

Experimental Physics - III
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Lecture - 40
Zeeman Effect (contd.)

We will continue our discussion. We have seen that we will use mercury source and from that source we will take dean line wavelength is 5 4 6 nanometre. We will use filter to select that wavelength. Now we have a spectral line. If we apply magnetic field what will happen. Let us see theoretically first.

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Quantum Mechanical description of Zeeman Effect

Magnetic field interacts with magnetic moments, μ of the electrons, which possess additional potential energy
 $U = -\mu \cdot B$

Bohr magneton: $\mu_B = \frac{e\hbar}{2m_e} = 9.274 \times 10^{-24} \text{ J/T}$

Orbital magnetic moment: $\vec{\mu}_L = -g_L \mu_B \vec{L}/\hbar$ $g_L = 1$
 Spin magnetic moment: $\vec{\mu}_S = -g_S \mu_B \vec{S}/\hbar$ $g_S = 2$
 Total magnetic moment: $\vec{\mu}_J = -g_J \mu_B \vec{J}/\hbar$ $g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$

In magnetic field for normal Zeeman effect
 $\nu = \frac{(E_2 - \mu_B B m_{L2}) - (E_1 - \mu_B B m_{L1})}{h}$
 $\bar{\nu} = \frac{E_2 - E_1}{ch} + (m_{L2} - m_{L1}) \mu_B B / ch$ $\Delta m_L = 0, \pm 1$
 $\bar{\nu} = \bar{\nu}_0 + \Delta m_L \Delta \bar{\nu}$ $\Delta \bar{\nu} = (\mu_B / ch) B$

Quantum mechanical description of Zeeman Effect. Say you have two energy levels. Transition between these two energy level, it will give you the spectral line. It is a frequency ν will be equal to E_2 minus E_1 divided by h ok. Whatever the energy difference. That is equal to $h \nu$. ν equal to this ok.

Now, if we apply magnetic field; if we apply magnetic field then magnetic field with interact with magnetic moments μ of the electrons of the atoms, which possess additional potential energy that is U equal to minus $\mu \cdot B$ ok. In this case, here this μ magnetic moment actually I have defined magnetic moment. Magnetic moment a Bohr magneton it is a μ_B it is the universal constant. this μ_B equal to $e \hbar$ cross by $2 m_e$. how Bohr magneton know this this magnetic moment orbital magnetic; moment of

an electron in a Bohr orbit ok. For n equal to 1 in hydrogen atoms. What is the magnetic moment of this electron hydrogen atom in ground state in n equal to 1.

That is the; that is magnetic moment for hydrogen atom in ground state. That is the Bohr magneton say its value is this this is the universal constant. We express the magnetic moment in terms of this Bohr magneton. As you know this electronic charge we or charge we express in terms of electronic charge e . e equal to 1.6×10^{-19} coulomb yes. When we express charge so sometimes, we tell $5e$, $4e$, $3e$. Similarly, when we express magnetic moment we use this unit. $4\mu_B$, $3\mu_B$ like this we use. This is the Bohr magneton.

Now, orbital magnetic moments spin magnetic moment, total magnetic moment. Magnetic moment is coming because of the angular momentum. However, the angular momentum is there will be magnetic moment. Due to orbital angular momentum L . corresponding magnetic moment will be μ_L equal to $-\frac{e}{m} \mu_B L$ vectors ok. Here minus sign is because the magnetic moment and the angular momentum they are in opposite direction ok.

that is why negative sign and the g_L it is a 1; g_L equal to 1 and spin magnetic moment μ_s equal to $-\frac{e}{m} g_s \mu_B s$ by \hbar cross ok. This this is spin magnetic moment and this g_s it is an it is a g_s equal to 2 and total magnetic moment μ_j equal to $-\frac{e}{m} g_j \mu_B j$ \hbar cross. How the g_j is coming. From here, this you can see from symmetry it is a g lambda g factor is always with all magnetic moment ok. Where g_j is equal to a value it is not 1, not 2 these value ok.

Whereas, L magnitude of L is one can write an $m_L \hbar$ cross in space quantization form actually here a dot product we have taken $\mu \cdot B$. I think μ when you express this way $\mu_L \cdot B$. $L \cdot B$ this $\cos \theta$ term will come ok. That $\cos \theta$ term actually we have included. That is why the magnitude of L we are writing we should write $L \hbar$ cross. Now, L have space quantization $m_L \hbar$. that is we have written $m_L \hbar$. these include this dot product ok.

S equal to $m_s \hbar$ cross. J equal to $m_J \hbar$ cross ok. Now in magnetic field for normal Zeeman effect. Now, I have applied magnetic field this was the energy levels. Now additional energy will be included ok. E_2 plus this, this. Here minus sign is there. minus $\mu_B B$ m_L minus μ_B minus $\mu_B B$ m_L B is higher B is this one B is this one and

this from here $m_l h \text{ cross } h \text{ cross } h \text{ cross}$ will go m_l will be there. $\mu_B B m_l m_l 2 I$ have written minus E_1 . E_1 minus $\mu_B B m_l 1$.

I told here normal Zeeman Effect because here we are not considering the J , we are considering L . we are considering L one can consider S also but there is no coupling they are independent, but if you consider S $n \Delta m_s$ if you consider there is no additional splitting we get. That is why we are not considering m_s , but here we are considering m_l means we have considered that we have assumed that there is no $l-s$ coupling ok. Then if there is no $l-s$ coupling or magnetic field is such that the $l-s$ coupling will be broken ok. $l-s$ coupling effect will not see in the splitting of the spectral lines ok.

So that is why here we have written we have taken this not μ_J , we have taken μ_L here. This ν now ν will be here ν equal to this. This by h this by h . Now, here I can write E_2 minus E_1 by $c h$. Here ν bar I have written so ν bar equal to 1 by λ ; 1 by λ ok. Plus $m_l 1$ minus $m_l 2$ then $\mu_B B$ by $c h$. that c is coming from here $c h$ ok. This I can write. this one the original one, this one the original one ν_0 bar plus $m_l 2$ minus $m_l 1$ or $m_l 1$ minus $m_l 2$ whatever. This I can write Δm_l ok.

Δm_l this is the centimetre inverse. This is the number. this part will be this part I can write the $\Delta \nu$ bar ok; this $\Delta \nu$ bar is change due to apply magnetic field because of the applying magnetic field what will the change of the wave number of the spectral lines ok. This is the original spectral lines with respect to this what will be the change of the spectral lines. That we are telling $\Delta \nu$ bar ok. where $\Delta \nu$ bars $\Delta \nu$ bar is equal to $\mu_B B$ by $c h$ into $b \mu_B B$ by $c h$ $\mu_B B$ by $c h$ and m_l value m_l value it can take 0 plus minus 1 .

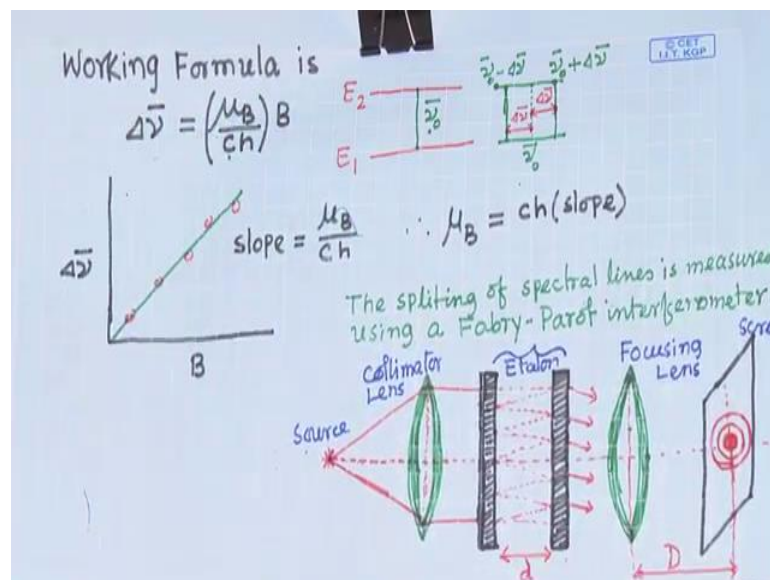
That is the selection rule. Now, here see m_l value if you put 0 . you will get ν bar equal to ν_0 plus 1 . ν_0 bar plus $\Delta \nu$ and minus 1 ν_0 bar minus $\Delta \nu$ ok. Clearly, you are seeing this 3 wave number, 3 frequency, 3 wavelength ok. Initially, it was 1 wavelength, 1 frequency. Now, in presence of magnetic field you were you will see 3 frequency; means 3 spectral line. 1 spectral line will be splitted into 3 spectral lines. One will be original one and other will be symmetrically plus ν higher and plus ν these $\Delta \nu$ $\Delta \nu$ higher and $\Delta \nu$ lower than the original one.

And this $\Delta \nu$ is proportional to the B magnetic field. If you apply, higher magnetic field this splitting that is wave number separation will be higher. Now from this you can

design the experiment. If I take a spectral line, say mercury that line 5, 4, 6 ok. Now if I apply magnetic field, I will apply different magnetic field and for that, I will measure this this splitting their separation ok. A del nu you remember that is from the original line what is the separation of this one or other one ok. There is the del nu ok.

Now, you put a separation of these 2 nu lines. Then that is 2 del nu or if you take this one del nu, but. Then divide by 2. That will give you whatever del nu here we are writing. Now, for different value of B, if you can measure del nu bar then we will plot graph del nu versus B you will get a straight line you will get a straight line here I have shown.

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Working formula forever experiment will be del nu bar equal to mu B by c h into B ok. If I plot the graph then I will get straight line slope of the straight line will give you mu B by c h ok.

You can calculate a mu B C is light velocity. 3 into 10 to the minus 8 meter per second and h value is Planck constant you know. Slope will measure we will get from here. You will find out the mu B ok. This experiment will give us; will give us the flavour; flavour of the of the of the Zeeman Effect. How magnetic field affect the spectral line of atom and also we can calculate the fundamental constant that is in this case it is the mu B, Bohr magneton we can calculate. And, then you can compare with the standard value of mu B.

Our task experimental task is to choose source that already we have chosen mercury and then we will use filters to choose the green line 546 wavelength. Now, I will apply magnetic I have to apply magnetic fields. Electromagnet we will use we have to apply magnetic field to the source. We will use electromagnet we will use electromagnet ok. Then line will be splitted now that splitting we have to $\Delta \nu$ we have to calculate we have to find out ok. For that, the splitting of spectral line is measured using a Fabry-Parot interferometer. We will use Fabry-Parot interferometer for measuring this splitting for measuring this $\Delta \nu$ ok.

We will apply different magnetic field and we will measure the splitting using the Fabry-Parot interferometer what is Fabry-Parot interferometer. This is the geometry of Fabry-Parot interferometer. Source then we will use a collimator a lens. Which will give us the $\Delta \nu$ always. Now, it is called Etalon it is called Etalon 2 2 semi silver plate it is the reflectivity is 90 percent and transmission is 10 percent ok. This type of 2 mirror you can say is separated by d this parallel to mirrors separated by d ok.

It is a reflecting power is 90 percent and transmission power is 10 percent ok. If these two type of mirror is separated by d . that is called Etalon, here light will come, and then there will be multiple reflection and then then transmission on the other side ok. This transmitted light is here. It is different sets of panel raising will get. They will interfere and you will get interference fringe.

Interference fringe on a screen. Again, these are again parallel rays. You have to use one lens is called focusing lens. This focusing lens. This parallel rays will be focus on a on a screen. Whatever the Etalon this parallel is formed it is a circular rings it will form circular rings. That this lens will put that that put if this lens is not there this image will come in infinity.

Therefore, if you put lens then it will form in a finite distance ok. On a screen, you will get these type of concentric circular rings ok. These the interference pattern from the from this Fabry-Parot interferometer you are getting. the distance here the capital D we are using these the screen distance is from etalon these distance, but it is since it is parallel rays are coming ok. From here to it is a centred here. This distance $\Delta \nu$ we take the focal length of this focusing lens ok. That is equal to this screen distance capital D ok.

This is the Fabry-Parot interferometer for measuring the for measuring this (Refer Time: 17:45). these lines interference know that for a particular wave length you will get a set of fringe ok; you will get a set of yes fringe it is called fringe; set of dark either ring or all lines. In this case, you will get ring. This you will get for a particular wavelength. If this light, have 2 wavelengths. You will get 2 sets of 2 sets of concentric ring for 2 wavelength, for 3 wavelength is set of concentric rings. Measuring the diameter of these rings, we can calculate this del nu.

When you will apply the magnetic field. One spectral lines it will be a splitted in to 3. One is original and other one is other 2 different wavelength. We will get three concentric ok. Here each order each order say nth order. nth order will have 3 rings. For 3 wave length ok. Now, we have to measure the diameter of those of those rings depending on the wavelength and from here, you can calculate the you can calculate the del nu ok, and magnetic field will apply from electromagnet. Then you can find out your this mu B.

For measuring the spectral splitting we will use Fabry-Parot interferometer sometimes we tell just Etalon. We should understand the theory of the Fabry-Parot interferometer. This so this is the interferometer means there will be interference ok. I have shown the schematic diagram of the Fabry-Parot interferometer. The condition for getting fringe.

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Theory of Fabry-Parot interferometer (Etalon) © GET I.T. KOP

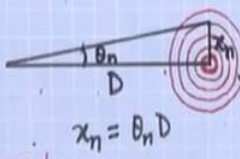
$2d \cos \theta_n = n\lambda$

For $\theta_n = 0$ $\cos \theta_n = 1$
 $2d = n_0 \lambda$
 so the order of central fringe is maximum, say $n_0 = \frac{2d}{\lambda}$.

The order of rings decreases as one goes outward from the center.

$2d \cos \theta_0 = n_0 \lambda$ zeroth ring from centre. ($\theta_0 = 0$)
 $2d \cos \theta_1 = (n_0 - 1)\lambda$ 1st ring from centre
 $2d \cos \theta_p = (n_0 - p)\lambda$ pth ring from centre
 $2d (1 - \cos \theta_p) = p\lambda$ pth ring from centre.

condition for the nth bright ring on the screen for transmitted rays from etalon.
 $\lambda =$ wavelength; $d =$ separation between the etalon
 $n =$ order of rings
 θ_n is the angular position of nth ring



$x_n = \theta_n D$

$x_n =$ radius of nth ring
 D is the distance between screen and etalon (focusing lens).

That $2d \cos \theta_n$ equal to $n \lambda$. d is the separation between the two mirrors in the Etalon. θ_n is the angle incident angle there is the incident angle of the light on the mirror. this here this d is constant. Angle has to be vary angle has to vary on the incident angle I have to vary. To get the different order of the interference fringe.

This equal to $n \lambda$. This is the condition for the n th brings on the screen for transmitted rays from etalon ok. Therefore, however, λ is wavelength these the separation between the Etalon, n is the order of rings, θ_n is the angular position of the n th ring ok. It is an inside angle or angle is this this is the angle. If this this is the Etalon position and this is the screen position. etalon means here as I these in Fabry-Parot interferometer there is a after etalon before etalon there is a lens; there is a lens which is called the collimator lens and after etalon there is a lens is called focusing lens.

This focusing lens focusing lens store. This focal length of this focusing length that is we take as a distance of the of the screen from etalon. This capital D is focal length of the focusing length ok. In addition, x_n it is an n th ring x_n is the radius of n th ring and D is the distance between screen and etalon of that is focal length of the focusing lens ok. Now, if this is the relation for the for the n th birefringe n th brings now you see if θ_n is 0. $\cos \theta_n$ is equal to 1 so that means, $2d$ equal to $n \lambda$ ok.

That I have written n_0 . n_0 is $2d$ by λ . Here you can see this n_0 is the maximum order number ok. In addition, where you are getting we are getting that is at the centre. At the centre, this is the maximum number order number. Then if you go outwards. Then if it is n_0 . There next one is $n_0 - 1$ $n_0 - 2$ $n_0 - 3$ etcetera up to 1 you can go ok. The order of rings be decreases as one goes outward from the centre.

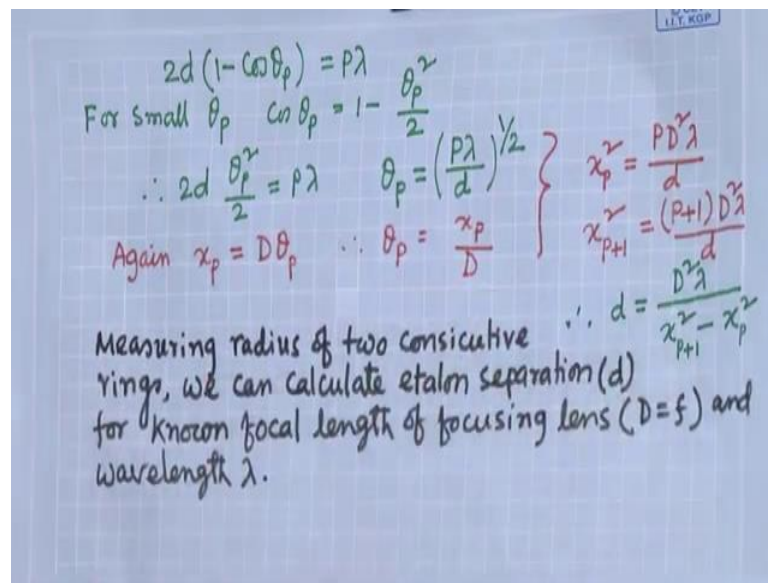
So $2d \cos \theta_0$ is equal to $n_0 \lambda$ $\cos \theta_0$ is 1 because θ_0 is 0. Here, what about θ_n I have written. Here 0 I have written. This I can tell whatever here I am writing this I can tell 0 th ring from centre ok. n_0 is this is the order of the centre. now number of rings so when θ_0 these are centre one θ_1 we are writing that is where $n_0 - 1$; this first one from the centre this first one from the centre then second one will be $n_0 - 2$ that is for θ_2 .

Now θ_p ; obviously, $n_0 - P$. it will be number of rings from the centre. This this is very important. What about P here we are writing. That is the number of rings from the

centre we are counting from the centre. That we have converted and we are getting this relation ok. This is the P th ring from centre ok. This is the condition for the birefringe and when we are counting the ring number from the centre ok, and this is n these we are counting from the outside. When we are coming towards the ring centre. this is a number 1 2 3 4 up to n n plus 1 n plus 2 up to n 0 ok.

This now from the centre, we will come to the ring. They if we if I use p. this is very important now. We have this $2d(1 - \cos \theta_p) = P\lambda$

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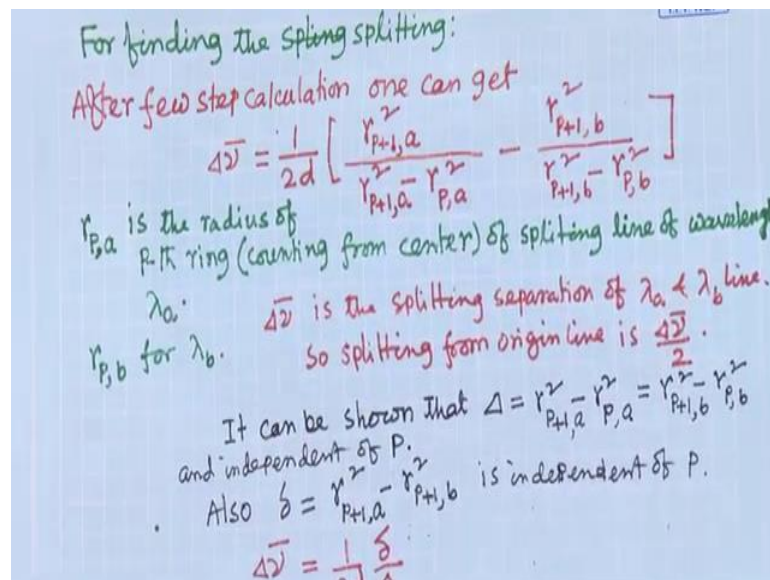
This I can write for small theta P cos theta P equal to 1 minus theta P square by 2 we can write ok. If you put this on here, you will get 2 d theta P square by 2 equal to P lambda. theta P we are getting theta P equal to P lambda by d square root half ok. What is theta p? Theta P is angular position of the of the ring. Now, if you want to express in terms of radius. This this from this you can you can see from this you can see. An x n I can write theta n D ok. x n is equal to theta n D, x P here x P equal to theta P D.

Now theta P equal to x P by D. if I use square if I use this here ok. x P square I can get x P square equal to P d square lambda by d. from here you can find out the x P square equal to P d square by d square lambda by small d. In addition, x P plus 1 next ring from the centre. Square is this so if we take difference of these 2 we are getting d equal to D square lambda by X P plus 1 square minus X P square ok. if I can measure without applying magnetic field if I can measure the radius of P th ring from the centre and P

plus 1 th ring and square up them take the difference of square of them then you can calculate these d ok.

Measuring radius of two consecutive rings. If it is, P is 1. Second ring and first ring P is two first ring and second ring. That have 2 consecutive ring we can calculate etalon separation d. that d value we need for our for measuring the measuring the spectral difference. First, we should calculate this d and capital d of course, as I mentioned it we know and lambda or wavelength of the light whatever we will use that also we know. we can calculate small d. then we have to go for the for measuring the fringe width; not fringe width splitting of the fringe splitting of the spectral lines in terms of fringe of course, interference fringe.

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After few step calculation one can get this kind of relation del nu bar. Just I have not continued. Just this the relation del nu bar means the separation or splitting of the fringe separation of the spectral lines from the original and these a splitted one. What is the separation in in centimetre inverse? That it is del nu bar equal to 1 by 2 d then this is r P plus 1 a square. Now spectral line is splitted ok. in our experiment of geometry we will choose in such a way that one spectral line under magnetic field it will be splitted into; it will be splitted into 3 components 1 is original and other 2 are the just plus minus mu 0 plus minus del nu ok.

We will use polarizer to discard this original one discard this original one ok. Because this that 3 spectral lines will splitted lines we will get one of them this for $\Delta m = 1$ equal to 0. That will be that will be pi polarization and other two $\Delta m = 1$ equal to 0 or equal to plus minus 1. That will give you the sigma polarize light. pi polarize light and sigma polarized light their electric component are perpendicular to each other ok. If you use a polarizer, optic axis is this. It will allow only say if it is allowing the sigma polarize light. pi polarize light will not be allowed or other way also possible.

In our experimental geometry will use the polarization polarizer in such a way that this 2 sigma polarized light will pass through it ok. That is why we will get we will under magnetic field we will get 2 spectral lines. one spectral lines will be splitted into a 3, 1 is omitted, but we will see on the screen this 2 ok. This we are doing because of the to reduce the clumsy of the fringe in on the on the screen. Now, here. This we have telling this a and other one is b. wavelength is λ_a and wavelength is λ_b ok. One ring will be splitted into a and b. first ring pth ring. It will be splitted into two ok. One is a is b.

Here $r_{P+1}^2 a^2$ divided by $r_{P+1}^2 a^2 - r_P^2 a^2$. here we are taking the this this difference is this first ring say; first ring and second ring $P = 1$ $P = 1$. It is a $r_2^2 a^2$ and $r_1^2 a^2$ ok. First ring and second ring. For a this one also for a that radius we have to measure and then we will get this this term and this other one for b ok. first ring and second ring for b splitted into two a and b b 1. We will take b 1 ok.

That means, for ring number 1 2 3 4 5 each one will be splitted into two a b a b a b. for each we will measure the radius r_a and r_b r_a and r_b for which number of fringe. That is or which number of rings first ring second ring third ring fourth ring ok. That data we have to note down then we can calculate this and d also we have calculated we have to calculate as I discussed earlier. From here you can calculate this now it is seen it can be shown that it can be shown that this these difference $r_{P+1}^2 a^2 - r_P^2 a^2$ this this one is equal to this one ok.

It is independent of a and b it is independent of P ok. Also this we are telling del correct it. We will we will measure all and we will take average. That is ΔR and the small del it is the radius square difference of radius square of a and b of the same ring.

$R^2 + a^2 - r^2 + b^2$. This separation is for same ring of a and b and square of them of course, and here the separation of thus of a of first ring and second ring or second ring and third ring ok. This capital Δ and this is small δ then this Δ is equal to one can write $1/2 \delta$ divided by capital Δ . , if you if you can measure small δ and if you can measure radius of ring a and b ring of different number of rings 1 2 3 4 5 at least 5 rings 4 rings if you take. From there you can calculate the average capital Δ and average small δ and then you can find out this Δ .

Now, this Δ as I mentioned here whatever I think one thing; here this in the theory we have not considered here Δ is the splitting separation of λ_a and λ_b line. splitting from original line is $\Delta/2$ that we have to we have to consider. During measurement original one is we have emitted using the polarizer omitted. Did it using the polarizer. That is why here actually we are getting 2δ or whatever δ at telling. Originally, it will be $\delta/2$ ok. That we should use for the calculating the μ_B . I think I will stop here.

Thank you.