Electronic Theory of Solids
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Lecture - 24
Integer Quantum Hall Effect (IQHE)

Hello and welcome again. So, we have been discussing low dimensional systems.

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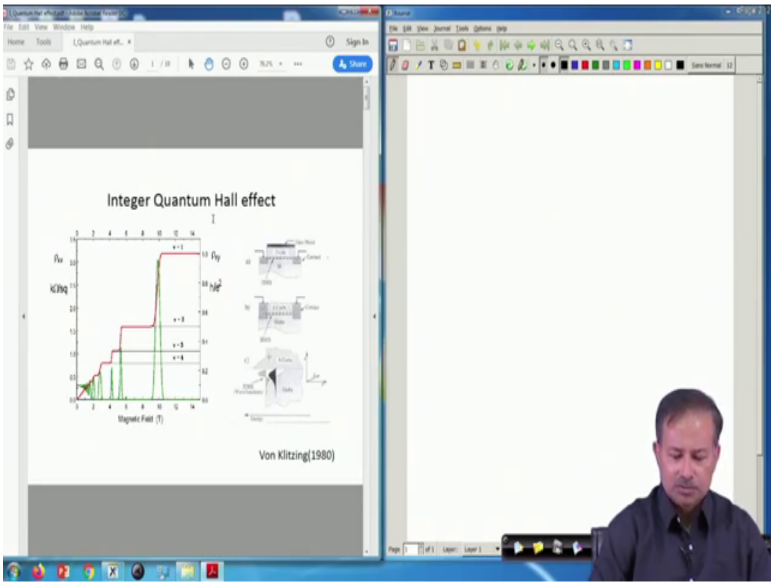

And, as I promised there is this extremely interesting discovery that actually led to several Nobel prizes that involves the two-dimensional electron gas. So, let me first tell you what the discovery is, it was called Quantum Hall effect. Now, what is Hall effect is something that we have already done. And what is quantum about this?

The quantum about this is really spectacular in that as you can see from this picture, the in a two dimensional electron gas under strong magnetic field at low temperature. These conditions I will come back to what was found in 1980 by Von Klitzing was that there is a there are this plateaus the red one. Look at the red ones there are these plateaus in the in rho xy and they correspondingly when there is a jump the, the plateaus the, the rho xx rho xy jumps from one to the other.

And at that point rho x x which is the diagonal conductivity the diagonal resistivity also jumps, but in the during the plateau the diagonal resistivity remains 0. Now this plot is with respect to rho xx in kilo ohm per square remember in two dimension. If you remember your R equal to rho L by a then you will find that a which is the area and L have the same dimension. So, this becomes independent of the area and kilo ohm per square is the[unit].

So, resistance and resistivity have the same dimension. So, it is generally expressed in terms of kilo ohm, but kilo ohm per square area. So, in two dimension that is the convention whereas, this rho xy as I will show later has a dimension has a and it can be expressed in terms of two fundamental constants which is h by e square.

And so, the rho xy is expressed in this unit and these nu's are the values of the quantization. See, these are the numbers number density of the Landau levels which I will again discuss, but basically what you have to look at now is that, that there are these plateaus which are extraordinarily quantized, I mean that is the real these are look at the rho xy value it is exactly 1 here and this value is really.

So, if you plot sigma xy for example: this will be 1, this will be 2, this will be 3 and so on; so, , these, these values one here in rho xy one half and so on. These are quantized to almost few parts in billion. So, these numbers are not approximate numbers.

So, that is where the surprise was, because if you remember resistivity is the property of a of a material and the material can have defects, can have impurities, can have any other things. In fact, if you keep the same material exposed to air for two days, you will see that your measurement or resistivity has changed. Whereas, here the value of the soft diagonal resistance or resistivity is extraordinarily fixed and it is fixed to integer values which is this the inverse of that is in integer value and that that integer value is integer up to a few parts in a billion.

So, that is why this thing for example, if you know the value of h to a high precision then from these quantization these values of rho xy you can actually find the value of e square and; that means, e. So, the application of that was first thought to be in metrology which is the standard bearers for measurements and units and so on.

So, so, like we have in India, we have the national physical laboratory keeps our standards the measurements standard measurements. And, similarly the internationally there are agencies which keep the standard and it was thought that this, this discovery will be very important in finding the finding new ways to, to fix fundamental constants. Now the constants like a electric charge and so on.

Now, the thing that is interesting here is that the discovery although it opened up many new avenues for applications and all that the main attribute of this is fundamental physics. And this physics is so extraordinary that we should discuss it a bit, ok. The, before you going there let me just in introduce you to where it happened and then I will come back to Hall effect and then I will show you how this is done.

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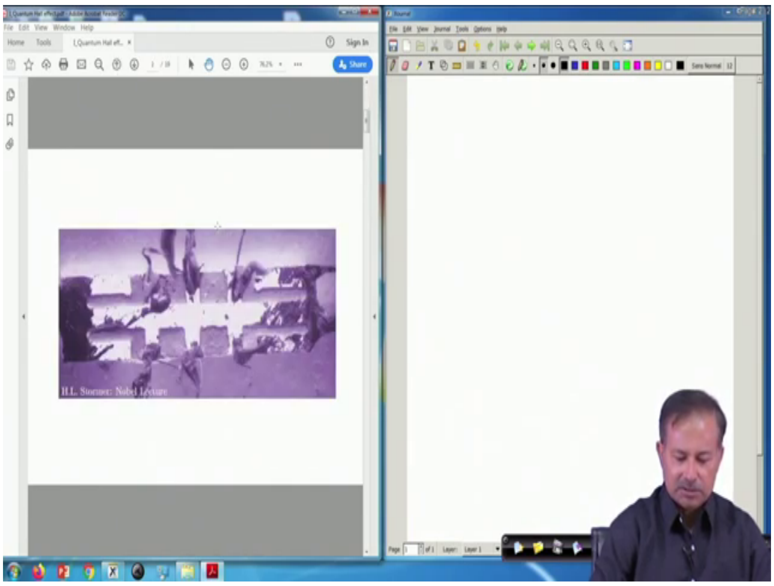

So, this was the original device in which this, this say this was done its from Stormer's Nobel lecture.

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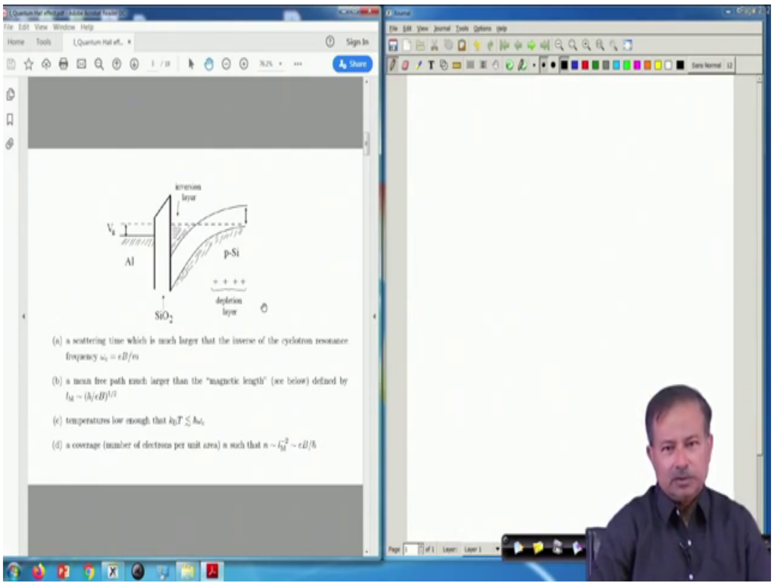

So, this is how the system worked, there was a in a metal which was aluminium below that there is a silicon dioxide layer and there is a p type silicon after that and you confine the electrons in these shaded regions. So, and this shaded region is, the inversion layer and here what is happened is that the we you can see that the nature of the potential is such that the potential cuts off this electrons movement along the along this axis along this direction.

And; that means, the, the carriers are confined within the region from this solid line to this up to this potential after that it just decays. So, there is a confinement of the, carriers below this SiO2 layer within a very narrow region. So, and that region depends on what potential you are applying. I will show you how to, how to do it and this is physics is very simple to do this, but of course, technically it is a challenging problem to keep the electron to keep the carriers confined in this region.

Now, what are the requirements for this these to, to show up the, the system the where the electron moves which is this region must have a very clean sample. So, this region must be defect tree as much as possible. So, that the scattering time must be much higher than the so, this scattering time should be the relaxation time which is tau that we have used must be very-very high. So, that there is almost little scattering from defects and so on.

And the scale for that is it should be much larger than the resonance frequency. So, it is called these cyclotron resonance frequency inverse of that see cyclotron resonance frequency is the inverse of cyclotron, cyclotron resonance has a frequency and that frequency inverse will give you a time and this time the glass session time must be much larger than that.

Now, the mean free path should be very large and it should be much larger than another scale in the problem, we will we will find out what the scale is, it is called a magnetic length and so, the mean free path has to be much larger than this. The temperature has to be very low now again how low it is the means in a region where h cross omega c appears to be the gap of this spectrum and the temperature should be such that this energy scale corresponding to the temperature should be lower than h cross omega c.

So, this is this is the condition and of course, the number of electrons per unit area should be of the order of eB by h cross all these numbers lm h cross omega c and so on., We will come out of the Schrödinger equation that we solve for an electron two dimensional electron gas in a perpendicular magnetic field. This is actually called the Landau problem, Landau was the first who worked out this thing this calculate worked out this problem and found that the energy levels are quantized, we will we will do that as we proceed.

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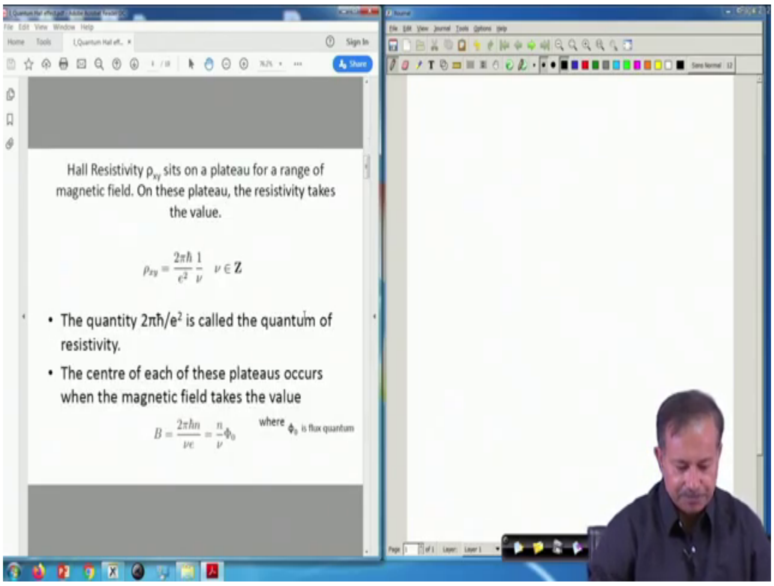

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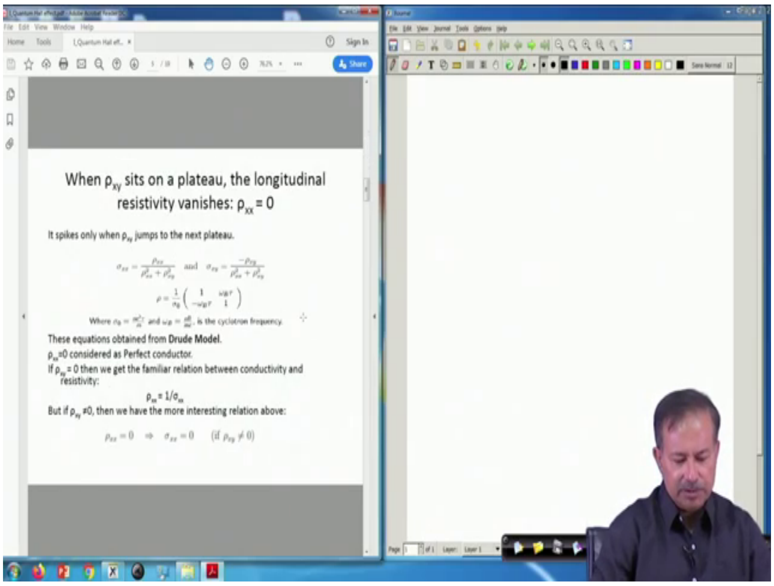

So, I will digress since I will be using the formulas for Hall effect and all that I just quickly digress back to Hall effect and let me just show you what we did in Hall effect and that is.

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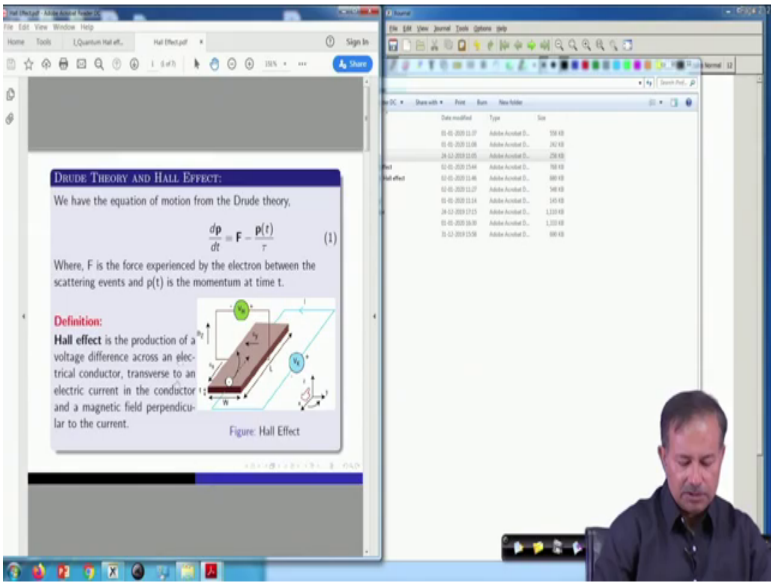

Here remember, your Hall effect diagram this was the geometry. So, you had a, had a sample and a perpendicular magnetic field and then the electrons move here of course, this is three dimensional, but there is a sufficient thickness of the of the sample, but suppose the thickness of the sample becomes exceedingly small then you are approaching a two dimensional limit.

So, but of course, for Quantum Hall effective require more than this. So, let us just understand the Hall effect a bit again this was the geometry in which we said that there will be hall potential developed across this sample, ok.

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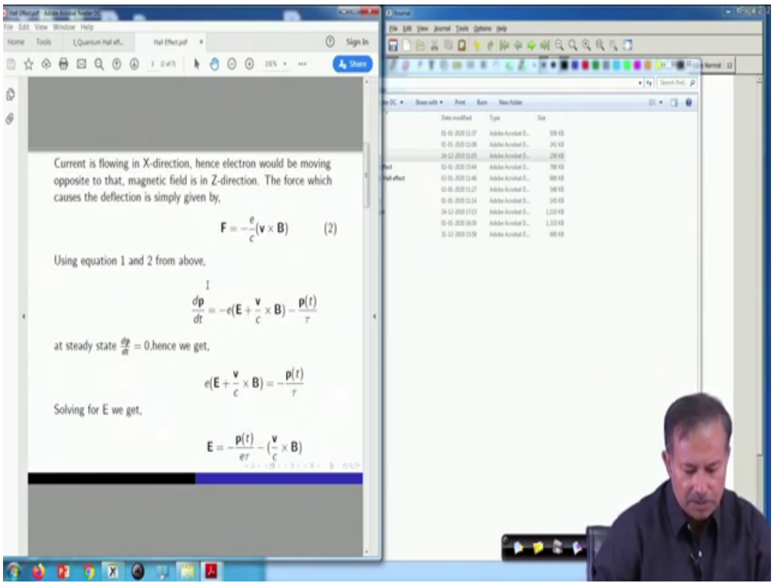

So, what was what was the equation that we, we solved. It is simply this again the, this is the classical Hall effect I mean simple classical old, old Hall effect of the 19th century. So, this today formula is what you used under the force of crossed magnetic and electric field, ok.

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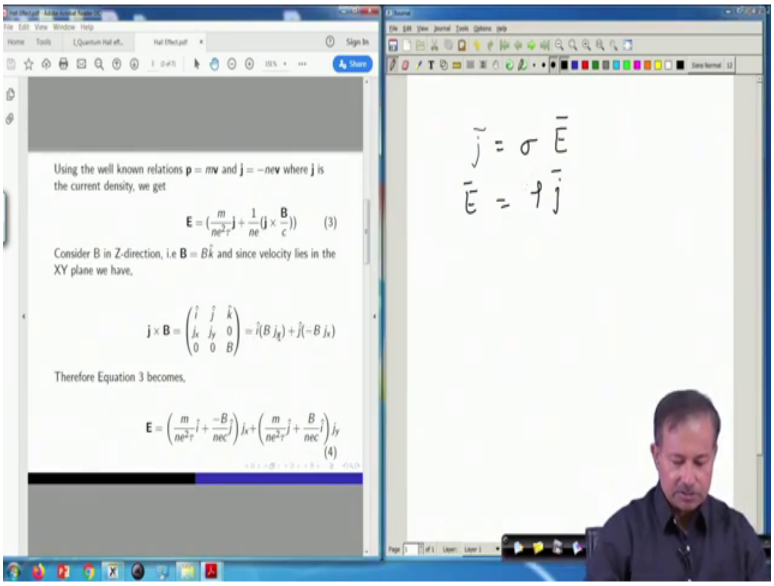

And, we landed up in getting an equation for, for electric field in terms of the current density j x and j y. So, the so, E is remember E and j are connected by a by j j equal to sigma E. So, E equal to rho times j, that is how you can define the rho. So anything that connects E on the left hand side to, to j on the right hand side that object is the resistivity. So, let us see what comes out of this calculation.

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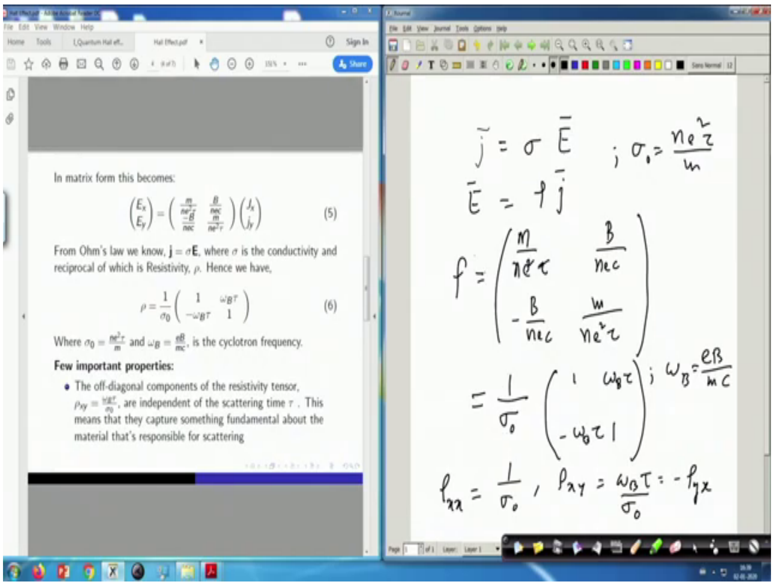

So, this is what you get. So, E which is ex ey to components is a matrix times the j x and j y. So, the rho matrix is therefore, remember any square tau by m. So, let us write it m by n square, tau, tau is the relaxation time B by nec B is the magnetic field minus B by nec and m by ne square tau. So, that is your rho now what has been done in equation six is that just m by ne square tau has been taken out as and then you can write this as 1 by sigma naught. Remember our Drude formula sigma naught used to be ne square tau by m, ok. So, that is so, that is why this is one by sigma naught and if you if you do that then you will get one 1 omega B into tau minus omega B into tau.

So, rho xx equal to 1 by sigma naught and rho xy equal to omega B tau equal to minus rho xy rho y x,. So, this is the formula that we obtained for resistivity. So, resistivity is no longer a scalar quantity, it is a matrix and that matrix couples connects the current density which is two component here jx and jy, two electric field which also has two components one component is the applied one.

And the other one is developing because of the motion of the electrons away from the direction of magnetic the direction of electric field due to the magnetic field. So, that is e y the transverse field that generates, ok. So, this was what we did. The one thing that you should notice is that the rho xy is omega B tau divided by sigma naught. Now this sigma naught has a tau in it. So, this will cancel the tau that appears here right. So, that; that means, the, the off diagonal resistivity is independent of the scattering time, ok.

So, that is interesting because this then becomes independent of the property of the system right, because omega B is equal to eB by m c c is a fundamental constant m is the mass of the carrier and B and e are B is applied externally e is also a fundamental constant charge of a of an electron. So, this quantity rho xy becomes a fundamental kind of quantity which does not depend on material properties.

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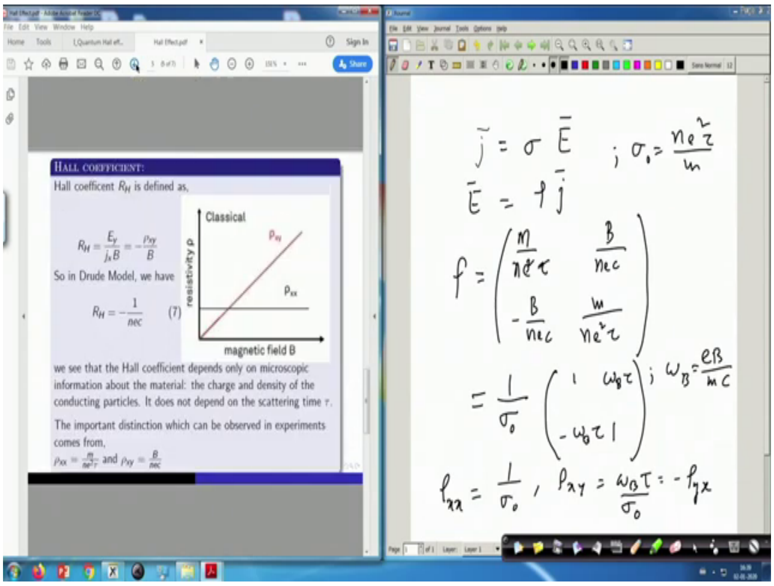

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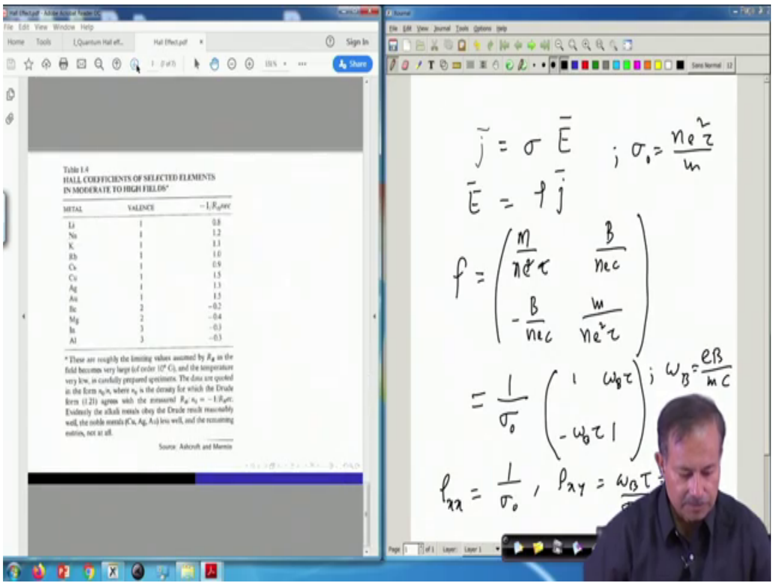

So, this was the classical Hall picture that we got. So, let us now close this.

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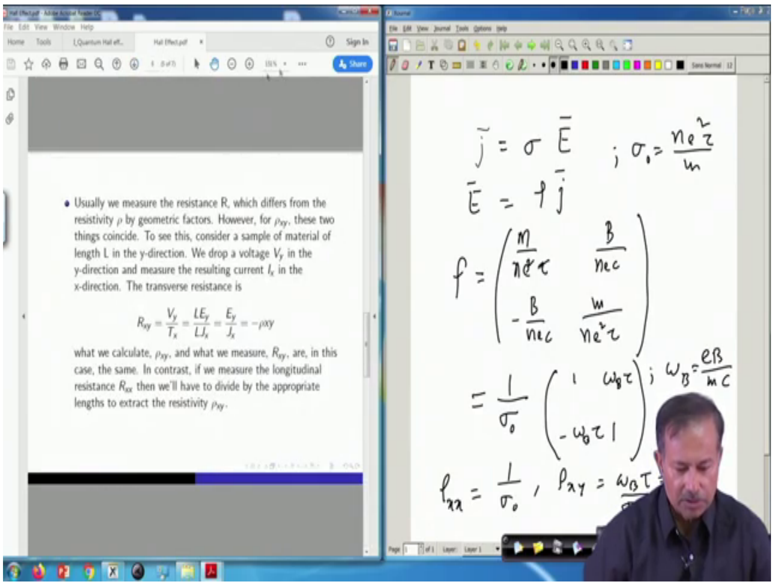

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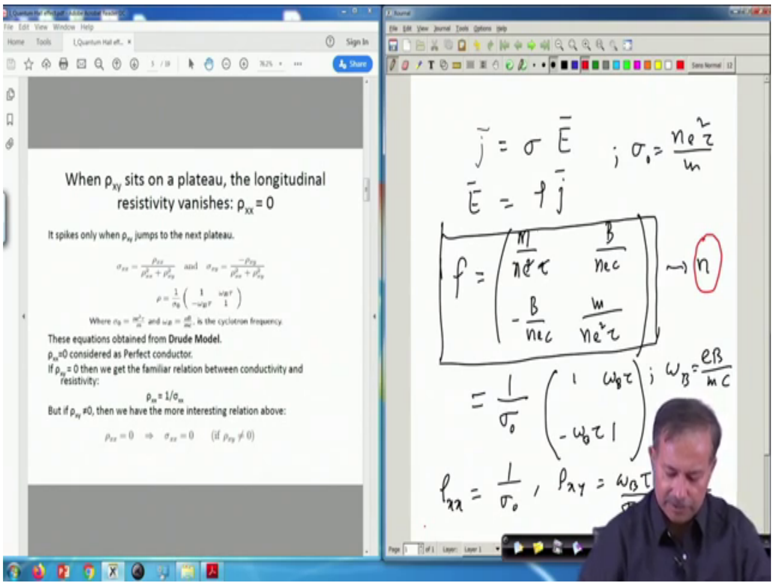

And we go back to the quantum, quantum calculation which is which actually is very similar to the classical calculation up to this far we will do the same thing as you did in classical mechanics. This formula works even in quantum mechanics.

So, there is nothing quantum over this formula the thing that is going to affect in quantum mechanics going to be affected is this n the n in qm is the culprit that gives us the, the all the properties that a quantum hall system that I showed in the beginning that a quantum hall system has comes from the quantization of these energy levels. And, therefore, the n is a I can show that there is a n is multiple of some number times and integer. So, let us proceed there is a interesting caveat that is here.

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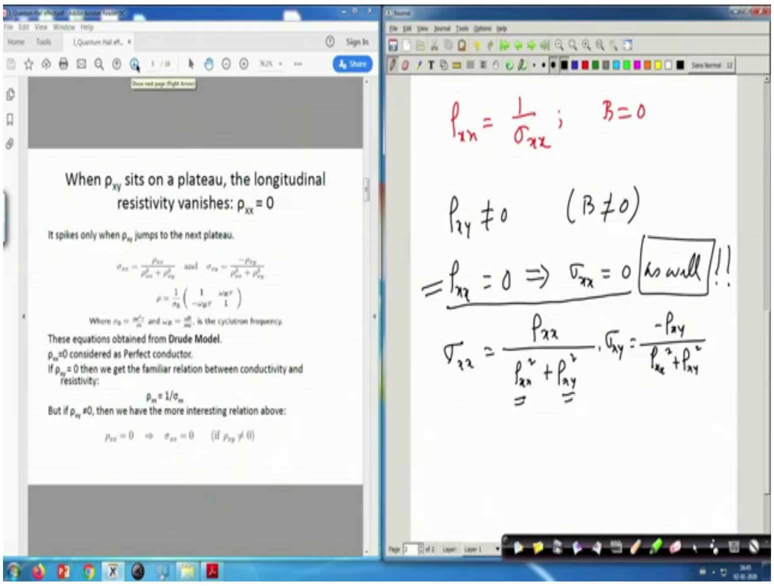

Remember that if you have a scalar relationship remember sigma x sigma rho xx equal to 1 by sigma xx, if you did not have B equal to 0, ok. If you had B equal to 0 then this relation goes through. So, then, you remember this is one, I am saying suppose there is no off diagonal component B by nc is then 0.

And then of course, then rho x x goes through as 1 by sigma that goes through even here, but that is now and there is no rho xy. Then whereas, what is interesting here is that when B is when b is non-zero; that means, your rho xy is non-zero, x y not equal to 0 which is b not equal to 0.

Then you see what happens then you have rho x x equal to 0 implies rho sigma xx equal to 0 as well. This is really interesting here for example, in the top relation for B equal to 0, if sigma if rho x x goes to 0, sigma xx will be have infinite conductivity whereas, here and the reason is very simple that if you look at this expression for sigma xx it is just the you have to just invert the matrix and then you will get rho x x by rho x x square plus rho xy square, ok.

And sigma xy equal to minus rho xy by rho this the denominator comes from the determinant plus rho xy square. So, that is what exactly what this says this one says that if you set rho x x equal to 0 then sigma xx also goes to 0, because sigma xy is non-zero sigma x rho x x is 0. So, the denominator is finite where the numerator becomes 0. So, this implies that rho x x and sigma xx can simultaneously be 0. So, this is really remarkable, this says that in presence of a magnetic field you have a situation where both diagonal conductivity and which is longitudinal conductivity and the longitudinal resistivity can vanish together can become 0, if one becomes 0 the other will become 0.

So, that is remarkable and when does that happen well it we will see when that happens that happens, because again the, these sigma rho x x vanishes because there is no dissipation in the system it is there is absolutely no dissipation in this in this system whereas, sigma xx going to 0 means there is no conductivity which means that there is a gap in the spectrum.

So, these two things come from two different sources, but it is remarkable that compared to a single, compared to a situation where you do not have a magnetic field which makes the resistivity or conductivity a tensor you have to rethink about your physics in a way which is nontrivial already even at the level of classical mechanics, ok.

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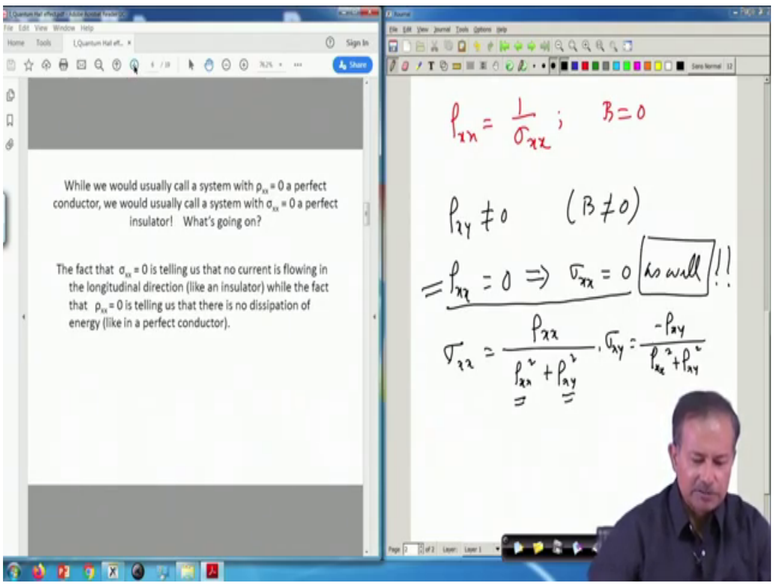

So, let us now go forward and see what we; so, this is what you said. In fact, that sigma xx is equal to 0 is telling us that no current is flowing in the longitudinal direction like an insulator which means there is a gap in the spectrum.

Whereas the rho x x is equal to 0 is telling us that there is no dissipation of energy like a perfect conductor. So, it is a perfect conductor no dissipation, but it says it has is sigma x x equal to 0, because there is a gap in the spectrum which is like an insulator.

So, that is, that is really remarkable and that happens without any introduction to quantum mechanics or anything it just, because of the presence of b. So, b magnetic field does something fundamentally different brings in new fundamentally different physics, ok.

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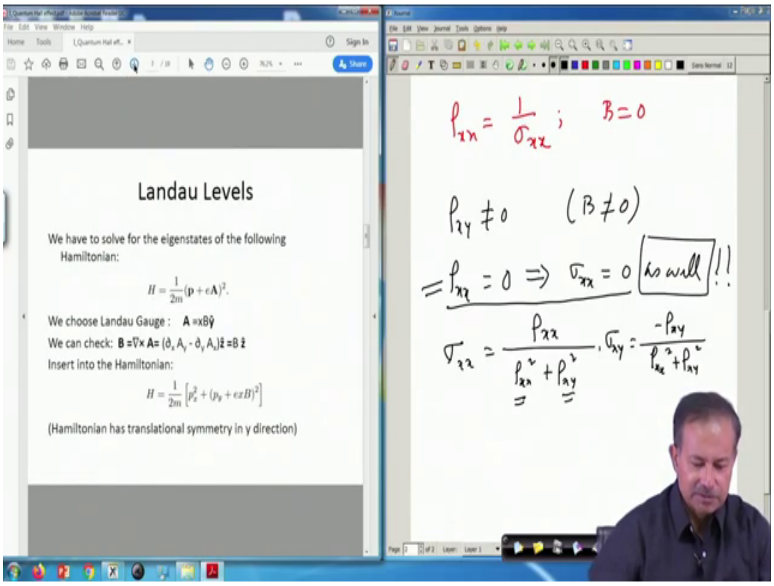

So, let us now try to understand the problem, but before that let me first show you how the, the confinement is achieved in these systems.

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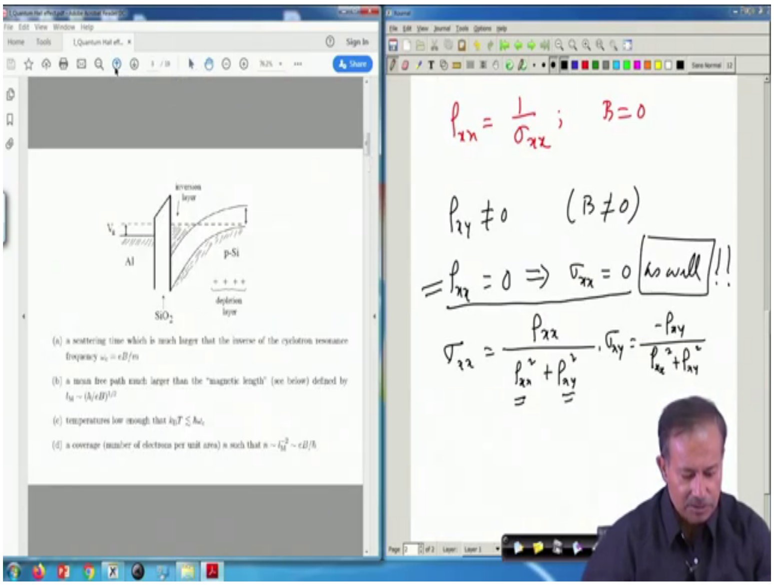

So, let me go back to this picture and I will show you how these carriers are confined in this geometry. Now, let me repeat this confinement means that just below the silicon oxide layer there is a small region few angstroms or few nanometers which is where the electrons are confined typically few nanometers and in and that region has to be small enough, so that the, is less than the mean free path there the this length must be lower than any other length scale in the problem which one on length scale which is the magnetic length scale which is there. And, the other important thing is that the scattering time the relaxation time must be very-very large. So, that means, there is very-very little disorders catering from the disorders in this system, ok.

So, I will show you in this next lecture as to how to get this confinement and then we will proceed to, to work out the Landau levels and the physics of Quantum Hall effect in the integer Quantum Hall effect. There is also a counterpart of this where these plateaus occur at fractional values of some definite fractions with odd denominators, because nowadays you also get even denominators. But mostly odd, odd denominators and these are called fractional Quantum Hall effect and that is another remarkable area of physics where huge amount of efforts are being paid.