Experimental Physics - II
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Lecture - 07
Basic Analysis (Contd.)

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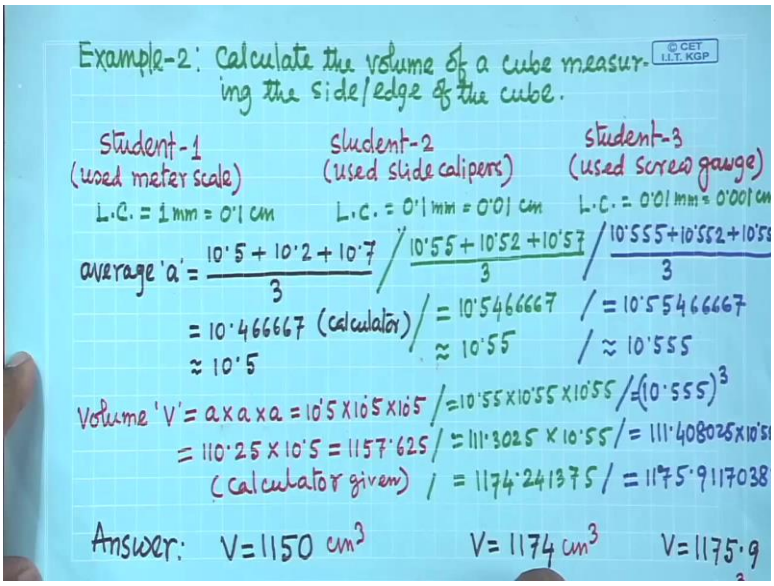

In last class I was discussing about the calculation of the volume of a cube measuring the side or edge of the cube. So, this student 1, student 2 and student 3, they used the meter scale, slide calipers and screw gauge and they calculated the volume. So, this for student 1 this answer should be the 1150 volume cm cube, of course, if it is in centimeter cm cube and then for student 2 the answer will be 1174 cm cube; and that for student 3 the answer is 1175.9. So, actually calculator was giving this, this type of number, this type of number right, and how to write the result that is what we are discussing.

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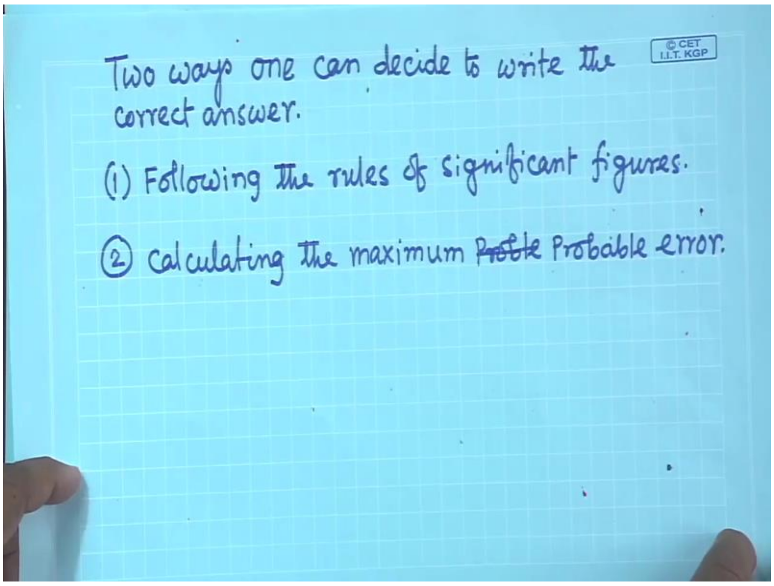

As I told that is there are two ways, there are two ways to decide how to write the correct answer. So, one is following the rules of significant figures, and another is calculating the maximum probable error. In fact, we need both of them to write the correct answer.

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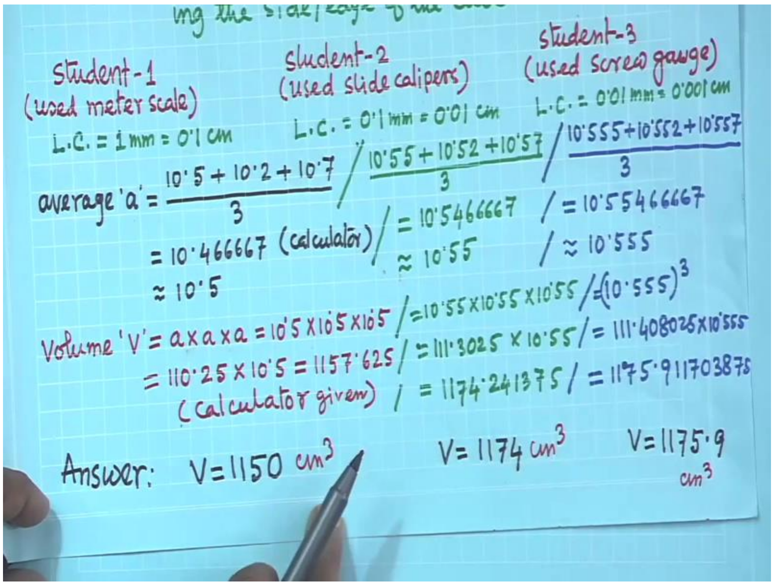

From significant figures, we have learned that this answer should be like this that we have discussed, and uncertainty will be uncertainty in these measurement or in these result will be in the last significant figure. For this last significant figure, so the basically it is a significant figure for this number is 3. Last significant figure is 5, 0 is not significant figure significant digit. Uncertainty will be in this in this last significant figure. It is a result, this is a error in this measurement using the meter scale. So, this it can be 1150, 1160, 70 whatever depending on the.

From this result, from these significant figure, we cannot say only we can say this what is the order of the error in this result but exactly you cannot say what is that a either it is plus minus 10 or plus minus 20 or plus minus 30 that we cannot say, but error will be in last digit ok, there is the uncertainty in the last significant figure ok.

Here this significant figure is 4. So, error will be here at the last significant figure that is 4. So, error will be plus minus 1 or plus minus 2 or plus minus 3. So, 1 to 9, it can be 1 to 9, any number error. And similarly, here this error is so here significant figure is 5. So, last figure is vocalized-noise] 0.1. Here the error will be in this decimal point So, to know the exactly what will be the amount of error in these in this volume.

Whatever this will be write plus or minus some error. To find out that one, we have to calculate most probable error and we have to know how to calculate that one. So, that is that is what I will discussed today. Maximum probable error how to calculate maximum probable error.

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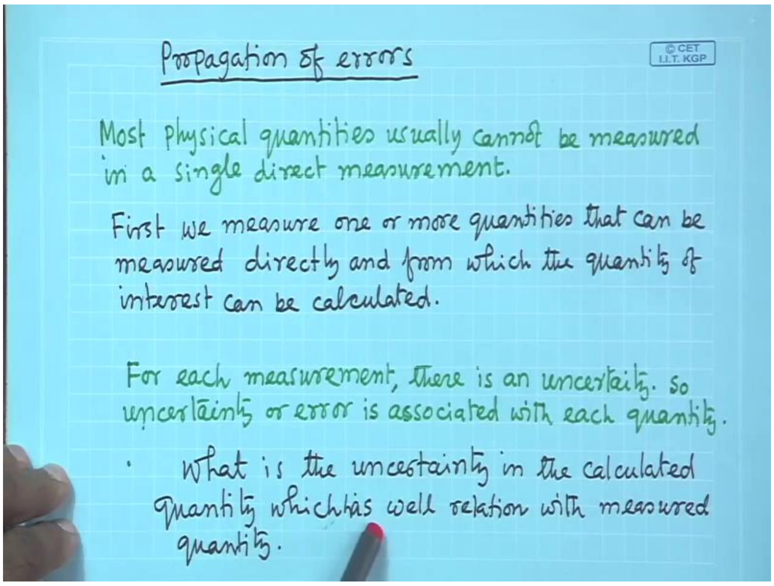

Actually we measure some parameter but calculate something else depending on the relation of the measured parameter with the calculated parameter right. Error although it is directly involve with the measured parameter, now to get the error in calculated parameter, so there is a procedure. There are basically error will propagate from the measured parameter to the calculated parameter, so that is a propagation of error.

Most physical quantities usually cannot be measured in a single direct measurement. First we measure one or more quantities that can be measured directly and from which the quantity of interest can be calculated that is what I was telling for each measurement there is an uncertainty. Uncertainty or error is associated with each quantity.

Now, what is the uncertainty in the calculated quantity which is or which has well relation with the measured quantity. That relation can be in terms of summation, in terms of in terms of difference, in terms of multiplication, or division or together some relation ok.

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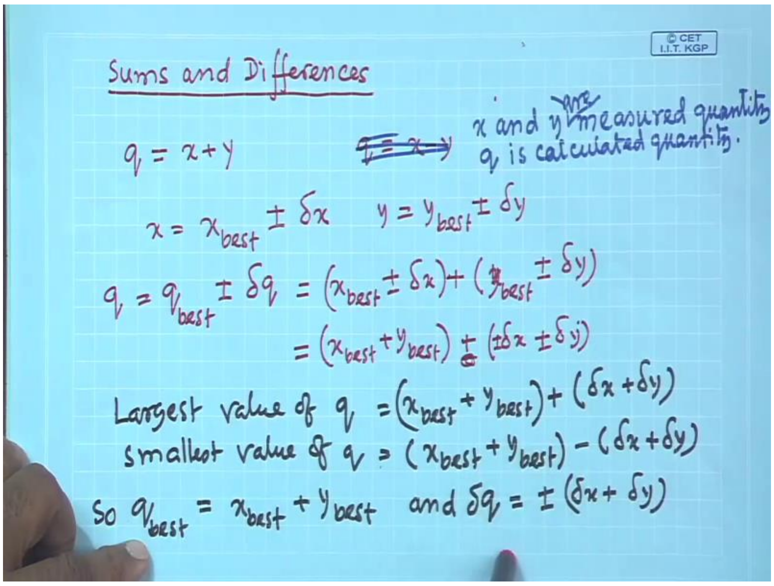

In case of summation and difference say q equal to so equal to x plus y ok. x and y are the measured quantity, and q is the calculated quantity ok. So, x and y are measured quantity, so that we express generally this x best value plus minus uncertainty y similarly y, y best plus minus y uncertainty. q and q you can write q equal to q best value plus minus del q delta q ok. So, this equal to basically x plus y q equal to x plus y, so x best plus minus delta x plus y best plus minus delta y. From here you can write this ok.

If you compare with these, so q best is basically x best plus y best, and error what will be what will be the; what will be the here basically q equal to this, so these plus these. So, what will be the largest value of q; these plus del x plus del y right and what will be the smallest value of q, so this plus this minus of q del x plus del y. It can be del x minus y, it can be minus del x plus y. So, there are four possibilities ok. Out of these four possibilities, so largest value will be this and lowest value will be this.

Then we can tell that this most probable uncertainty most probable. So, other value will be between, between these two. Most probable uncertainty del q will be plus minus del x plus del y. So, q best equal to x best plus y best, and uncertainty in q that will del q equal to plus minus. So, similarly for difference also we can show that whether it is summation or difference in both cases the uncertainty is basically in calculated parameter uncertainty is basically sum of the uncertainty of the measured quantity right.

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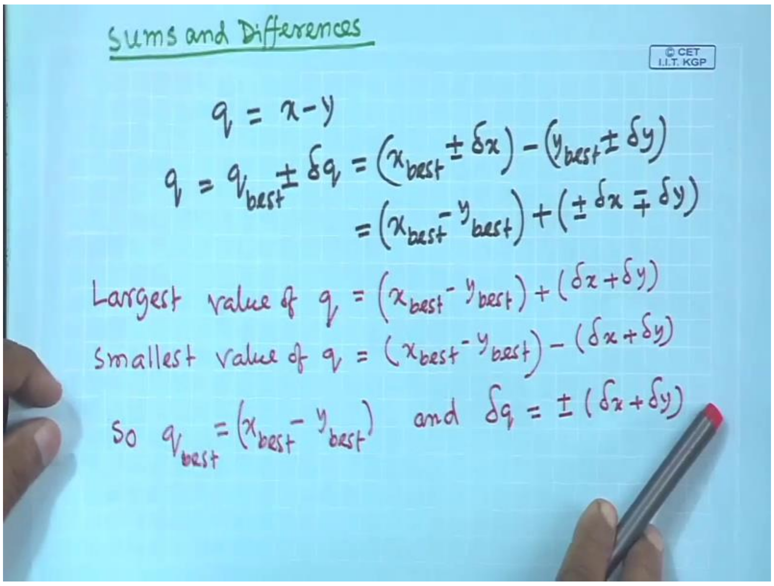

That is what in next I can show you that if you proceed same way q equal to x minus y, if you proceed same way here also, you will see that largest value of q is this, and smallest value of q will be this. Here this is the value but uncertainty is whatever individual uncertainty that will be added for the uncertainty, uncertainty of the calculated quantity. For summation and difference in both cases, uncertainty is maximum probable error is basically summation of the individual error.

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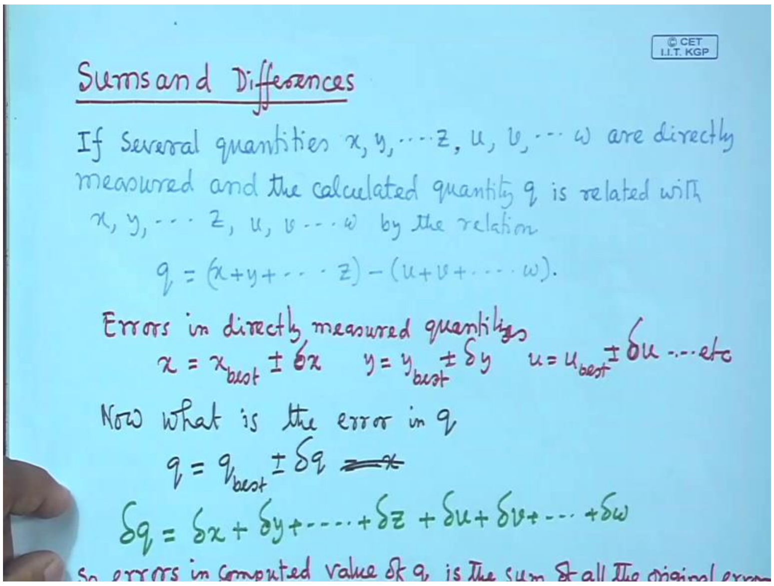

You can take this in general way. If some relation q equal to x plus y plus z minus u plus v plus etcetera. What will be the error in delta q. This delta q will be del x plus del y etcetera del u plus del v etcetera.

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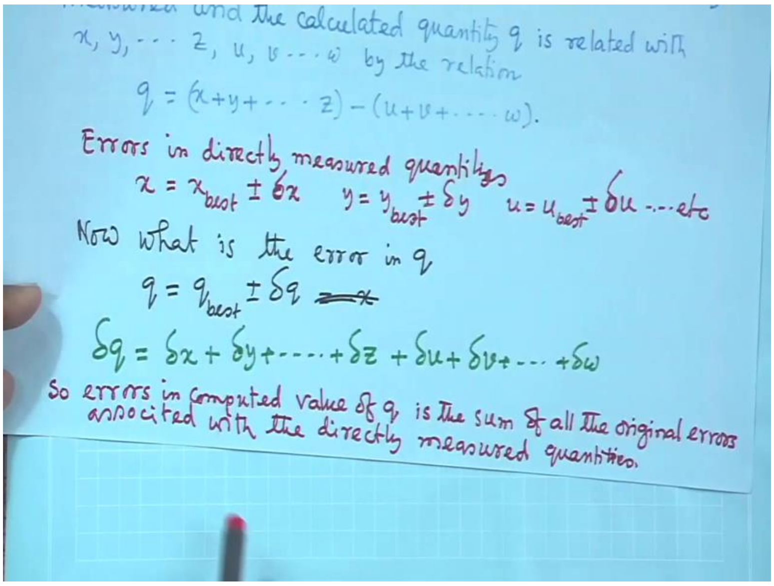

Errors in computed value of q is the sum of all original error associated with is associated, associated with the directly measured quantities. This will tell the absolute error. In case of summation and difference, so this error we summation of individual errors keeps the basically error in the calculate quantity that is absolute error. And relative error basically we tell this del q by q, del x by x, del y by y. These we tell relative error. So, in case of summation, and difference; subtraction, so absolute error are added ok. But in case of product and division, you will see there this not absolute, this is the relative errors are added ok, so that I will show next.

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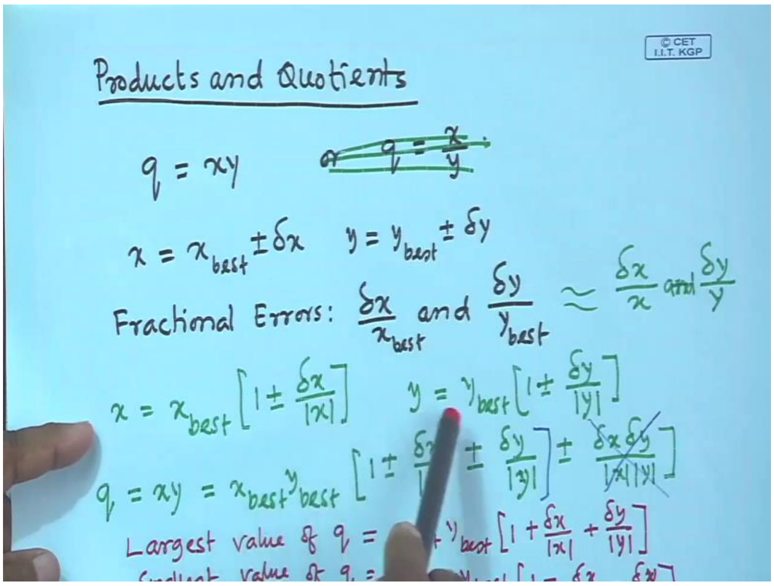

Here product and quotients division. If q equal to x y, again q x equal to x best plus minus del x, y equal to y best plus minus del y. Fractional error or relative error as I told this del x by x or x best del y by y or y best ok. So, this briefly we write del x by x del y by y ok. So, x equal to x best if you take problem so 1 plus minus del x by x ok. Similarly, for y you can write.

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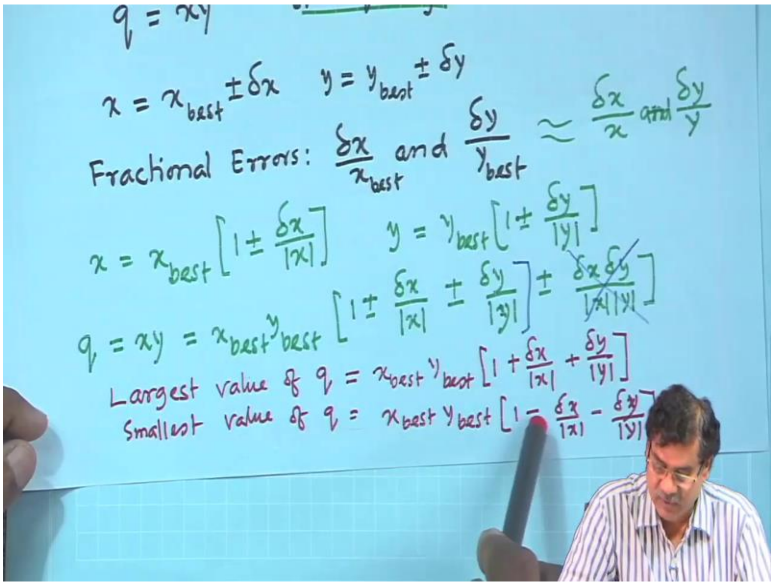

So, q equal to x y, so if you put these two x best, y best and 1 plus minus del x by x plus minus del by y, del x and del y generally this a small value. This is a second order error. We can neglect it. From here what we are saying this again the same way if you proceed largest value of q will be this plus this 1 plus this plus this and smallest value will be 1 minus this minus this. Again you can write that this q equal to q best plus minus delta q.

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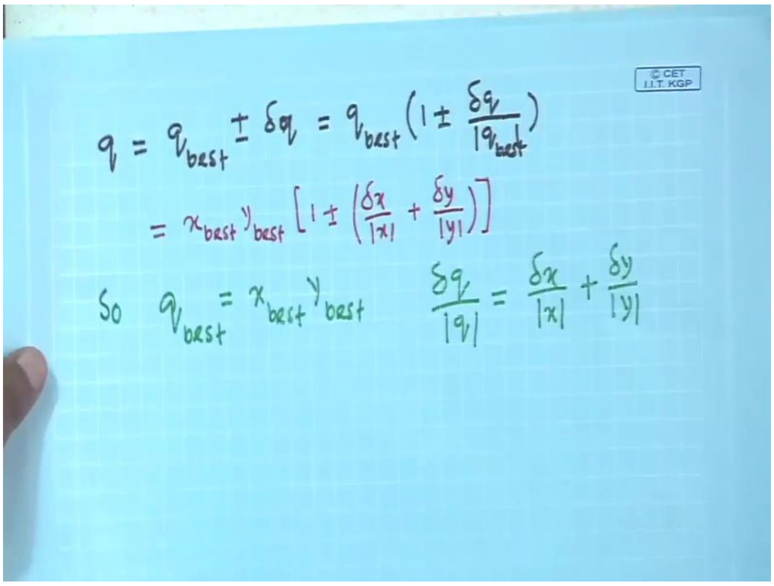

If you compare q equal to q best plus minus delta q equal to q best 1 plus minus delta q by q best ok, so it is the in mod. So, if you compare with this one, then you can see that delta q by q equal to delta x plus x plus delta y by y. In this case, so relative errors are added. For product we are seeing that relative error of individual measured parameter, so that are added which gives the error relative error of the calculated parameter.

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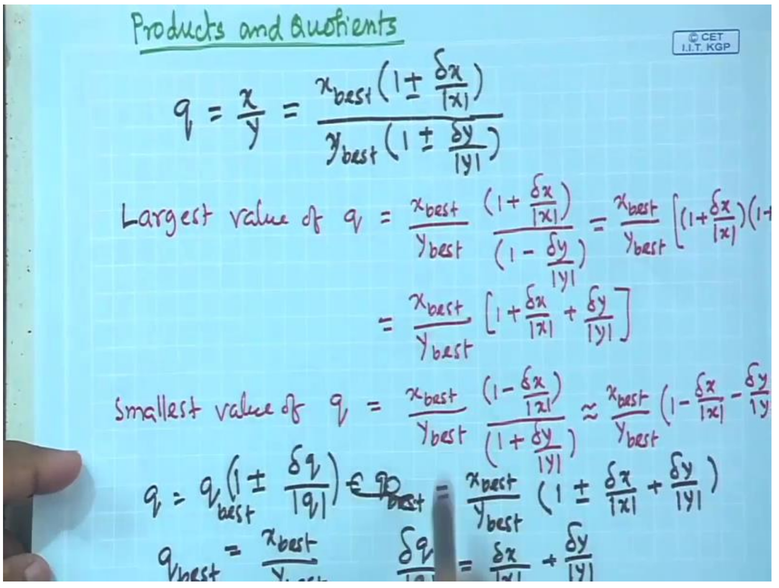

Similarly, if you choose for this division; q equal to x by y, if you proceed same way.

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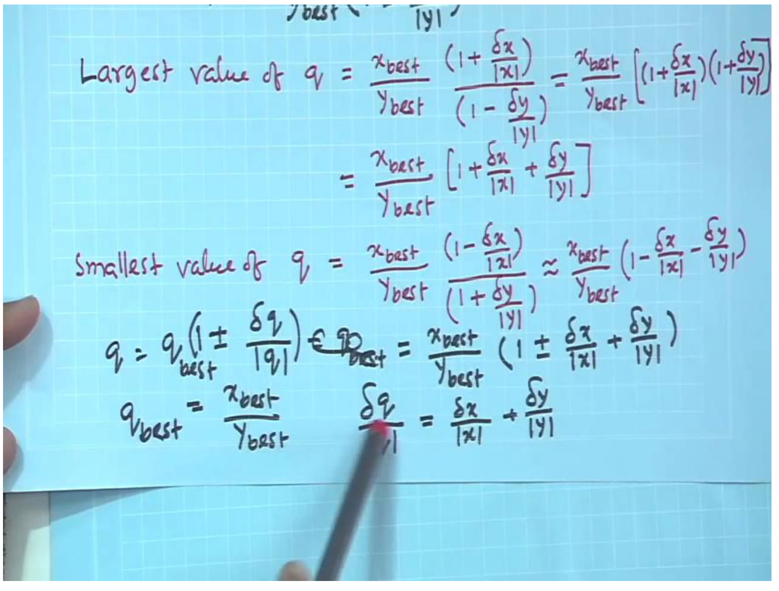

Here also you will see del q by q equal to del x by x plus del y by y ok. So, either division or multiplication in that case the relatives or fractional errors are added to get the relative error of the calculated parameter.

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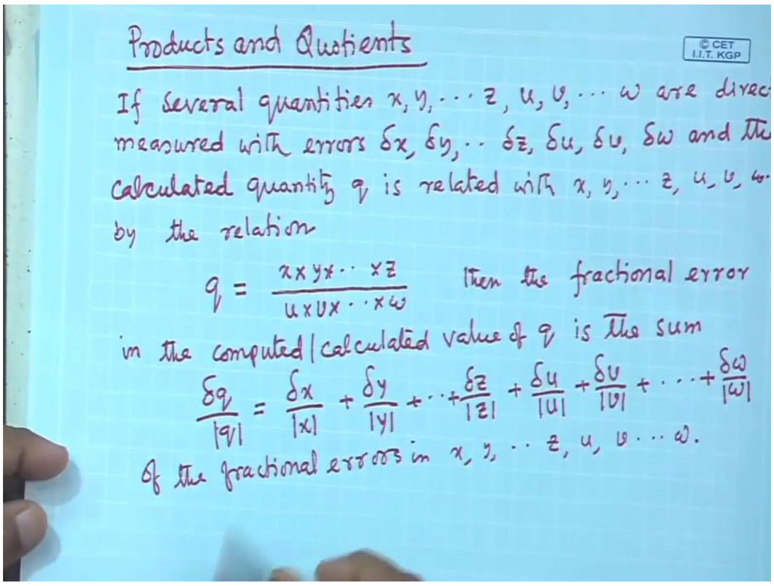

If you generalize it, so this q equal to x into y into z etcetera divided by u into v into w etcetera, then the error will be del q by q relative error equal to so these are measured quantity. So, del x by x plus del y by y plus del z by z plus del u by u plus del v by v etcetera ok. So, as I told this fractionals error or relatives errors of the measured quantity are added which will give the relative error of the calculated quantity. For summation, and for subtraction, for product, multiplication, for division, or the mixture of them in a in relation, there this rules will follow to calculate the error in the calculated parameter.

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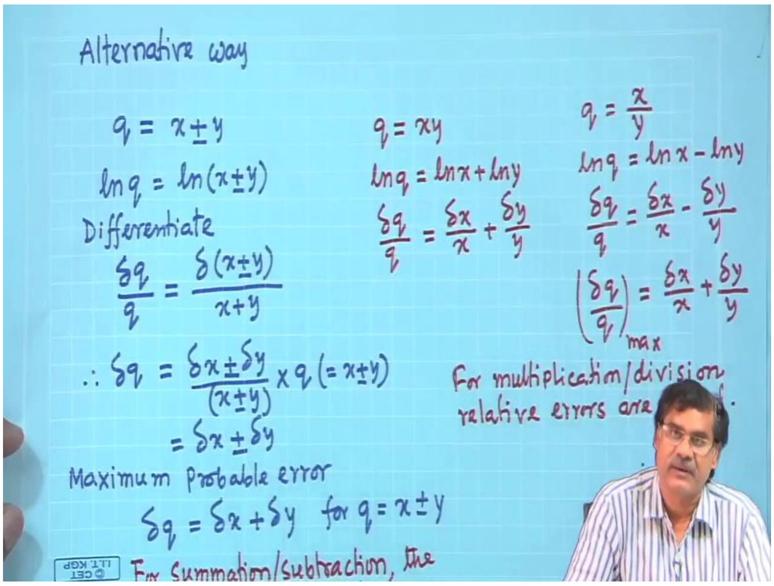

There is an alternative way to find the error this is basically it is very simple. So, say q equal to x plus minus y. Just take log in both side. Log q equal to log x plus minus y, then differentiate. So, del q by q equal to del x plus minus y by x plus y. Then del q equal to, say q we multiplied here. q is basically x plus minus y ok. So, this will go, so this equal to del x plus minus del y. We are going to find out the maximum probable error, so this here del x plus del y and del x minus del y.

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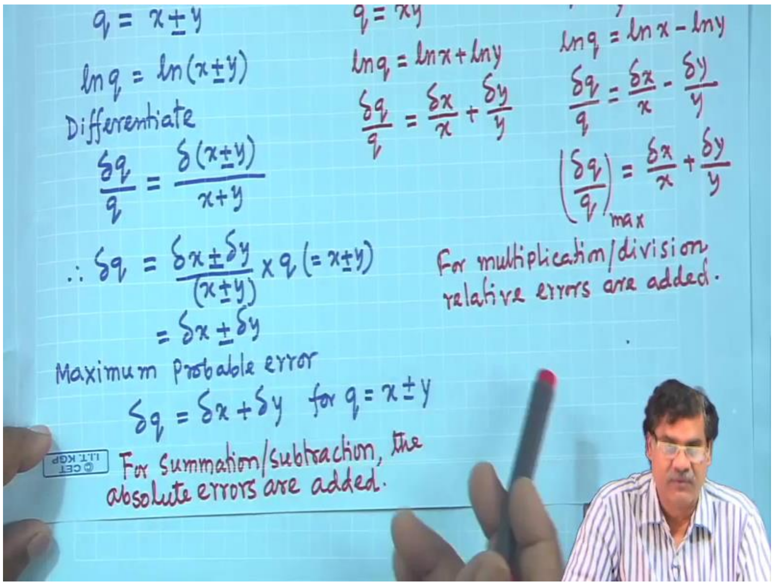

Which one will be maximum probable error. Del x plus del y, so that is why we write maximum probable error equal to del q equal to del x plus del y ok. So, this for x plus minus y, so this; so whatever we have learned that for summation and subtraction, these absolute errors are added for getting the error in the calculated parameter.

Similarly, for q equal to x y take log q equal to log x plus log y. So, del q by q equal to del x by x plus del y by y ok. Similarly, q equal to x by y. So, this log q equal to log x minus log y. So, del q by q equal to del x by x minus del y by y. Now, again this logic is maximum probable error, we have to find out maximum probable error. So, these are the relative individual error.

We will not write this minus sign. We will add them; so del q by q maximum will be del x by x plus del y by y. So, these are very simple way we use just quickly to find out the form of error of the calculated parameter from the measured parameter. Now how to calculate error; so that we have learned, this will apply for our example that to find out the volume of a cube.

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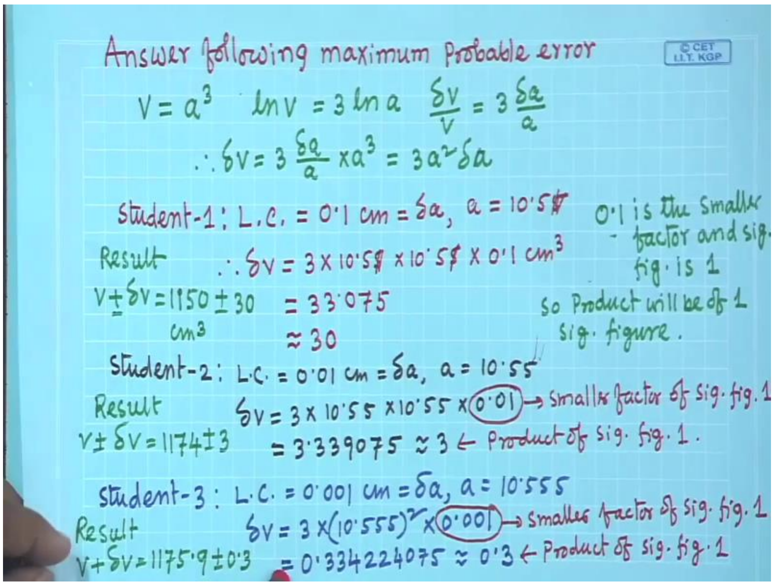

Let me go back there. This answer now we are writing following the maximum probable error. What is the relation what with volume and the measured parameter this side or edge of the cube. Volume v equal to a cube ok. Take log ln v equal to 3 log n log a. Del v by v equal to 3 del a by a right. Del v equal to 3 del a by a into v into v. So, v is a cube. So, basically 3 a square del a. Error in the in v, that will be 3 a square del a.

Now, for student 1 del a is basically least count of the instrument used to measure the length. So, LC - least count for student 1 it was point 0.1 centimeter right, and side was 10.5 right. So, a we know and del a also we know. So, if you put the 3 into 10.5 into 10.5 into 0.1 cm cube ok. So, this is coming 33.075. So, del v is coming 33.075 right. So, this we are writing approximately 30, why we are writing 30, why not 33? Reason is that; so, v we got from calculation (Refer Time: 23:12) the significant figure this v is 1150 right 1150.

Now, error we are getting here this thirty three point something. Now, this we are writing 3 reason is that the error will be generally your whatever result this rule is that this order of the last significant figure whatever the order of the last significant figure ok, and the error. Error has to be also of same order. This is the volume, in this volume what is the order of the last significant figure. Error also whatever we are writing, that has to be of the same order. Here this is the 5 whatever the order of this position of this 5, it is a.

It has to be of same order ok, so that is why it is that I cannot write 33, I cannot write 33. This is a significant figure 30 is the one right of this. Here this is a, so this last significant figure ok. So, it is a whatever order of this one and this error it has to be of same order, so that is why this.

Now, you see from significant figure, we are able to write correctly the answer also we are able to tell correctly the error in which order ok, in which digit the error is there and after calculation we are seeing exactly that is 2 right, so, but that time I was not able to tell this whether it will be plus minus 10 or plus minus 20, plus minus 30 or plus minus 40 ok, but now to gave that one we have to calculate the this error ok. So, it is basically plus minus 30.

Similarly, for student 2, least count is 0.01 centimeter that is delta a, a is 10.55. So, if you proceeds del v equal to 3 into a into a into this one. So, your result will be this. So, here actually I forgot to tell you actually in you remember for significant figure, what is the rule; this whatever the multiplication these parameters are there ok. So, these factors are there.

Simaller factors whatever the smaller factor will have the significant figure. So, result will have the same significant figure. Among these so 0.1 is the smaller factor and significant figure is 1. So, this is the result, I cannot accept that one. I have to write. Result has to be also of significant figure 1, so that is why it is 30, yes. Similarly here so smaller factor, this one the smaller factor and significant figure of this one is 1. So, this is the calculating result ok. Result it has to be of significant figure 1. So, it has to be 3 right. So, 3 is having significant figure 1 right.

If for student 2 result will be 1174 plus minus 3. Earlier we came to know that error will be at this last significant figure. It can be 4, it can be 5, 6, 7 whatever uncertain here. So, exact what amount of error that we are not able to say but now we are able to say it is plus minus 3. Similarly, if you proceed for student 3 then you will see the error, this error for this case is 0.3 ok. Following the same rule, following the same rule you can say it is a 0.3 ok. Result will be 1175.9 plus minus 0.3 ok. Actually from significant figure itself, we can find out this error it will be 30 or 3 or 0.3.

Now, that rule whatever rule I told you. From this result you can see you can conclude that the error and the result it has to be of same order, it has to be of same order. It has to be so of the last significant figure ok. Whatever the result, whatever the order of the last significant figure, error will be of the same order, error will be of the same order, error will be of the same order last significant figure error will be of the same order.

I think this is the good example and from this single example, I told everything; how to write the result, how to find out the error, and this is very important; generally most of the time, we do mistake for writing after calculation. From the calculator whatever result we get, from there up to which decimal point we should keep, etcetera.These are always confusing.

In experimental physics when I discussed this about significant figure; but I saw, there was confusion among the students and they requested me to repeat this class. That is why I repeated this again in this class, but in different way. Hopefully it will be useful for you. I will stop here.

Thank you.