Electronic Theory of Solids
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Lecture - 01
Free electrons: Drude Theory

Hello. Today we will start with this very well known approximation that went about explaining many of the phenomena in metals for nearly about a century and this is called the free electron approximation. Although most of the materials are insulating in nature, it is the metals that are of central interest because many interesting phenomena happen in metals.

The earliest ideas about metals before the birth of quantum mechanics was introduced in very interesting paper by Drude, which constitutes one of the basic postulates of the free electron approximation. Now, what does this approximation tell us. I will tell you the set of approximations and then I will work out one or two problems based on these approximations. Then you will realize how this approximation is used to calculate certain transport phenomena.

So, the basic tenet of this approximation is that the electrons that form the conduction electrons in a metal come from the atoms. The valance electrons of the atoms in the solid wonder about living their parent atoms and they wander about throughout the solid and then they basically behave like a free electron gas.

And, what one does in free electron theory is to assume that this is like a free electron gas of a density typically of Avogadro's number. Number of electrons is typically Avogadro number per mole. And then these electrons are basically classical, you can use kinetic theory, Boltzmann statistics and all that thermodynamic principles to explain their behavior. So, that is the fundamental tenet of free electron gas model.

Later on of course, when quantum mechanics came in, people modified these classical assumptions and introduced quantum mechanics and we will come to that as we go along. Now, the one thing that in addition to this free electron assumption of a classical gas one has to also assume and find out how the electrons move about in the solid and since there are heavy ions sitting static at their positions in this approximation, the electrons will collide with them. The ions carry a positive charge, the electron carries a negative charge, so the collision is fairly strong and although the Coulomb interaction between electrons and the ions are not included in this approximation. so, are the electron-electron interactions.

Remember, the electrons are negatively charged; they also interact with each other via Coulomb interaction. So, both these interactions, electron ion interaction and electron-electron interaction are neglected. Only thing that happens is collision of electrons with the ions.

Now, the other assumption is that between two collisions the electron moves freely. If there is an electric field applied to the system then of course, the electron will move under the influence of this field. So, between two successive collisions it is under the influence of the applied field, electric or magnetic and sometimes both.

Now, the collisions are assumed to be instantaneous, this is the only way the electron will achieve equilibrium in the solid because there is no other interaction allowed. So, the only interaction that it has is this collision and through these collisions the electrons will reach their equilibrium.

Another very important approximation is that immediately after each collision the electron emerges with the velocity that has no relation to its velocity before the collision. So, an electron comes with the certain velocity, collides with an ion and then it loses memory of its previous velocity completely.

Therefore, it emerges from a collision as if its velocity is completely new in the sense that it has no memory of its previous velocity and the direction of its emergence is completely random. So, the velocity gets randomized after each collision. This is a very strong assumption that it has no memory of its configuration before the collision as it emerges from every collision.

So, with this assumption one proceeds further and then assumes of course, that between two collisions, there is a certain time the electron remains free and on an average this time is called the relaxation time tau. Therefore electron collides in a time tau, so it collides with the probability 1/tau in unit time. The probability that it collides in a time delta t is delta t divided by by tau. So, under these assumptions one can now calculate the transport property of an electron as Drude did (Refer Time: 07:16) . Let me show you how this calculation goes.

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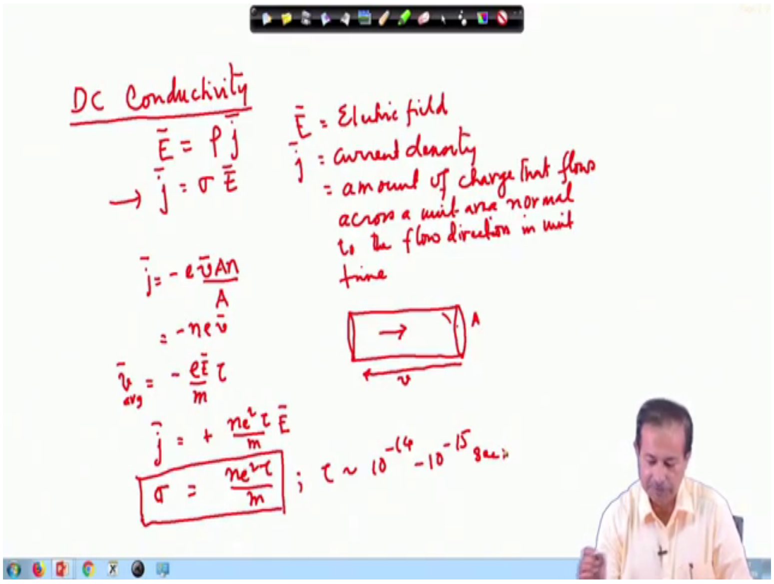

So, what we want to calculate is the DC conductivity under this free electron approximation along with the relaxation time approximation. How does one do it?

Well, it is fairly simple. Now we know that if there is an electrical field E, then it is connected to the current density by the resistivity, j is the current density, E is the electric field which also implies that j equal to sigma times E where sigma is the conductivity, inverse of rho. E is electrical field and j is current density, which is basically the amount of charge that flows across a unit area normal to the flow direction in unit time. So, let me just show you what it is.

Suppose I choose a cylinder whose cross sectional area is A, current is flowing in the direction along the cylinder. In that case, the amount of current that flows in unit time across this area A will be given by j. So, that is the definition of j.

So, let us calculate j in this geometry. How many electrons will pass through this area A in unit time? Well, it is simply those electrons that have started from a distance equal to the v from A inside the cylinder because, that is the distance an electron covers in one second. Those many electrons will cross this area A in unit time.

So that means, the current through area A is v times the area A (vA is the volume) times the density of electrons. These many electrons are going to cross A in unit time. I now need the current density. I have to divide the by A to get current per unit area. So, this is my current density for the system where you have a current that is flowing along the cylinder. So, A and A will cancel, gives me n e v. Let me repeat it: the number of electrons that cross this area per unit time divided by the area is current density. Clearly the number of electrons which will cross in unit time are those which are within a distance of v from this end within the cylinder.

This distance is v, all of these electrons will cross A in unit time. So, that is exactly what I have done here. I multiplied this volume v times A, the magnitude of v times the area is the volume, times the density of electrons gives me the number of electrons crossing A from left to right in the figure. The charge of an electron is negative.

What is v? Now v of course, in relaxation time approximation, is the velocity that an electron acquires between 2 collisions. So, once a collision takes place you are losing the memory of velocity. Then what happens is that there is an electric field. After emerging from one collision, this electric field drives it along the direction of the field.

So, that v is what I have to calculate in relaxation time approximation. I can easily calculate it, because that is the charge times the field (=force) times tau divided by mass. So, that is the velocity. That is the velocity that an electron acquires on an average between two successive collisions. I put in average sign here. So, j is then equal to minus n e square tau by m into E.

Now, from this relation I can read off easily that sigma is equal to n e square tau by m. The 2 negative signs cancel to give n e square tau by m. So, that is the formula that one uses quite a lot even today more than 100 years after it was written down and it works for many metals quite well. Of course, it has its pitfalls, it will not work in a large number of materials, also metals particularly, but it is certainly a good formula to start working with.

Now, tau is typically of the order of 10 raised to minus 14 to 10 raised to minus 15 second in a metal typically. One then goes further and one can actually find out the equation of motion of an electron in this relaxation time approximation. So, let us just try to do that.

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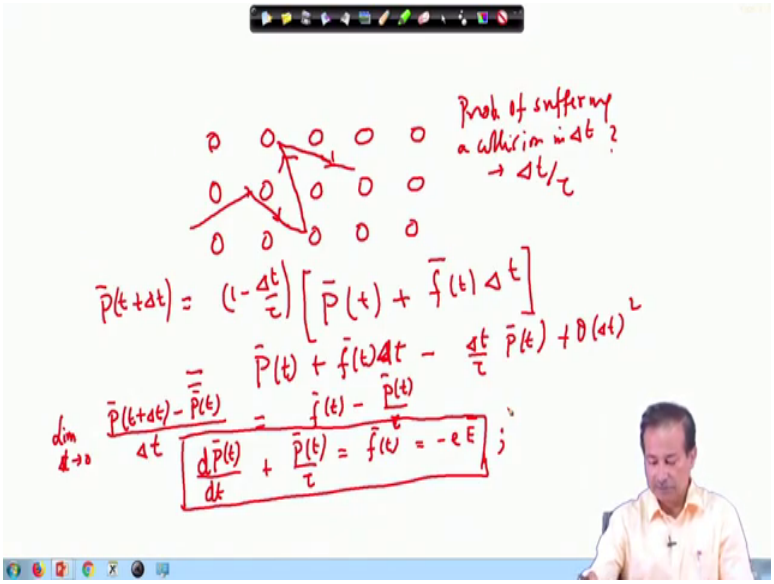

We have this solid and zillions of electrons were scattering off this solid. We know they lose their memory the moment they scatter. Memory of their previous velocity as it emerges from a scattering is lost. So, under these assumptions what I need to know is suppose an electron scatters at time t then in the next interval of delta t, which is very small comparable to tau, what is the momentum of this electron.

So, suppose the electron has a collision at time t. Its momentum at time t just emerging from the collision is P(t). It is has to be a function of time. Then there is a force due to electrical field which is f(t) times delta t, The momentum of the electron, if you follow Newton's law of classical physics is P(t)+f(t) times delta t.

Of course, this would be fine, but the fact that not all electrons will survive to keep increasing the momentum, because at the next collision this momentum history will be obliterated for some (that collide). So, you have to find out how many electrons survive the time delta t, that is, they have not suffered another collision in time delta t. Now, what is the probability of suffering a collision in delta t? This is simply delta t by tau. Then the survival probability that it has not suffered collision is 1- delta t/ tau and these are the electrons that have not suffered collision and will pick up the momentum in the time interval of delta t.

So, that will be my momentum at t plus delta t, remember delta t is very small, replace by dt. There are corrections to it which let us not bother about. The corrections can be shown to be of the order of dt square, but from this equation let us just expand and keep terms only up to linear in dt because dt is small. So, (P (t) + f dt) (1- delta t/tau) = P (t+ delta t). I neglect (delta t) square term now.

I rearrange terms and take a limit dt small. So, dt 0 means with respect to tau it is very small. So, in that limit I can write this as dP/dt = - P/tau + f(t). Now, f (t) is minus eE and that gives me the equation of motion for the electron under relaxation time approximation.

Now, you can easily check that if there is a steady state then I can neglect this dP/dt term, because there is no more time dependence of the momentum and then of course, we will get back the equation sigma equal to n e square tau by m (Drude formula), you can just check for yourselves. So, this equation is actually a central equation in the Drude model. You can write down an equation for the evolution of the momentum of the electron under this approximation. So, this is called the relaxation time approximation and this is how it works.

In the next class we will work out certain properties of this equation and find out the Hall Effect for example, and the conductivity at finite frequency and so on.

Thank you.