

Experimental Physics - III
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Lecture - 14
Hall Effect

Today, I will demonstrate Hall Effect Experiment. This is very important experiment fundamental experiment and, as I demonstrated that giving this effect Hall Effect we have derive for magnetic measurement, or magnetic field measurement, we used also gauss meter. today I will demonstrate the Hall Effect experiment to find out different parameters of meter and semiconductor.

if I give you a semiconductor you know the semiconductor are three types; one is intrinsic semiconductor means, number of concentration of whole length concentration of electrons are same that it (Refer Time: 01:45) that intrinsic semiconductor and other type is extrinsic semiconductor. If, that is intrinsic semiconductor is no doped with trivalent or pentavalent atoms, then we can get N type semiconductor or P type semiconductor.

now we what should we if I give you a semiconductor, whether it is P type semiconductor or N type semiconductor, you will tell, what is the carrier density of that semiconductor or (Refer Time: 02:35) what is the carrier density, how we will find out, how we will find out the mobility semiconductor you know this mobility is very important parameter, how we will measure the mobility of carriers in semiconductor?

Hall Effect is the Hall Effect experiment is the experiment, which can measure the carrier concentration in (Refer Time: 03:13) or P type semiconductor, or N type semiconductor, it can tell the type of semiconductor, whether it is N type semiconductor or P type semiconductor. And, we can measure the mobility of carrier, from this experiment in combination of the resistivity measurement or resistance measurement, and I will we have one experiment how to measure the resistance or resistivity giving the group of method.

either this resistivity or conductivity of this semiconductor will be supplied or you can measure the resistivity of the sample and then if you do this Hall experiment of this

sample, then we can find out the mobility. type of carrier, carrier density mobility of carrier in semiconductor, those parameters you can find out from this experiment.

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Hall Effect experiment

Theory:

$$V_H = R_H I_z H_z / z$$

↳ Hall Voltage

$$R_H = \frac{1}{nq} = \frac{\mu}{\sigma} \rightarrow \text{mobility}$$

Lorentz Force

$$\vec{F}_L = q(\vec{E} + \vec{v} \times \vec{H})$$

$E = \text{electric field}$

$$F_{Ly} = q v_x H_z = q E_y = F_{Hy} \quad v_x = E_x \mu$$

$$E_y = v_x H_z \quad J_x = \sigma E_x = nq v_x$$

$$V_H = E_y \cdot y = v_x H_z y = \frac{J_x H_z y}{nq}$$

$$V_H = \frac{I_x H_z}{z} \cdot \frac{1}{nq}$$

Sample: metal/semi-conductor
 length: x
 width: y
 thickness: z
 Current: $I = J \cdot y \cdot z$
 $J = \text{current density}$
 $H = \text{Magnetic field}$
 $\sigma = \text{conductivity}$
 $n = \text{charge density}$
 $v_x = \text{drift velocity}$

you know this Hall Effect if we have a sample say (Refer Time: 04:41) shape sample also this is x axis, this is x direction, this is y direction, and this one this one is z direction, sorry this I think I had mistaken. this will be Z and this will be Y direction.

if current goes constant current goes along the X direction, I and if magnetic field is applied along the Z direction along the Z direction then there will be voltage or electric field there were along the Y direction. then that electric field or voltage or Hall voltage.

this Hall voltage is depending on some constant R H is called Hall coefficient and this current along the X direction R H with I x and H H this is the applied electric field along the Z direction at this R H is $1/nq$; n is the carrier density and q is the charge it can give the plus minus E.

in case of N type for electron q will have electronic charge and it will be minus sign. R H sign of the R H will be negative where this P type. n will be Hall charge, n charger Hall. it is the it can be the electronic charge, but positive sign. R H will be positive. Hall coefficient is positive first one Hall voltage will be positive of P type carrier and for N type carrier Hall voltage will be negative ok.

from the sign of the Hall voltage one can decide, one can tell, which type of semiconductor it is, whether it is N type or P type. how this formula has come? this we know that you know that Lorentz force, F_L Lorentz force is equal to $qE + v \times H$ ok.

in the electric field and magnetic field what is the force acting on the charge carrier? that is the Lorentz force. in the electric field Lorentz force is qE . now, into magnetic field the force is $q v \times H$. Now, we have to tell this v is v , this v receiving of voltage this v is the velocity v into velocity of carrier effect where this electric field is applied along the X direction. current will flow along the X direction; that means, the carrier will move along the X direction, it will have the velocity. that is called drift velocity Other drift velocity or velocity there is a difference without any electric field this velocity of carrier will be scattered in different direction.

Now, when we apply electric field this there will be challenge in more direction that is that velocity in that direction is we tell drift velocity, it is directed in a direction. there will be velocity along the X axis. that is v_x , that is v and if magnetic field along the x direction, into magnetic field there will be force on that carrier along the y direction, this is $v \times H$. if it is x direction H in Z direction, then this force will be along the y direction ok, along the y direction.

during this force, now we (Refer Time: 10:06) sign of the charge. in case of go whatever the direction of the force or electron it will be in opposite direction. what are the carrier inside the sample? this Hall will (Refer Time: 10:25) say along this direction and they react (Refer Time: 10:29) or if it is in case of metal of N type of carrier is there. when electron along in this direction, this other direction other surface of the sample. it is there that will lack of electron only positive charge will be there.

then positive charge will be in this surface. other surface will be populated in the negative charge. there will be separation of positive and negative charge just like battery. there will be a voltage along this voltage will develop along this y direction. this voltage it is spelling Hall voltage ok.

here you know that electric field along the x direction current is flowing due to voltage due to electric field it electric field is E_x along the x direction, (Refer Time: 11:36) Hall

electric field; it just electric field for constant current they are passing through the sample. E_x now this length H this length if it is H , E_x it will be x that will V_x voltage.

voltage along the V along the x direction V_x . Now J_x current density along the x direction that is equal to σE_x , σ is the conductivity. J_x is equal to σE_x equal to nqv_x current density is nqv_x . this V_x is velocity drift velocity of electron of carrier and this V_x is the voltage, can tell V_x is the voltage.

this is the relation this is the relation this yeah general relation for between electric field and the current density, from there we can connect with the current and the voltage, because current is current density into the cross sectional area. in this case cross sectional area is YH , this is the cross sectional area YH . multiply with J that will be current I . And, voltage you know this electric field, we do this length that is the voltage. V see we can express in terms of voltage and current also.

here we see this, now what happens that due to Lorentz force due to Lorentz force along the Y direction that is $qV_x H_z$ $q_x H_z$ this force during this force electron or carrier will move in Y direction when they will move. there is charge separation with charge separation, that voltage will develop. When voltage will develop, then this charge into this voltage it will, be affected, it will fill force due to the voltage due to the voltage or corresponding electric field.

it will fill the force that force is qE_y . E_y is here electric field produce above an along the Y direction, it is the Hall electric field the same charge will fill force during the what is called this electric field is along the Y direction is developed during that electric field again this force this charge carrier will fill the force in opposite direction. there will be balance in balance between this two opposite force, Hall due to the Hall electric field and another is due to the Lorentz force before of the magnetic field.

this is Lorentz force due to the magnetic field along the Y direction, which force will be balanced due to the force, due to the force, coming from due to the force, coming from the Hall voltage of Hall electric field along the Y direction. that is qE_y . this is the Hall force, Hall force $F_H y$ along the y direction where $F_L y$ will be $F_H y$. that time it will get it will this to the equilibrium, and from this equation, from this equation, you can find out E_y Hall electric field it can E_x, H_z .

Now, V_H is Hall voltage again electric field E_y into this length that will be y . that will be Hall voltage that equal to E_y equal to V_H / z into this y . from here I can write this V_H equal to V_H equal to see V_H is equal to we can write J_x by nq , J_x by nq into $H z y$. now, J_x I can write in terms of I_x . multiply with this. J_x I have written I_x by $y z I_x$ by $y H$. $y y$ will go and $I_x H z$ by z into 1 by nq . Hall voltage we will get.

1 by nq is R_H then $I_x H z$ $I_x H z$ and I think R_H , now here there will be another parameter that is z that is will be there $I_x H z$ by z into 1 by nq 1 by nq is R_H , R_H . from this expression and they mentioned that this Hall voltage will depend on the I_x current, constant current with the sample along the x direction. this I_x carry we keep constants, we keep constant at an optimized value, we cannot apply (Refer Time: 18:05) I_x value, because it will I think there will be heat with this current.

heating up the sample will change the parameter. we have got the heating up the sample. I_x will be a small value optimized value, that without generating heat we can do the experiment.

Now, go down to with our sample geometry go down to these are sample geometry. for a particular sample z is constant. Now, this is I_x is constant and $R_H z$ R_H is we have density for a particular circuit (Refer Time: 18:58) carrier density is constant. R_H is constant V_H is the proportional to $H z$ ok.

if you do the experiment just with any Hall voltage as a function of magnetic field then for a particular current. then we can plot V_H as a function of H , then it is expected that it will be linear term linear term of V_H versus $H z$.

from that term you will get slope you will get slope and that slope will be equal to the $R_H I_x$ by z ; z value is known to you and I_x also known to you. V_H you are measuring experimentally you are measuring. you will get the R_H . if you get R_H then you can find out the carrier waves you can find out the carries waves I will come later on about what are the how to find out the parameter. this is the basic theory of this Hall effect.

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Note: In general, Hall Voltage $V_H = R_H I_x H_z / z$ is not linear function of magnetic field H_z . Because the Hall coefficient R_H is not a constant, but a function of magnetic field H_z .

However, our derivation is based on ~~two~~ assumptions:
(a) ~~carrier~~ that all carriers in the sample have the same drift velocity v .

(a) One type carriers: metal or n-type semiconductor or p-type

$R_H = \frac{1}{nq}$ → Sign of R_H depends on the sign of q .

$V_H = R_H I_x H_z / z$ → For a fixed I_x and H_z , V_H is proportional to $\frac{1}{n}$

$R_H = \frac{\mu}{\sigma}$ → Measuring $R_H \in \sigma$, μ can be obtained.

note that in general it is Hall voltage V_H equal to R_H by I_x R_H into I_x into H_z divided by z divided by z that is V_H . is not linear function of magnetic field of H_z ok? according to this formula, it is expected that it will be linear, but it is not linear in general because here if R_H is constant then only you have it will be linear, but R_H is Hall coefficient is not constant, but a function of magnetic field H_z .

this coefficient Hall coefficient it is not constant in general, it is a function of magnetic field so; that means, R_H will also change with the magnetic field. that is why you will not you may not get the linear function. if it is not exactly linear, you should not be worried, but if it is not linear there is no problem you just for a particular on the curve for a particular point. you can find out the slope and from that slope for 10 magnetic field you can highlight the find out the R_H and corresponding (Refer Time: 22:08) ok.

So; however, our derivation is based on assumption. why R_H is not constant, which means that all carriers in the sample have the same drift velocity v . there we have assumed we have assumed that all carriers will have the same drift velocity, but it is not the case, because velocity have some distribution and drift velocity will not be same for all carriers. if they are not same then because of that this R_H will be field dependant P dependent. but here we are taking keeping as a constant and expecting that we get the we will, this Hall line based on this assumption because of that looks R_H is constant.

But, it slightly varies if the magnetic field and these thing is that the if velocity of all carriers are called equal, but we have considered equal that is why this formula is linearly dependent on the now it will Hall voltage linearly dependent on the magnetic field, but in reality there is a slight difference. the difference into the variation of the drift velocity and, one need correction of this formula, but we are not going to that, we will use this actually with that this Hall velocity will be into this here same. that is why from the linearity it will slightly vary, if you (Refer Time: 24:11) you could get not exactly linear. if you do not need to be worried.

why it is not exactly linear? It is debited from linearity, that is the reason you know, because of the variation of the drift velocity. now, say for one type of carrier sample if you take metal or n-types, semiconductor or P type semiconductor, then if you then R_H equal to $\frac{1}{nq}$ sign of R_H is depends on the sign of q as I already told and V_H equal to $R_H I \times H_z$ by q by z .

for a fixed current and H_z V_H is proportional to $\frac{1}{n}$, depend V_H proportional to $\frac{1}{n}$. now, R_H equal to $\frac{1}{nq}$ R_H equal to $\frac{1}{nq}$. as I showed you.

measuring R_H and σ measuring R_H and σ , one can also either the term. we this expression are telling that these are the carrier (Refer Time: 25:41) we can find out, sign of the carrier, concentration of the density of the carrier, just measuring output voltage on their Hall voltage and function of magnetic field from there, then calculate the find out the R_H . If you find out the R_H then you will get sign (Refer Time: 26:09) the (Refer Time: 26:10) electron not electron carrier velocity. And, if you measure R_H then only conductivity of resistivity of that sample then we can find out the μ load μ .

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(b) Two type carriers: Intrinsic or lightly doped semiconductor.

In this case, both type of carriers contribute to the Hall voltage in opposite sign.

The weighted average R_H for such sample (both carriers present) is

$$R_H = \frac{\mu_h^2 p - \mu_e^2 n}{q(\mu_h p + \mu_e n)^2}$$

p and n are carrier densities of hole and electron.

μ_h and μ_e are mobilities of hole and electron. They are not constant, but function of temperature.

check: for one type carrier p or $n = 0$, then $R_H = \frac{1}{nq} \propto \frac{1}{pq}$

R_H may become zero and even inversion of sign may happen only if $p > n$, since $\mu_n > \mu_h$.

in this case well we will continue in the continued in the mobility sorry in Hall voltage as well as electron also we will have continued in the Hall voltage, but this Hall voltage for both an electron, they will give the opposite direction, Hall voltage will be the opposite direction, in Hall case, if it is this is the opposite direction, but other case it will be in opposite direction.

contribution from these two carriers in Hall voltage will be in opposite sign, and their contribution of Hall and contribution in Hall voltage from electron and from hole, there are same there will be weightage, there will be weightage in contribution of hole and electron.

If, you consider this weightage, considering the weightage, weighted average R_H Hall from Hall coefficient of this R_H Hall coefficient for such sample is can be written as R_H equal to $\mu_H^2 P$ minus μ_x^2 or $\mu_e^2 n$ divided by $q \mu_H P$ plus $\mu_e n$ whole square; μ is the mobility, P is the Hall concentration, n is the electric concentration, q is the charge ok.

this one can find out design this formula. check if it is a one type of carrier. either n equal to 0 or P equal to 0, if n equal to 0 or P equal q , just consider this term will go say, you can put 0 this term will go. $\mu_n^2 P$ divided by $q \mu^2 H P^2$. $\mu^2 H$ will go P and this is P^2 . there will be 1 by $P q$ or 1 by $a q$ this $q \mu$ all the time it

is positive we have considered. this negative sign will come in case of electron; the positive sign will come in case of Hall ok.

your R H will be either 1 by $P q$ or minus 1 by $n q$ you are getting the same formula as we have seen for the derived for the one type of carrier. this formula that we kept that is fine. Now, here you can see that contribution from Hall carrier $P n$ are as I told density of Hall electron (Refer Time: 30:21) mobility's of Hall electron.

Now, this R H, not R H this mobility is very important character in semiconductor. mobility of electron and hole, they are not same. Mobility of electron is higher than the mobility of the hole, in semiconductor and they are not constant, but function of the temperature. with temperature this mobility affected a lot, it is mobility varies. that is why that is why earlier I told this mobility, earlier I told this R H is not constant; reason is that magnetic with magnetic field with temperature effects and, missing of variation is as I prove this that drift velocity are not same ok, drift velocity is not same at on magnetic field.

there drift velocity then I have made in terms of mobility. mobility is defined by the defined by the drift velocity divided by divide by electric field ok.

for unit electric field what is the mobility, what is the drift velocity, that is for mobility? drift velocity and mobility are same except that in one case this is (Refer Time: 32:10) electric field, what is the velocity? That means, the mobility ok.

now, this mobility is very sensitive to the temperature with temperature is varies in case of here you can see for friction μH is there here is the μe these term will be higher than these term for in case of intuitive semiconductor. If, P and n are same, but hole mobility is less than the electron mobility this term will be will dominate ok.

Now, in case of P type sample, if P is greater than n if P is greater than n in that case what will happen? This term is higher; this term is higher than this term Now, with temperature, with temperature as I told with temperature this term this term will dominate this term will dominate because this μe that will with temperature with temperature initially this term is greater than this term. it is R H positive. Now with temperature, this mobility will decrease in both cases mobility will decrease.

this term will decrease, this term will decrease, here it is negative term alit will change. this resultant term for room temperature whatever the value, it is positive, now with temperature what will happen? This value will decrease this value will decrease this value will decrease and we get inversion of this coefficient R_H , how coefficient inversion means from positive sign to negative sign change. it imposes is to be pass through the 0, $R_H = 0$. in that case when R_H become 0, in that case what will happen? this term will be equal to this term. from there you can get the mobility ratio of hole and electron in terms of the carrier concentration ratio ok.

Hall coefficient versus the magnetic versus the temperature or Hall coefficient versus the temperature, if you measure the Hall voltage function of temperature then from there you can find out the (Refer Time: 35:37) ratio of the electron and hole of that sample what are the experiment we can do or what are the important characterisation of the sample?

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Important characteristics:

(1) Hall coefficient inversion for P-type Semicon for as a function of temperature

$$R_H = \frac{\mu_h^2 p - \mu_e^2 n}{q (\mu_h p + \mu_e n)^2}$$

At $R_H = 0$

$$\frac{\mu_h^2}{\mu_e^2} = \frac{n}{p}$$

So measurement of R_H as a function of T for P-type semiconductor gives the ratio of mobilities of hole and electron.

in important characteristics are from object inversion curve. from there you can write down the mobility ratio and the mobility ratio and the (Refer Time: 36:21) carrier means ratio. measurement of R_H as a function of P for P type semiconductor the ratio of mobility's of all the electron can be obtained. this is one characteristic of this Hall effects.

second is Hall coefficient for one type carrier as a function of magnetic field.

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(2) Hall coefficient for one type carrier sample as a function of magnetic field.

$$R_H = \frac{1}{nq} \text{ or } \frac{1}{pq} \quad q = -e \text{ or } +e/h$$

So measurement of R_H as a function of H gives carrier density, type of carrier (n-type or p-type)

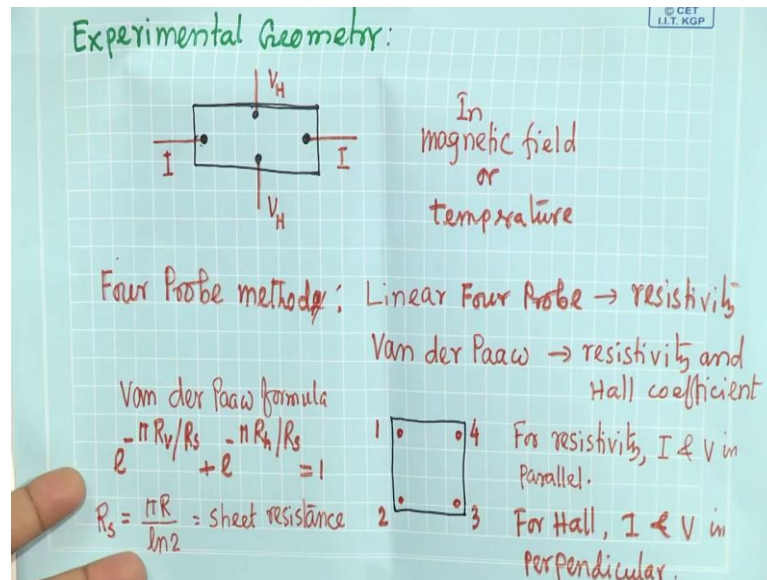
(3) Since $R_L = \frac{\mu}{\rho} = \rho\mu$, so measuring resistivity and Hall coefficient, mobility of carrier can be obtained.

that follows this as I mentioned. measuring of R_H as a function of magnetic field each carrier density type of carrier, as, I described and since R_H equal to μ by σ equal to σ is 1 by ρ . $\rho\mu$. measuring resistivity and Hall coefficient mobility of carrier can be obtained for one type of carrier one can find out the mobility, if you know the resistivity of the sample or you can measure the resistivity of the sample.

combination of the measurement of Hall coefficient or Hall coefficient and the resistivity or conductivity of the sample one can find out these carrier density type of carrier and mobility, where one type of carrier you know. And, other case you can find out the mobility ratio in both carriers and there you can find out the mobility ratio, especially in case of P type semiconductor ok.

our experimental geometry, our experimental geometry from for this Hall (Refer Time: 38:14) experiment would be like this, would be like this. you may have a sample this connection in Hall (Refer Time: 38:26) this connection is there will be 2 probe for current and 2 probe for measuring the Hall voltage ok.

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in case of measuring resistance measurements, we have used linear 4 probe method linear 4 probe method for measuring the resistance, linear 4 probe method measuring the resistance. for measuring the resistance, another way to use the probes that is 4 probe, but that is for van der Pauw geometry for van der Pauw geometry. there the probe is put like this, 4 probe are put like this two all are not linear on the same line. there 2 are 2 are in perpendicular to the other 2 these two are perpendicular to this other two.

this here I have shown this van der Pauw geometry. 4 probe 1, 2, 3, 4 they are going to this way these 4 probes van der Pauw (Refer Time: 39:52) advantage is that this one you can use for measuring the resistance as well as for measuring the Hall voltage or Hall voltage.

van der Pauw geometry gives us the facility for measuring both resistivity and the Hall voltage Hall coefficient. when we will measure the Hall resistivity, then we wish sparrow this 2 probe quickly used for current. this 2 probe will Hall voltage. it is in same line. parallel, I and V in parallel and for (Refer Time: 40:44) geometry, we will use this, if this 2 probe are used for the I and V, it is other probe 2 and 4 will be used for the voltage measurement this they are perpendicular. I and V perpendicular these are that probe can be used for the externally thus (Refer Time: 41:08) means it have 4 probe now, which 2 probe will be used for voltage and which 2 probe will be used for the current that we can decide externally.

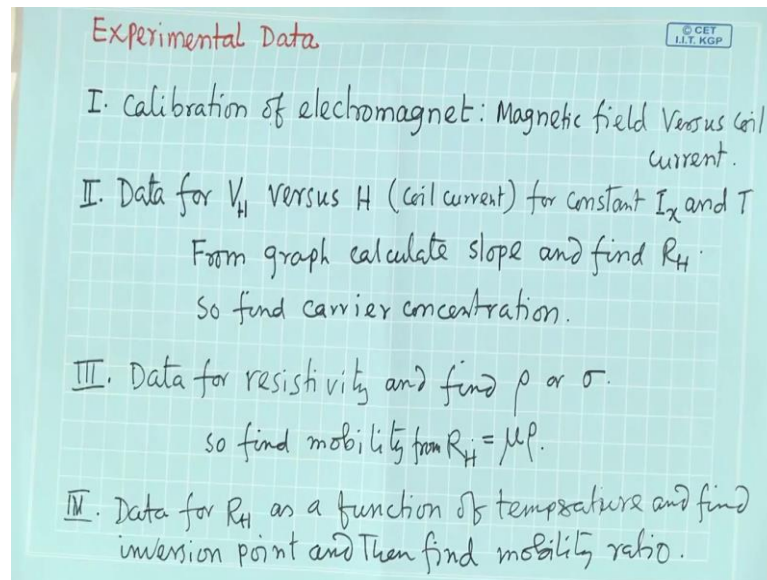
van der Pauw geometry is useful for the measurement of the resistivity as the value of the Hall Effect. but only linear four probe that it cannot use for the linear term output is only can use for measuring the resistivity or resistivity. but in case of van der Pauw, this different formula is used for resistivity measurement. van der Pauw formula is called the power πR_v by R_s plus e to the power minus πR_a by R_v , we can prove not $R_v R_s$. R_s is called the sheet resistance and one can derive this formula $R_s = \frac{\pi R}{\pi \log}$ lateral log this R is the resistance.

from this formula you can see, if you can measure the R_v and R_H , what is the R_v ? R_v is vertical resistance when there will be vertical column, means this way these 2 probe for current, these 2 probe for resistance sorry voltage or vice versa, this is for voltage this is for (Refer Time: 42:54). you get this, you calculate from the measuring voltage and current, we can calculate the resistance. there we (Refer Time: 43:05) and R_H is in horizontal. these two you are reading for R_H and these two voltage or vice versa ok, from there this current and current and voltage in horizontal direction. whatever resistance you are getting that is arrange ok.

this we need to measure. if you measure this one from this formula you need R_s . Now using this formula, you will get R . we will get R , we can calculate the resistivity or conductivity anyway we are not going to use this one, but I just told you that generally in research generally we use above this van der Pauw technique, for measuring the voltage, for measuring the resistivity as well as the Hall voltage. but in our lab we use just different measure 4 probe method for measuring the resistivity and this Hall geometry, we will use this way for measuring the Hall voltage ok.

what it makes to move? It makes to move experimental data collection. that explain the data collection is unit varying field unit varying field. calculation of electromagnetic that is formal.

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magnetic versus coil current that one has to do this experiment and (Refer Time: 44:50) it then you may get calibration curve. for coil current you will be telling, what is the magnetic field.

Then, next data you have to take that is Hall voltage versus H magnetic field. we use coil current and from this calibration we will get the magnetic field. for constant I_x and the temperature, for constant current and for a constant temperature particular temperature room temperature, we do this experiment, we will measure the Hall coefficient and constant Hall voltage constant of line will. Now, if you plot that one then from the slope will get final coverage.

And, from R_H we will be find out the carrier concentration. And data for resistivity for resistance measurement one can do the resistance measurement you need the 4 probe method or of this resistivity will be supplied for that sample. from there you can find out the mobility of the carrier.

data for R_H as a function of temperature and find out inversion point ok, where R_H is will be equal to 0 and from there one can find out the mobility ratio we will demonstrate experiment based on this theoretical crisis. let me stop here. Thank you. And, I_x and t temperature for constant current and for a constant temperature particular temperature room temperature we will do this experiment we will measure the Hall coefficient.