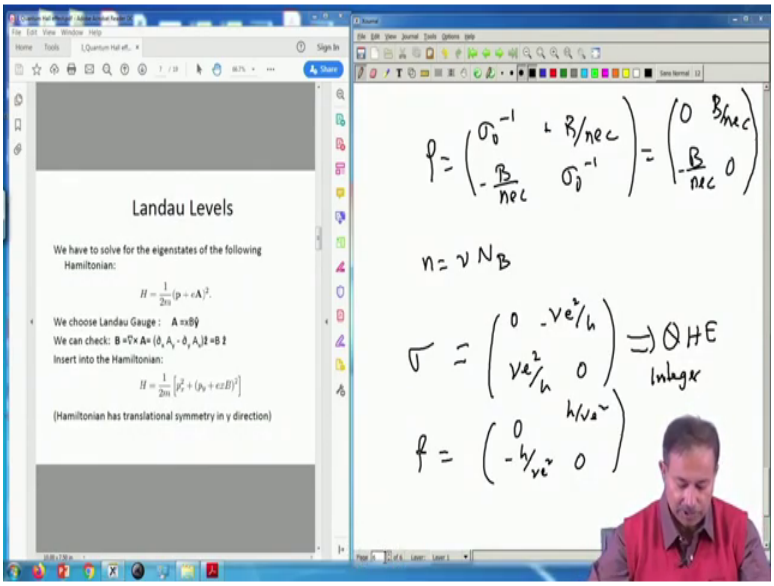
So, if you divide by n b by h cross omega c which is the suppose the these levels are broadened out like in a two dimensional electron gas free electron gas without a magnetic field then you can easily check that this is m by 2 pi h cross square, the same result that we had for 2 D electron gas without there is no dependence on e and this is the pre-factor m by 2 pi h cross square.

So, the other way to, to represent it is that you had this these hugely degenerate states 3 E suppose I plot E by H cross omega c then all if you have some gives some if you are B equal to 0, then you have these are all occupied, the all these states are occupied. And density of states is a constant whereas, when b is non-zero then of course, you will have these states all these degenerate states will converge to just one level. And so, this is B not equal to 0.

So, these become now hugely degenerate all these states have gone here all these states have gone here all these states have gone here and so on. So, then the point is that if you now have disorder then of course, these levels will just become a bit extent there will be a width to the each of these levels. And that is actually what happens in real system there no matter how pure your system is there will be some disorder always.

And, this is what will happen the center will be of course, at the value given by the Landau level values and n plus half h cross omega c, but they will just extend a bit and there is a disorder and in two dimension disorder does something interesting as I said some of these states will become localized and so on. So, we will not go into that complication. So, this is this happens when the when the there is a little bit of disorder which is inevitable in any real system that you consider. Then of course, let us just understand what is our quantum Hall effect its now very simple.

(Refer Slide Time: 31:24)



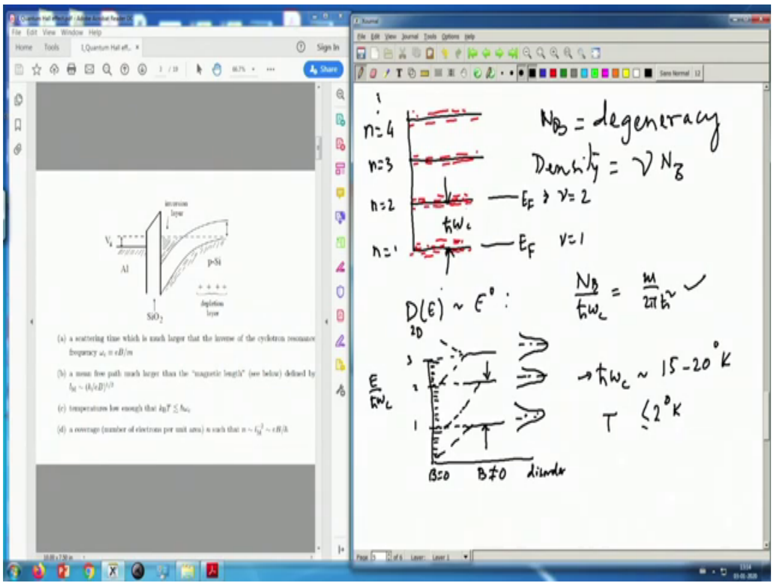
Remember, rho was sigma 0 inverse n e square tau by m inverse minus B by n e c sorry, B by a e c minus B by n e c and sigma 0 inverse. And, if you have no diagonal conductivity then just like write this B by n e c minus B by n e c 0, ok. And now what you do is that right n equal to nu times N B, because now you have a quantized level.

So, you have you can you have to fill up either the first one the second one or the third one depending on your chemical potential and if you put that in then just, just do this calculation and see that your rho will be equal to 0 nu e square by h sorry this will be sigma. So, rho will be h by nu e square and this will be sigma, sigma will become 0 minus nu e square by nu e square by h 0.

Remember the first slide that we, we started with see this is the sigma that we have obtained by doing this calculation Landau level calculations and that is exactly what we have achieved, ok. So, and correspondingly rho will be 0 0 h by nu e square minus h by nu e square.

So, this is the idea of quantum hall effect and this is the integer version of it, because these nu s are integers. So, this is called integer quantum Hall effect and that is what Von Klitzing had observed and the typical see remember the constraints we had was that you had to have large magnetic field. So, why do you need that for two reasons there are these four constraints that I wrote down the other day remember that four conditions.

(Refer Slide Time: 34:15)



Yeah, scattering time has to be very big much bigger than the cyclotron time, because this if the becomes very large you cannot have then they will overlap. And, you have this going back to the old this kind of a density of states which you do not want then the quantization will be lost.

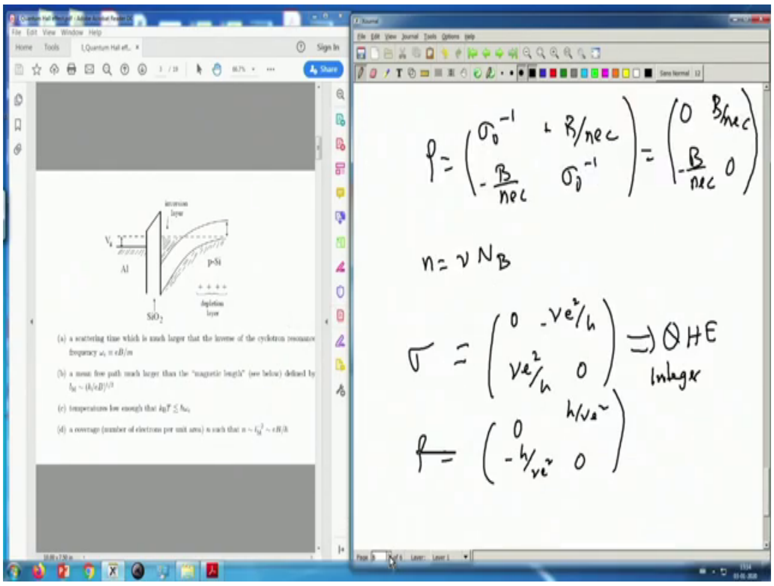
The mean free path has to be very-very large compared to the magnetic length which we already discussed. And that is again saying that the there be very little scattering of the electrons affective disorder will be less, temperatures have to be low. Because, these energy differences h cross omega c h cross omega c is typically at about say 20 tesla magnetic field is about 15 to 20 degree Kelvin in terms of temperature.

So, if I convert this energy difference into temperatures then this is about 15 to 20 degree Kelvin or even sometimes less. So, you have to be careful that your temperature should not be comparable to these values. So, your temperature that, you work with should not supply an energy which is more than few milli electron volts that means, your KBT even less than milli electron volts. So, it is T is of the order of typically considered 2 degree Kelvin or less.

So, that is the typical energy the temperature at which you have to work. Nowadays, people work for, for the for the other quantum Hall effect that I said fractional quantum Hall effect even at a much lower temperature. So, these are the conditions you that you have to satisfy to get your to see your quantum Hall effect this implies that your magnetic field has to be large, you can increase this by increasing the magnetic field.

So, its typically 20 30 tesla magnetic field that people use temperature is at less than 2 degrees or even 1 degree or even less. And, electron densities are also have to be reasonably large and that is ensured in by choosing the samples correctly the fed geometry and the gate voltage and so on.

(Refer Slide Time: 36:50)



So, that is under that condition you will land up with this is this phenomenon Spectacular phenomenon called fractional quantum Hall effect called integer quantum Hall effect and that is exactly that summarized in this sorry about this. So, this is rho equal to this is something wrong coming from the software, ok. So, that is the summary and that is what happens in integer quantum Hall effect.

So, spectacular remarkable phenomenon that fascinates physics community for over 3 more than 4 decades now.