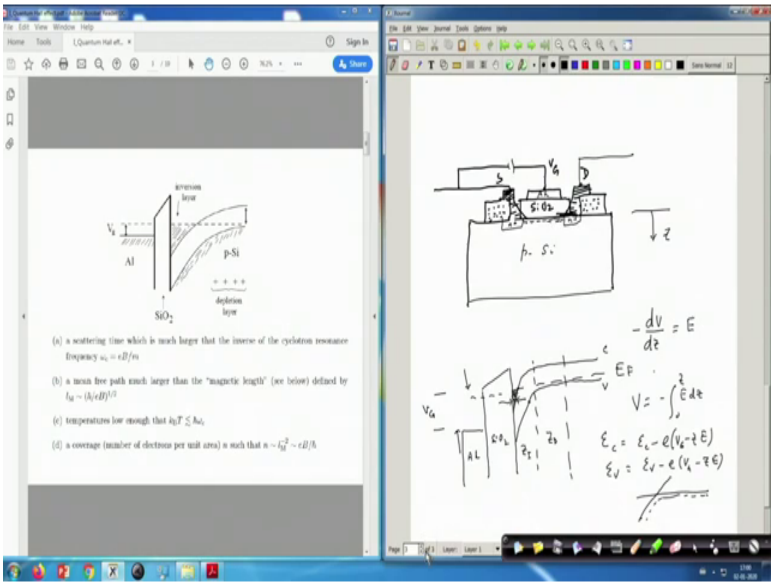
Electronic Theory of Solids
Prof. Arghya Taraphder
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture – 25
Integer Quantum Hall Effect Continued

Hello.

(Refer Slide Time: 00:23)



We have been discussing quantum hall effect which happens in a two dimensional electron gas at very low temperatures in a very clean system and under a perpendicular magnetic field which is fairly large. Typical magnetic fields are 20 Tesla 30 Tesla and so on and temperatures are as low as 1 degree or even less 1 degree Kelvin or less.

So, what I said was that the electrons first have to be confined and confined in one direction so, that they become two dimension the carriers become two dimensional carriers in they are moving only in two dimension. So, let me show you how it is done. So, the geometry of this device is this that let me draw it there is a MOSFET geometry MOSFET configuration and there is a source and there is a drain here sorry the ok.

And there is a silicon dioxide layer SiO 2, these are also SiO 2, these regions are also SiO 2 then you have this source and drain these are the source and drain. These regions are source and drain and so, what you do is that, you apply a here and then on top of this you put a aluminum metallic aluminum and this is these acts as the gate.

So, this is the v gate that you apply here. And in this case this is positive respect to this ok. So, this is the configuration from here on drains out. So, this is the typical [construct] and this is p type silicon. So, because of this configuration you will have a n plus layer here, n plus region here and there is a confinement of electrons the carriers in this region just below this SiO 2 layer and they are confined I will show you why and how they are confined.

So, the axis is such that the down side is z. So, this is my aluminum gate ok. If I draw the energy diagrams in this system; so, let us just draw it this is my aluminum gate, this is the SiO 2 layer and then there is a region and it is called inversion region and there is a depletion region z d which is like this and what you are doing is that let me just plot first and then I will show you what I mean ok.

So, I have this valence band bends and the conduction band which is like this and this is where the Ef is ok. So, this is somewhere the Fermi energy is. Now you put the gate voltage. So, there is a vg at here in this here at the aluminum. So, that you can now move this Fermi level up and down or other way to think about it is you can move this conduction and valence bands up and down.

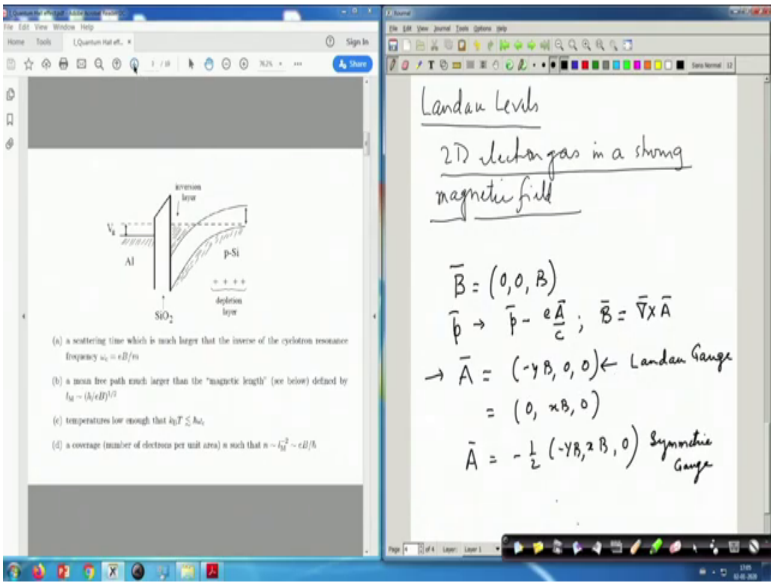
The better way to look at it is that the we move the Fermi level up and down by doing this. Now the valence band since the since this region of the conduction band has come below the Fermi level and it is a region of potential. So, why does the potential look like this? Earlier the before I applied a gate voltage the potentials were where straight horizontal lines this c and v were horizontal lines whereas, now what has happened is that minus because of the electric field coming from the gate, you can you have to solve this equation this is equal to your electric field.

So, there is a field inside that field bends these bands and so bending happens because this electric field is constant and your v then is v equal to 0 to z, see at 0 the value is the same as vg then E times dz ok. So, what you get is Ev Ev Ec then goes to Ec minus e times VG minus zE and Ev goes to Ev minus e times v minus v means VG at 0 it is VG minus zE.

So, that is; that means, there is a proportional to z change. So, the whatever potential was there suppose this was the potential and now you apply another one which is like this. So, the net effect is a potential like this ok. So, that is the way the bands are bent and then electrons get transferred from here to here and these are the electrons which remain confined within this distance ok.

So, that confinement is what has to be achieved within few nanometers or so, depending on the other parameters of the system. So, that is how the confinement works ok.

(Refer Slide Time: 10:36)



So, once the confinement is achieved, the rest is basically very simple we have to solve something called a Landau problem. So, these are called Landau levels and it is basically 2D electron gas so; that means, the it is confined in one direction which is z here for us electron gas in a strong magnetic field.

Strong is required for the experiment, but landau problem you can do it any magnetic field. So, that is the problem that one needs to solve now ok. See the B is basically a vector which is 0 0 B only the z component exists and its value is B. Now we know how to solve the Schrodinger equation for such a situation. How does one do it?.

The way to do it is that see the other thing that one assumes is that the spin degree of freedom is already frozen because the magnetic field is fairly high so, the spins have all aligned along the direction of the magnetic field. So, we will not bother about the spin part of the Hamiltonian that will not give us any new result. So, then the only the orbital part is necessary.

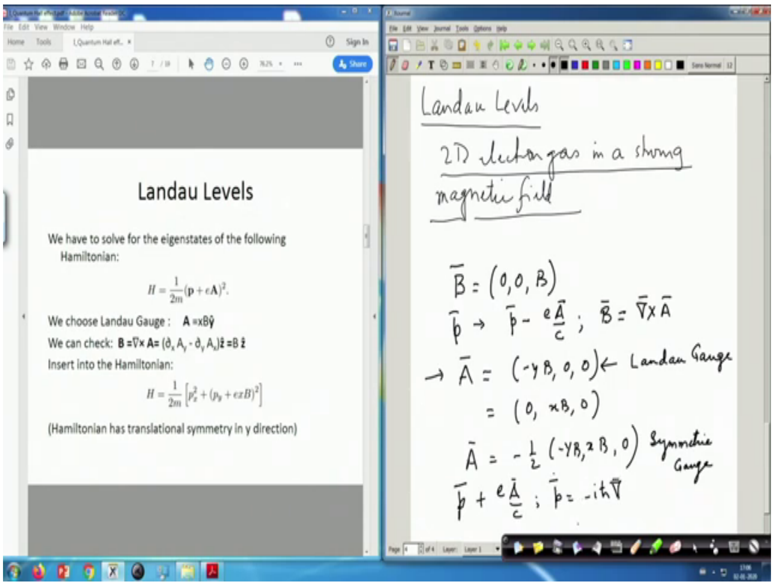
So, p we know under magnetic field goes to p minus eA by c right. So, e here is of course, negative and the other thing that we have to use is B where B is equal to curl of A ok. So, this is the vector potential that we have all done in our electrodynamics and p minus A by c is the canonical momentum in presence of a magnetic field. So, in this we have to just incorporate into our theory.

So, for that we need to know what A is; A of course, has we remember there your electrodynamics you can have a choice of [A] which is you can choose in several ways there is a freedom of gauge choice. So, one choice which is conveyance is. So, A you can choose as minus yB 0 0 or you can choose in another way 0 x times B and 0.

So, both the choices are perfectly all right this is called the Landau gauge. There is another way you can also do is something called a symmetric gauge which you can choose as a combination of both keeping both of them a for your A. So, symmetric gauge for example, is minus half minus YB x B 0. So, this is Y times B Y times B x times B and 0 of course.

So, this is called the symmetric gauge you can work in any gauge you like and different books of different notes you will find work in different gauges and we will not discuss much of it. We will choose one particular gauge which is the first one that I have written down this one and we will stick to that.

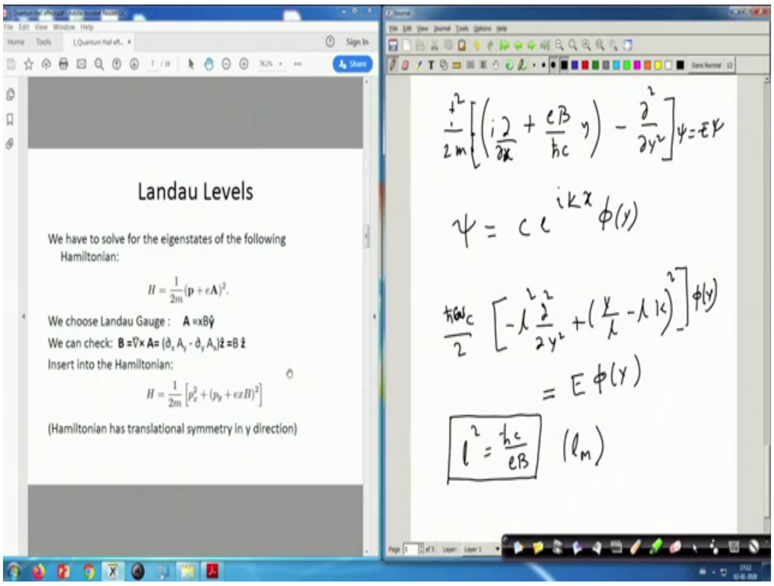
(Refer Slide Time: 15:45)



For example, this one chooses this example that the this viewgraph chooses the other gauge and that the y component is to be x x times b ok, but it does not matter because the results do not depend on the choice of gauge, the final results will not depend on the choice of gauge.

In fact, the symmetric gauge is also a very popular choice and the algebra becomes interesting in that. So, if you do this, then just replace the way to do is now the charge is negative. So, p plus eA by c for p you replace minus ih cross grad.

(Refer Slide Time: 16:51)



So, if you do that then of course, you will get back your Schrodinger equation that is what Schrodinger equation we look like now. So, let me just write down the result that you will get in the gauge that I have just informed h cross square by twice m into i del x plus eB by h cross c into y minus del 2 y del del 2 y. So, let me just write del del x instead of del x and del 2 del y 2 times psi equal to E psi ok.

So, that becomes your Schrodinger equation remember this z component is just a free fermion because its a the sorry the z component is confined and in that direction you do not have a motion of the electron. So, you are only concentrating on the x and y direction and that is the way this thing is set up ok.

So, for example, remember the Hamiltonian still as this says in our case the Hamiltonian has translational symmetry in the x direction ok. So, if this is the Hamiltonian, then you can choose a wave function you trial choice is not difficult to make, it is some constant e to the power i k sorry about it, e to the power i k x some phi times y.

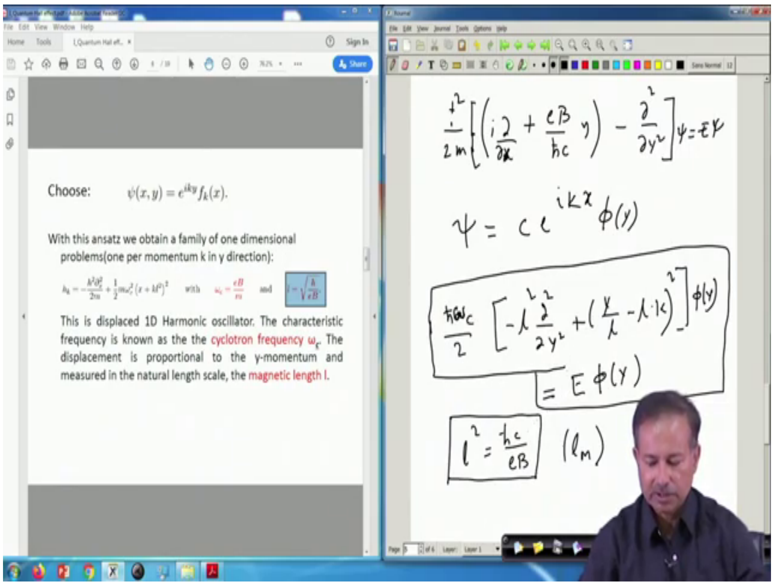
This is called the separation of variable you have done it in hydrogen atom problems or in many other cases. So, what one does is that one chooses a plane wave function along the x direction because the translational symmetry exists in that direction still as and so, one keeps that intact and right a plane wave state along the x direction whereas, in the y direction of course, you have to be careful because there is a term which you will change the nature of the wave function and what will happen you will just see.

So, they then you can we just put this solution in put this trial wave function in this and what you will land up is in is h cross omega c by 2 minus l square del 2 del y 2 plus y by l minus l k this is that k square into phi y, this is h cross omega c equal to E phi y. So, this is what you will get where l is l square is h cross c by eB.

So, that is called that is the magnetic length, this is the also written generally as l sub m to make it to explicitly mention that there is this is a magnetic length. It as you can see that these h cross c and e are all fundamental constants, and it is only inversely probe l square is inversely proportional to B. So, this defines a new length scale in the problem ok.

It is interesting that from the algebra in a magnetic field transverse magnetic field a new length scale has now emerged in the problem and that length scale is dictated by the strength of the field the stronger the field will be if you increase the field the length scale decreases as a square root.

(Refer Slide Time: 22:23)

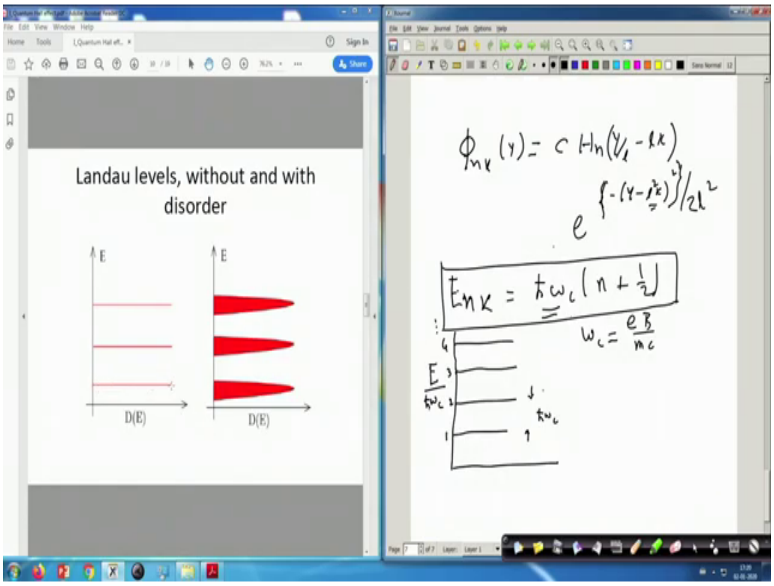


So, that is the there here also you can see on the this that l is l is the c has been taken to be 1 in this calculation, but the as you can see the result does not change in which gauge you work in and the length is h cross c by eB this is the magnetic length. Now if you look at this

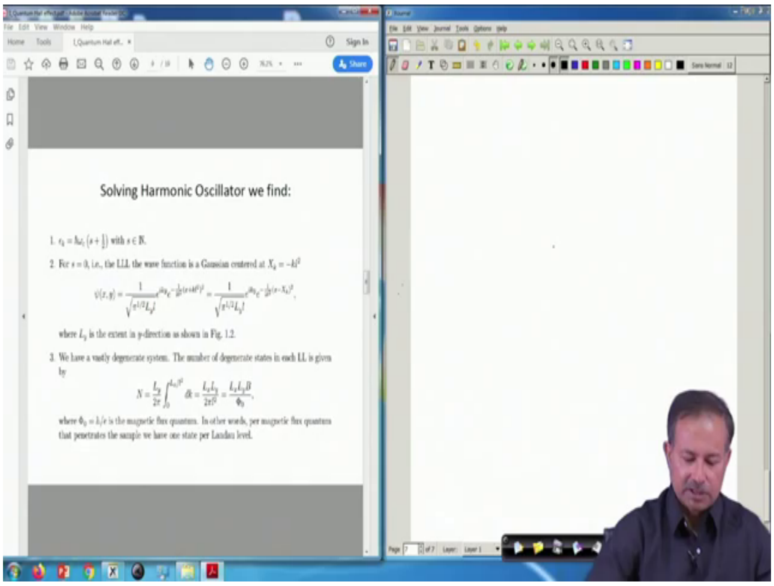
Hamiltonian that you got this equation h psi equal to e psi, the left hand side represents a simple harmonic oscillator in one dimension suppose this k was 0 then this would be just a simple harmonic oscillator right.

So, that tells us that this problem has an interesting symmetry that it becomes a simple harmonic oscillator problem in one direction and a plane wave in the other direction. So, the solution then appears to be can be written off and remember that because k this term is nonzero, the basically the center of that the oscillator has just been shifted from y equal to 0. So, that is the amount of shift l square k. So, the center is away from 0 by amount l square k.

(Refer Slide Time: 24:17)



(Refer Slide Time: 24:18)



So, one can write down the solutions and solution is basically a simple harmonic motion in one dimension simple harmonic motion in y direction and a plane wave in the x direction for my choice of gauge. And for this one for example, its just the other way its y along y direction, they have a this has a plane wave and along the x direction it has a displaced harmonic oscillator that is all.

And I purposely chose this example just to show you that two gauges do not differ in their final result the solution remains the same only the x and y gets interchanged. With the minus sign plus to minus as you have seen the choice of gauge also involves a sign. Now what does this lead to? Well this leads to.

So, wave function of the form phi n k what this n is I will tell you equal to some constant, sum this is a Hermite polynomial remember your simple harmonic motion y by l minus l k e to the power exponential minus y minus l square k square by twice l square. So, this is the one can write it as e to the power then if you can see the similarity clearly that this is a e to the power minus y minus l square k.

So, I had this part not been there, this would just be e to the power minus y square by 2 l square which is a simple harmonic oscillator with origin at 0. Well now you have the origin shifted by an amount l square k. So, that is the; that is the solution and the energy this n comes from this sorry about it is n k is actually independent of k h cross n plus half and this n is an integer omega c as before is eB by mc.

So, h cross omega c is the scale of energy, I mean this is exactly like a harmonic oscillator. So, the harmonic its like there is this zero point energy at n equal to 0 you will still have a finite energy as happens in any confined potential for example and oscillator potential is a confining potential and h cross omega c into n plus half has the same is the same form that you get from the from the oscillator.

So, that means, that the energies are quantized in this form that energy as a function of this is density of states written, but if you write the other way which is. So, let me just draw it the way we normally draw E is basically a series of lines and which is which are separated by an amount h cross omega c and they are the separation is constant goes on and on.

Now this x axis let me just keep it for the time being but this is the just the energy you look at the energy and this is how it turns out the result tells you that this is how discreteness of the energy as just like a harmonic oscillator. The interesting thing is that this is the way if you just divide this by h cross omega c then these will be 1, 2, 3, 4 and so on.

Now if you have B equal to 0 then of course, this will not happen this will be just the two dimensional case where we know that the two dimensional free particle case right with the density of states which is constant ok. So, this plot on the left for example, gives you the density of states as a function of energy and as you can see that there are fixed energies at which there is a finite density of states.

The density of states calculation is important here, because the number of states here depends on the degeneracy of this level. So, that degeneracy needs to be calculated and this that will do in the next class where we will try to understand the quantum Hall effect in terms of these degeneracies of these levels and this level diagram.

The right hand side is actually the picture that you really get in real life in real systems this is of course, the mathematical theory the Schrodinger equation solution gives you on the left hand side one, but real systems always has disorder imperfections. So, the relaxation time being very very large almost infinite cannot be true and then there are always scatterings and that scattering will develop will lead to some width of each of these levels.

And that is the real life picture, but even then if your h cross omega c our magnetic field is fairly large, then the width can be much smaller than the gap which is h cross omega c and you can still treat these as discrete levels with a of course, a large degeneracy which I will calculate in the next class and that leads to the quantum Hall effect the integer one.