Electronic Theory of Solids
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Lecture – 23
Low Dimensional Systems

Hello and welcome what we will start today is a slightly different topic, in the sense it has become extremely important in the last couple of decades or more that we normally called call a subject called nanoscience; which is about electronic properties of systems at a nano scale.

Now, some of these examples we have already highlighted in my previous talk, that in the previous lectures; we discussed graphene carbon nanotubes and this kind of things and these systems are actually nano systems in the sense; that they have one dimension which is confined. So, in that sense, they are also nano systems.

But, in general by nano system once may one means, systems which are which have a size of less than typically 100 nanometer or so in all directions. So, carbon nanotube is for example, a nano size system one can make a small graphene sheet also a nano sized graphene sheet that will also qualify as a two dimensional nano system.

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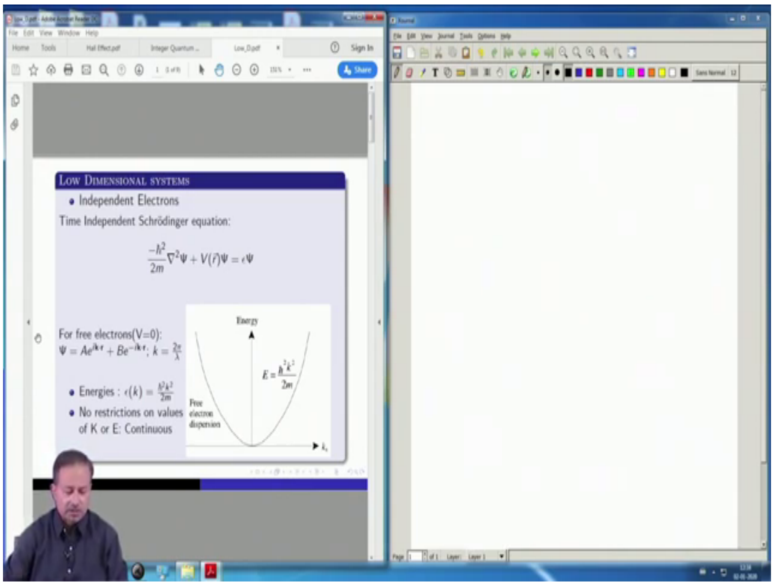

The reason that these small systems are important are many fold and they if you look at their applications they are now applied in the almost every branch of science including biology of course, chemistry they have a huge application and in electronics and physicsmainly physics applications or devices applications. And the name quantum device for example, is doing the round and these are systems which are also used for quantum devices.

The advantage of nano system is that they are their bandgaps can be manipulated and the bandgap depends on the size. That is one aspect of it, the other aspect of it is that; they are reaction capabilities. Sometimes, they are used as catalysis. And the reaction capability becomes very large because there is a large surface to volume ratio.

So, let me just illustrate these this section which I call a low dimensional systems not necessarily always nano systems, but it is low dimensional. Which means that it is not a 3 dimensional system, bulk system. These are these have dimensions lower than three. So, let me recap the whatever we have learned in quantum mechanics, because our my approach will be of course, from the physics side the reactions and chemical applications are of are extremely important, but that is a different story. So, we will confine ourselves only to physics applications.

So, let us just try to understand, why is it that the low dimension systems behave the way they do. So, this is the Schrodinger equation time independent Schrodinger equation that we all know how to solve under different conditions in most cases we cannot solve them in analytically, but certain fortunate cases like harmonic oscillator like coulomb potential particle in a box these kind of situations one can easily solve this problem analytically completely analytically ok.

Nevertheless with the power of computers today solving this kind of equations in any geometry is not very difficult. It is a linear equation, so it is and it is solved in almost any geometry that you like to solve it in. So, the simplest thing is a free electron. Where you have your potential 0 and states are like plane waves and then you know that the spectrum is p square by twice m and p being h cross k its h cross square k square by twice m.

So, the there is also in since it is completely free that k has no restriction right. So, there is no boundary condition that you have to apply that the wave function vanishes after a certain beyond a certain region and so on.

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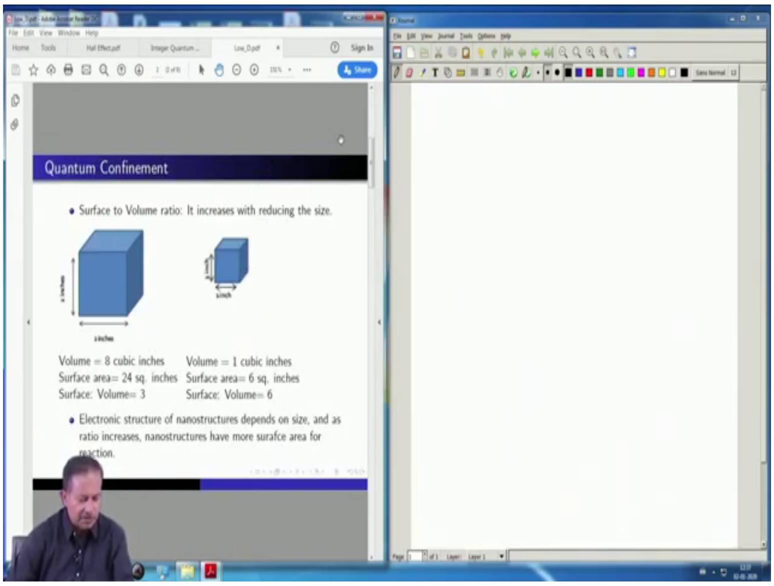

So, those conditions are not there. So, that k takes all possible values from anywhere to anywhere zero to infinity and so on. Now, so k is basically continuous. Whereas, if you have a confined system, we call it quantum confinement because, the Schrodinger equation is for a quantum system, quantum particle and quantum particle confined in a particular material whose size at least in one direction is very very small.

Now, this very small is a qualitative term sometimes you really need to understand what this small is and this small depends on your what your applications are and what kind of confinement you need for that particular application and so on. So, suppose this is a very simple example of surface to volume ratio. So, as you clearly see that the; this surface to volume ratio basically increases as you decrease the size of a system clearly right.

So, for example, volume is here if it is two inches on this one is for example, 2 inches on every this length is 2 inch and every side has a; side has a breadth length and height 2 inch. And so, this is 8 cubic inches, this is the volume of this object. Inch of course, is an American style American still use inches, but nevertheless whatever is this eight cubic whatever unit you want to write you can write centimeter also. But, suppose this is the 8 cubic inches. So, whereas, the surface area is 6 surfaces 4 square inch each, so it is 24 square inch. So, surface to volume ratio is one is 3.

Now, look at just reduce it by half of course, you know what will happen. The volume is now one cubic inch the surface is surface area 6 cubic inch and the surface to volume ratio is 6. And simply that volume changes by a cube power of 3 whereas, surface changes by a power of 2. So, volume changes much faster than the surface.

So, as you lower the size and that is exactly what has happened by. So, it changes by one length scale here you need a one length scale which is a half the size. I doubled the surface to volume ratio. So, and you can go on doing it you can make it one fourth then you will have 4 times and so on and so forth.

The interesting thing is of course, these have strong and important applications in chemistry because chemistry and some physical applications like adsorption and phenomena that depend on the surface reactivity in chemistry for example, adsorption capability. Because, the surface is where they adsorption mostly takes place. So, these that kind of applications required the large surface area. So, that is one side of the applications, one side of the story.

The other side of the story is that; they there is a quantum mechanical effect due to this if you come down to very small length scales. Again that very small we have to be clear about what it means. It means that compared to the electronic mean free path, the size should be comparable. I mean that is how a quantum confinement is generally defined. So, see if any dimension becomes comparable to the electronic mean free path then you should be in the region regime of a nano system or a confined system.

So, let us just. So, there is no rigid definition as such it also depends on what applications you want to do and so on. So we will show you some slides later on as to how for example, in optical applications what is the typical sizes that you require.

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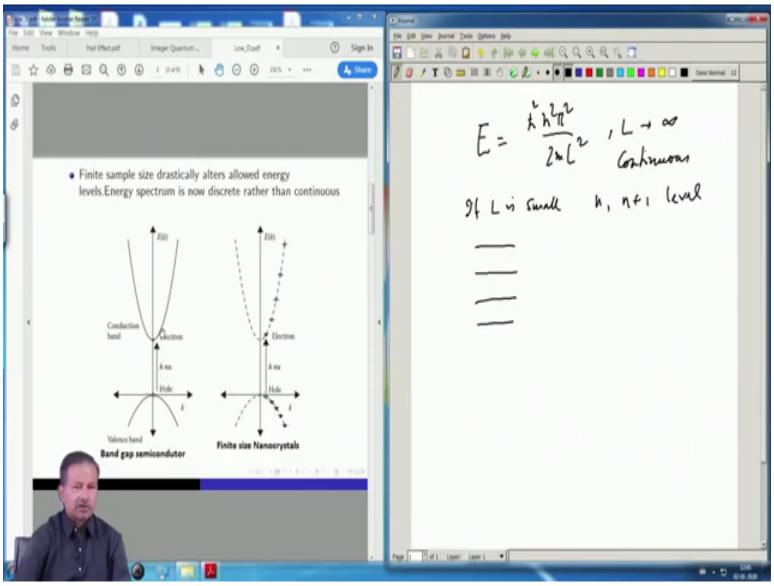

So, the next slide basically tells you the; what happens quantum mechanically. So, finite sample size drastically alters allowed energy levels as we know from a particle in a box case. For example, if you make the box size the energy for example, is n square pi square by 2 m L square. So, this kind of energy n is an integer for example, this kind of energy for example, if your L becomes very large L is say almost infinity then of course, you have the gap between states is exceedingly small.

So, it is like a continuous right and that is what we said when we said to a free particle it is like a continuous spectrum all k values are allowed. Whereas, if L is small then of course, the nth and n plus 1th level the separation becomes larger. And you can easily calculate it and that separation basically goes as its energy scale energy itself goes as one over l square. So, the that the separation will now increase from a semi continuous to a discrete spectrum that is exactly what we have seen in particle in a box case.

Now, that is that has an application I mean that is very interesting because, if you are doing a if you are for example, a normal semiconductor in bulk system you require two bands one band is full the other band is empty and require a gap. But, in quantum systems just reducing the size brings in gaps between states. And so you do not even a metal can behave like a semiconductor if you can bring its size down to a very small size. At least in one dimension you can always get a confined spectrum I mean discrete kind of spectrum.

And so, this is the example that shows that the electron here there in the left hand side is a continuous spectrum and electron can on optical excitation for example, the electron can move up and it can move to any of these states right starting from anywhere here.

Whereas, in this case it is a nano sized system and the same physics applies here, but you can only go from one set of discrete states to another set of discrete states. What it normally vertically goes up and the so; that means, there you do not have all choices of energies will have discrete choices of energies where you can excite an electron.

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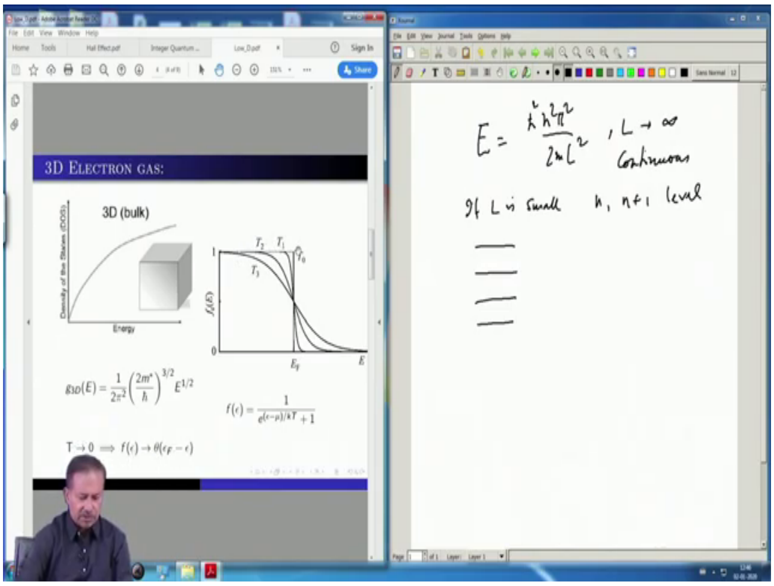

Now, we know that the 3 D electron gas of course, we have studied so far. There the density of states goes like this. It is like a square root of energy and the Fermi function of course, does not depend on the density it just depends on these function this function and the temperature and it looks like this. So, these are the two things that are necessary to calculate properties or electronic properties of a system. At T equal to 0 the Fermi function is like this T 0 is the this is the vertical drop at Ef.

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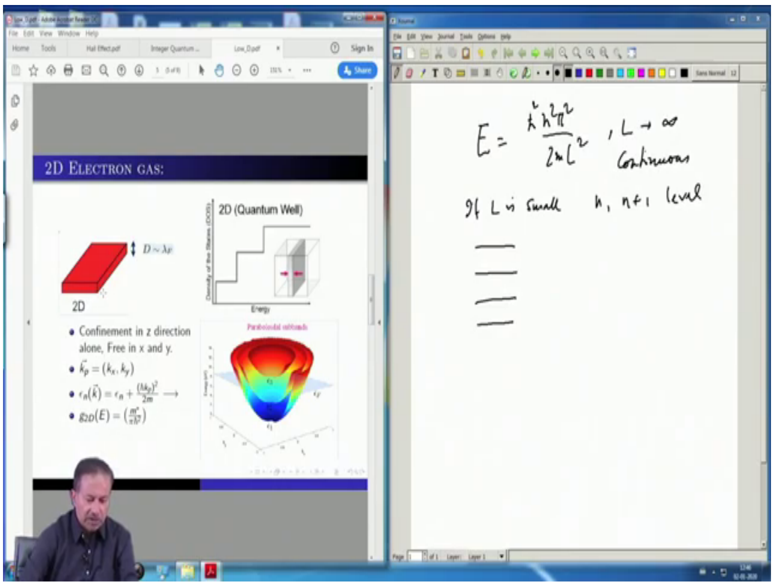

Now, these two things are all that you need, density of states and the Fermi function to calculate electronic properties. And let us now look at what happens in a low dimensional system. For example, this is a 2 dimensional system. Why is it 2 dimensional? Because, the dimension in this direction in the z direction for example, the vertical direction is very small and its of the order of mean free path of the electron that we are considering.

So, for example, that a typical scale for that is the wavelength of the electrons at the Fermi level for example. So, if that is what your dimension is comparable to that then of course, you are in a regime where one direction the direction vertical direction here is confined. But it is still free in x and y direction. So, it is like a free particle along kx and ky.

So, the spectrum is h cross square kx square plus h cross square k y square divide by 2 m. Whereas, there is a in the z direction of course, there is a confined part to the spectrum which is a discrete spectrum. So, that is what is being shown here and the density of states in 2 dimension is actually independent of energy. This is what we have we had done and that is what is being shown here.

For free for a k square spectrum this is how the energy goes in 2 dimension a density of states goes as a function of energy in 2 dimension ok. This is an example of a 1 dimensional system. Now, why is it 1 dimensional?.

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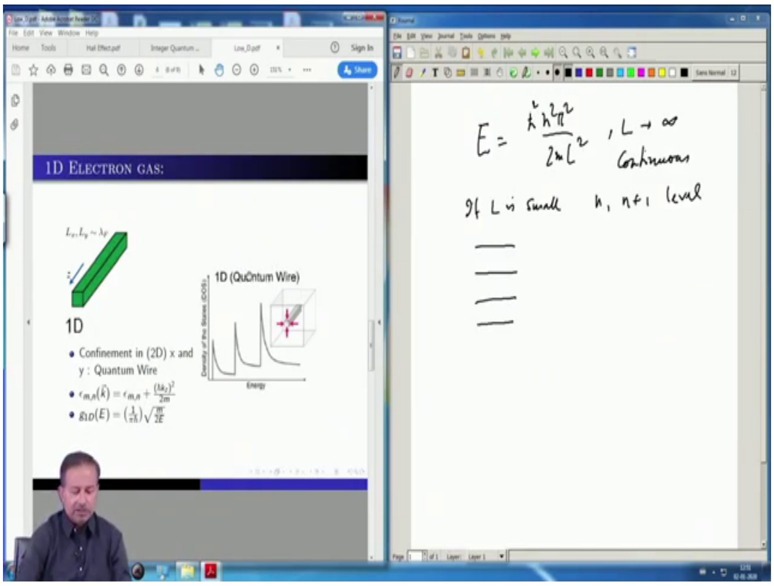

So, look at the direction in which the electron can flow freely. There is only one direction along this is written as z direction ok. So, along this direction the electron can move freely. And the other two directions the confinement is of the order of the wavelength of an electron which is or the mean free path depending on what how you define it, but that is the kind of length scale that one is confining the electron 2. So, this is an example of a, it is like a wire this is also called a wire. It is 1 dimensional 2 dimensions are confined and 1 dimension is free.

So, it is spectrum will be along k z it will still behave like a free particle h cross square kz square by twice m. Whereas, along the other two dimensions it will have this discrete energy levels and you need this two quantum numbers m and n to define the quantization along the x and y directions. And the density of states as we had done in the previous lectures is a square root of E inverse of that.

So, E to the power minus half and so, that is what is should given here this is the picture for the energy levels compared these 2 dimension; 2 dimension is basically flat completely flat. So, again you jump to a discrete level it is flat you can jump to another discrete level it is flat and so on.

Whereas, here from one so, here did each of these discrete levels will give you this spectrum along this. So, these density of states this root over E comes from this real part of this easy part of this respect to this free part of the spectrum which is h cross square k square z square by twice m. So, it in 1 dimension so, that is just one dimensional spectrum and that will give you one over root e.

Then again you come to the next energy level, discrete level; you have this square it will be a minus half dependence and so on. So, it goes on. Which in the 2 dimensional case was flat at each level you have a flat dispersion flat density of states because of this term the xy degree of freedom which is the 2 degree of freedom.

The discreet comes from the 1 dimension here this e sub n you need only one quantum number there ok. So, that is basically the story of what one does with this kind of see why are these systems interesting and important low dimensional systems.

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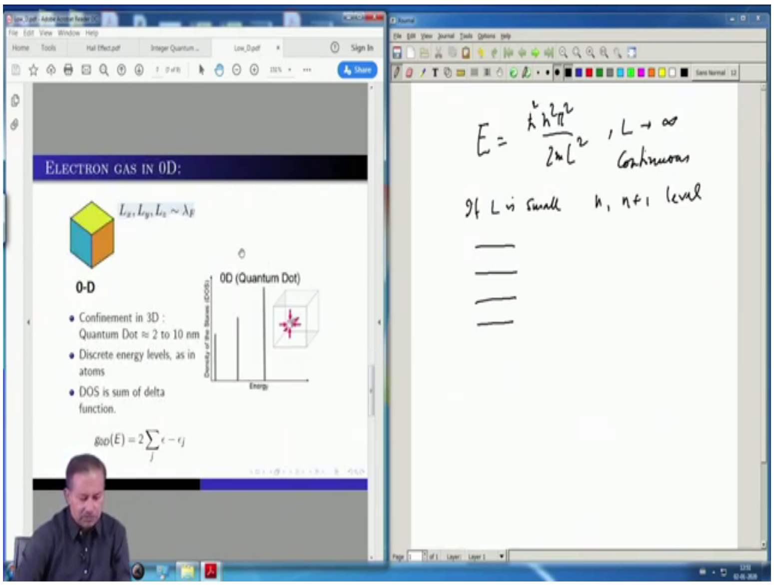

And there are many many different phenomena which one can study. This is for example, the extreme case, where all directions are confined. So, these are called quantum dots or 0 dimensional system. So, they are not literally 0 dimension dimensional, but there confinements are of this order in all three directions. So, then these are there is no degree of freedom along which they behave the electron, but the quantum particle can behave as a free particle. It will be confined in all three direction.

So, it is like a true particle in a box situation in all direction the particle is confined in a finite size. So, this is the confinement in three directions 3 D, This is 3 D confinement its quantum dot typically their sizes at 2 to 10 nanometer that also can change I mean; this depends on what application you are up to then you can change the size it depends also on the system itself and so on. But, these are these are the typical ballpark figures that it has to be around some nanometer.

So, the so it is like an atomic state. I mean; it is like a giant atom like in atoms we have these discrete states right, minus13.6 for example, in hydrogen then it goes up then it goes up and then so on. So, here also you have fixed energy levels for each of these systems.

So, there is a so, there is a complete discreteness now in no direction one is free and the density of states is basically your energy has to hit this value each of these values to get a finite number. So, it is like a histogram I mean you are now finding out how many states are available at what energy and you know that only a certain discrete energies states will be available.

So, you are you are summing these two comes from the spin degeneracy 2 into e minus ej ej's are these quantum states. They are defined by three quantum numbers. And this relation basically this is a there is a delta function here delta of e minus ej and that is what gives you the density of states and which is like a comb. I mean, it is like a comb that we used to comb our hair these are like spikes at fixed energies. And if you have large number of them then it will be just like spikes.

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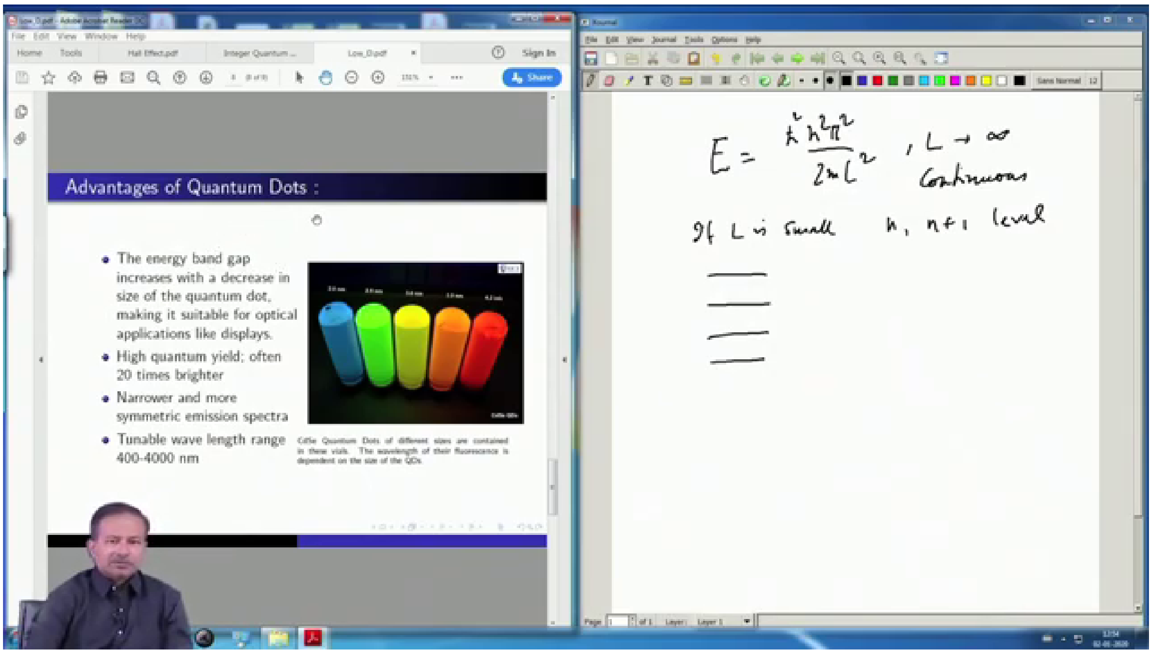

The one picture that you must have seen many times is this picture this is CdSe. And here this is a these are quantum dots you can see the sizes the size is 2 nanometer then 2.5 nanometer then 3 nanometer and 3.9 nanometer 4.2 nanometer. So, CdSe cadmium selenide quantum dots and look at the energy band energy.

So, as we say showed in a one of these previous view graphs that your optical excitations which is responsible for the optical the gap which is responsible for the energy that it emits which is basically the color of the light it falls in this case it falls in the visible range of the spectrum in all these cases for.

So, in this case one can easily see that the color of this CdSe quantum dot. It is a material is the same CdSe, but the color is changing because, with change in size your band these gaps are reduced gaps are increasing quantum. So, the inter level gaps these this if you look at the previous slide this gap from the first to the second maybe to the second to the third all these gaps are now increasing as you decrease the size.

So, if that happens then as you go down in size from right to left you will be your spectrum will now shift towards blue because you require more energy to the gap is large. So, the energy emitted corresponds to that gap and the corresponding color is more towards blue. So, this is the beautiful example of a same system look at the system it is a system cadmium sulfide. So, as you decrease the size the band gap is consistently increasing.

There are other applications which are high quantum yield. Basically, there because the states are so sharp and they are not disturbed by anything no nothing no broadening nothing. There they are extremely bright and very good sharp colors that come out and their quantum yield is very high. They are very narrow as I said and symmetric emission spectrum there is no as there is because, there is the width of these states is very low there is almost no asymmetry seen in the spectrum.

And of course, these are tunable as I said as you just decrease the size of the nano part nano dot, you can increase this I mean go towards blue color by just changing the size of the system. So, that is an example of tunability of the wavelength, emitted wavelength.

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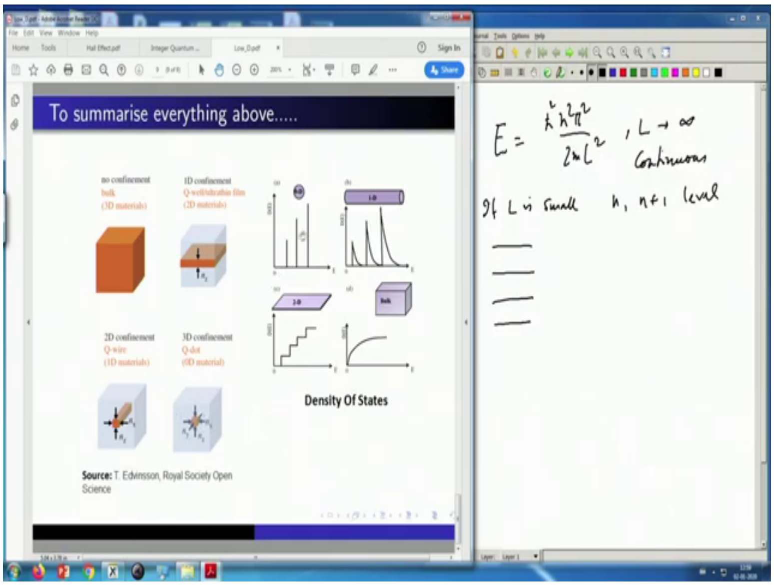

So, to summarize these are 3 dimension is of course, there is no confinement. Spectrum is like a free particle p square by twice m in all directions. Then you have you are confining one directions which is 1 dimensional 1 dimension is now confined. Which makes it a 2 dimensional system the corresponding density of states is like this staircase like. The 3 dimensional bulk one has this square route e this has a no e dependence the density of states is independent of e these are all for free particle spectrum that we are talking about.

So, then free particles confined in mean this is like a in 2 dimensions the object with the particles are free, but in 1 dimension they are confined. This is all 3 dimensions they are like free particles. This one is now 1 dimensional which means that you have wire which features confinement in two directions and; that means, that there is only one direction where it will behave like a free particle and for that the spectrum will be like this e to the power minus half 1 by square root e.

And the 0 dimensional object which is a quantum dot which has confined confinement along all directions and that has this spectrum of delta functions basically, you have a discrete energy levels and the gap increases between the states as you reduce the size of the dot which was exemplified in that example of cadmium selenide quantum dot.

So, the reason one studies low dimensional system is one of the thing is that the huge application of nano systems in physics chemistry math biology and so on. And the other one is that the physics is very interesting I mean now you have quantum mechanics playing out in front of you can actually see the discreteness that we learn in quantum mechanics for finite confined systems and that can be displayed right in front of you experimentally you can tune that energy levels and so on.

So, it is a lot of interesting new physics that comes in. The additional interest comes in when there are these electrons start interacting with each other and that is a subject that is of course, we will not discuss here. But, that is an it is an extremely important subject when you have a confined system in which electrons are confined and they are interacting with each other what happens then and there are beautiful interesting physics that is; that happens there and those also have fantastic applications.

So, in the next lecture what I will do is that I will show you one very major discovery which is which has tremendous had tremendous application it still has in metrology and it is the, in application terms it was metrology that was mentioned first. Basically, metrology means; the making standards, like the corner the weight, the time, length all these things are standardized. And these the subject that deals with the standardization of units measures is called the metrology.

And so, this subject this discovery basically shook people up in the sense that it told us that real quantum mechanics plays out in the transport and one has beautiful new physics that comes out in presence of magnetic field. So, that is what I will start doing in my next lecture which is called the quantum Hall effect.

We will only discuss the integer quantum Hall effect because fractional quantum Hall effect is a very different beast. Although, it is fascinatingly interested interesting subject, but we will confine ourselves only to 2 D electron gas in a magnetic strong magnetic field at ultra low very low temperatures. And we will discuss the physics of that and eventually show that the off diagonal resistivity can be quantized. We will come to that in the next lecture.