

Experimental Physics – II
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Lecture - 54
Zone-plate theory

today I will demonstrate Fresnel Zone plate experiment orbital zone plate experiment.

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I will show you this is a zone plate; this is a zone plate, you can see this that surrounding are black and inside this black there is a transparent part. In that transparent part you can see this at the center it looks black. And then this whole transparent part is divided into many concentric circles. Many concentric circles, ok, the area of this circle is called Fresnel's half period zone.

There are many concentric circles which are Fresnel half period zones, now if we just stop, we just make that of the alternate zones means alternate zones will be blocked and other alternate zones will be transparent. If then that we tell that it is a zone plate Fresnel's zone plate.

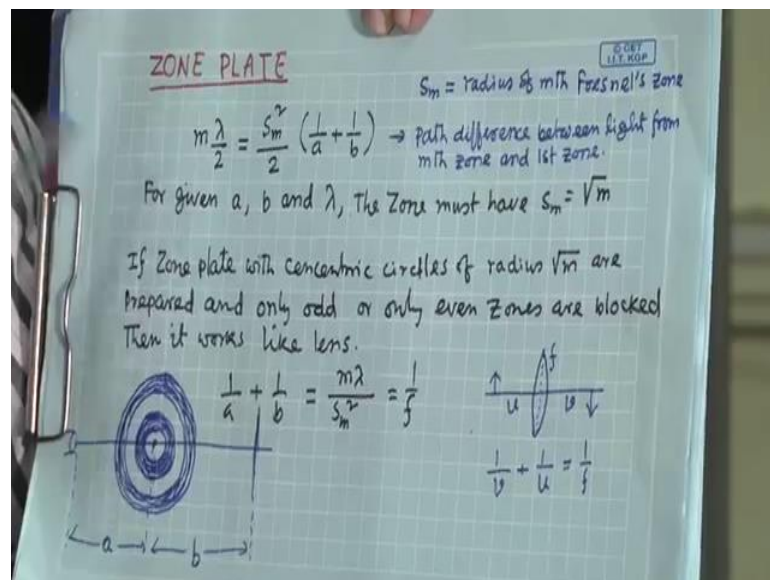
And now will use the zone plate for experiment. This zone plate is it acts like a convex lens, now convex lens have only one focal length, but this zone plate have multifocal lens. What are the focal length of zone plate? That you want to measure,

also we can find out what is the radius; what is the radius of the of these concentric circle? we will perform the experiment to find out to find out the radius of the; radius of the Fresnel zones.

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this very in computer we can make this dark by this kind of actually Fresnel zone this kind of you can make dark b alternate this circle will be black or blocked and alternate circuits next alternate circles will be transparent what will be the radius of circle actually that it should be the square root of m what is square root of m that I will discuss. we get

if you can construct this type of concentric circles with black circles and white circles and if you take print if you take print on this type of transparent sheet this type of transparent sheet

then that is the one of one pieces have we have cut from here and that we are using as that that piece we are using for our experiment. it is a is a very easy to construct this construct this on this zone plate and this plate will use for our experiment purpose. to understand that experiment actually we have to understand the theory behind this experiment that is very important.

here as a cold this these zone plate will behave like a convex lens, but it has multifocal convex lens. the relation we will get I will show you how it is has come. relation will get like this $\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$ and this S_m^2 is a S_m is the diameter of the m th zone m th Fresnel zone and this m is the order of the zone and λ is the wavelength of the light whatever we will use for that experiment.

what is a and b , a is the source distance light source distance from the zone plate and b is the screen distance from the zone plate. as (Refer Time: 06:41) it is the length this the a is the object distance and b is the image distance. you can see. like just $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ a and b if you equate to the u and v . it is a like $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ equal to $\frac{1}{f}$.

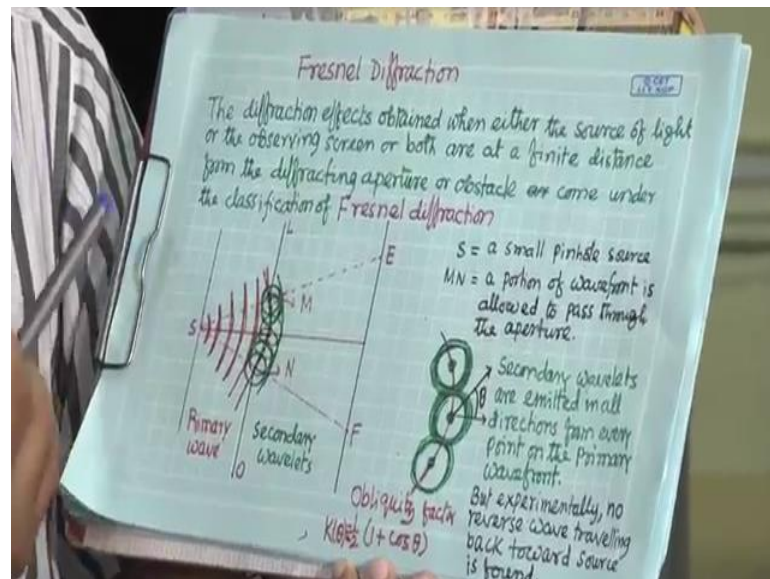
$\frac{1}{f} = \frac{m\lambda}{S_m^2}$ here $m\lambda$ by S_m^2 S_m is the radius of the image zone m th zone of the zone plate. this one here you can see this focal length for a particular λ it will focal length of these zone plate will the depend on the wavelength of the light. It will depend on the because your if a convex lens intake is does not depend on the wavelength focal length does the fixed and here you can vary the focal length and easily you can construct this type of m which we can use as a like a convex lens you know, so λ the wavelength and radius of these if you see here S .

this radius of the zone if you know the relation some relation of the radius of the zone with the of this number zone number m is 1 2 3 4 5 zone number. there is a relation I will discuss. this relation is this is same is equal to square root of a . square root that m is 1 to integer value 1 2 3 4 S_1 is 1, S_2 is square root of 2 means 1.414, then S_3 is square root of 3. m will be 1.732 theory has given us, the what will be the radius of the; what will be the radius of the zone plate radius of the half period zones of the zone plate.

And generally, the whatever I have showed you. using this formula one can in computer one can construct this type of zones and then we can take then in computer itself you can make alternative one dark and you can take print on this sheets and then it is a you can use as a zone plate.

experimentally we can find out what is the; what is the; what is the focal length of this zone plate as well as what is the; what is the radius of the of the zones of different number that you can find out from the experiment that is what we will demonstrate. how is formula has come in details I will discuss before demonstrating the experiment.

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you know this zone plate that is, I think. this whatever the it is walking like a convex lens. that this is diffraction phenomena it is because of diffraction. this type of diffraction there are 2 types of diffraction, one is Fresnel diffraction, another is Fraunhofer diffraction. this distinction between the Fresnel and Fraunhofer depending on the distance of the object on the screen from the getting; from the getting or from the yes.

in case of Fresnel diffraction, the object and the screen that distance are either both or one of them will be at highlight distance ok, if both are a at infinite distance then we tell the Fresnel Fraunhofer fraction. If one of them or both are at finite distance, then we tell it is the is then will see the diffraction that is called the Fresnel diffraction.

diffraction effects obtained when either the source of light or the observing screen or both are at a finite distance from the diffractive aperture or obstacle come under the Fresnel diffraction. whatever the experiment on zone plate will demonstrate that is Fresnel type of Fresnel diffraction type.

here high ends principle you know that if you have a source now this you will have the wavelength if this we tell the primary waves or primary wavelength then if you have a here whole aperture it is a dimension is very small compared to the; compared to the comparable to the wavelength then only you will be able to see that this diffraction phenomena if this whole that this apertures is very big compared to wavelength. you will not see the diffraction phenomena.

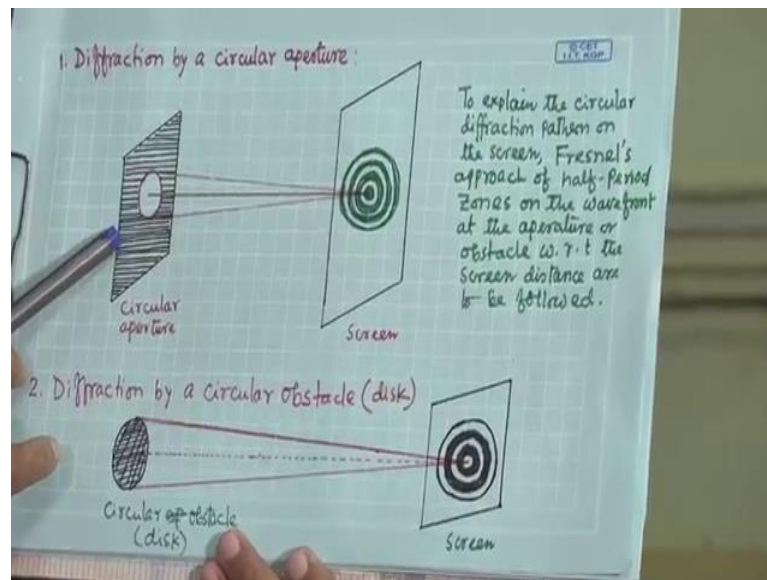
now, this it is a aperture it can be one obstacle also small size now; the wave front will fall on it and now this on this wave front each point will act as a new source and it will generate waves. that we tell wavelets. we will call them secondary wavelets. the secondary wavelets will generate secondary wavelets will generate from different points and they will interfere. actually, interference among the secondary wavelets or the diffraction

here it is a yes it is a small pinhole source and from there it is falling on a screen having a small apertures now, this N M portion wave front will be allowed to pass through it and then secondary wavelets will be emitted in all directions from here and they will interfere and on the screen you will get the diffraction pattern

here wave can move in all directions wave can move in all directions. it can go back side also, but experimentally no reverse wave travelling back towards the sources is found that is why this in the theory. obliquity factor is considered. that is $K \theta$ obliquity factors equal to $\frac{1}{2}(1 + \cos \theta)$.

So; that means, it is going in forward it is a $\theta = 0$ this $\theta = 0$. $\cos \theta$ is 1 it is the factor is $\frac{1}{2}(1 + 1)$ into half ok, now this if θ is 180 degree then $\cos \theta$ is minus 1. this obliquity factor is 0 this obliquity factor will be multiplied. this that will restrict the restrict or it will represent this experimental fact that reverse wave travelling back forward to the source will not be there ok, so that is called obliquity factor. Now so that diffraction Fresnel type of diffraction generally you can see by circular aperture

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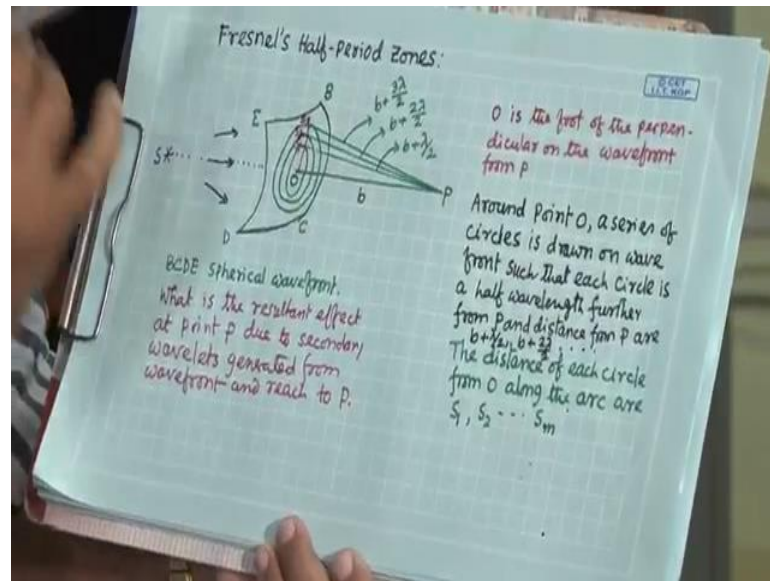
these are circular aperture and this light there coming it may be parallel light are coming. planer plane wave front source is at it at infinite distance, screen as is at finite distance. then you will get diffraction pattern on this screen like this concentric circle kind of diffraction pattern you will see if it is hole circular hole you will get circular pattern b dark b dark b dark pattern you will get. here depending on the I think it depends on the distance this center can be b of can be dark.

And also, you can see this type of diffraction pattern for a say circular obstacle this kind of things it will take a disc. light is falling no light will pass directly through this disc, but light will go from the surrounding if this is a very small object small disc then you will see the diffraction pattern and on the screen you will see this type of a gain pattern of the diffraction.

b dark b dark circular pattern you will see on the skin surprisingly you will see that; you will see that the center circle will be always b; will be always b. this is obstacle. no light passing through this; through this in this direction you know but at the center light is obstacle, but still you are getting that the center circle is b always this is b that is because of the diffraction and Fresnel type diffraction of course. zone late is this is of this kind this transparent region is divided; are divided. wave front will pass through this; through this circular aperture.

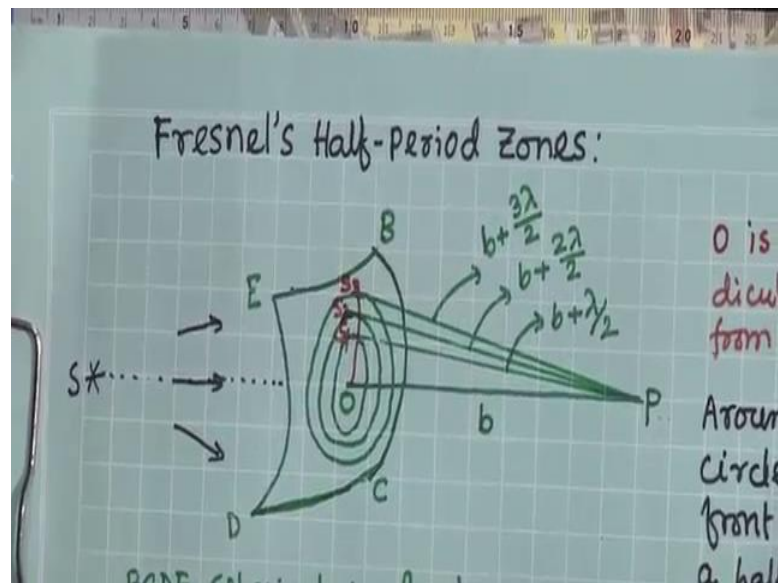
Now, this circular apertures on this wave front is will be of the shape now this wave front if we can. how is pattern or observe are found to understand that; to understand that one that Fresnel's half period zone that concept is introduced by Fresnel self. that I will discuss now how that Fresnel's half period zones are considered for explaining this type of diffraction pattern

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the Fresnel's half period zones. that wave front you have a wave front, and this is a point on the screen. like this from wave front secondary wavelet will pass in all the directions. Now, if I consider this point now some different points of the wave front wavelets secondary wavelets will come at point P ; that means, lighter coming from different distances that point P .

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Now, as I told interference among them will produce the pattern. superposition of the secondary wavelets at this point will give me the intensity of the resultant intensity at point P. amplitude here you know this that 2 lighter coming there in opposite phase obvious they will cancel each other we will not find any resultant light here.

If they come in phase so that will be that will increase the intensity of light, if they come in opposite phase, they destroy each other. if I know if I can calculate the light coming here what are the phase what the phase or what are the phase relation among them, then I will be able to calculate the intensity or resultant amplitude of light at this point and corresponding intensity I will be able to calculate. that is what Fresnel's that he suggested very beautiful way to do that.

what he has done. the from P this wave front or the aperture or the circular aperture center of the circular aperture if through which this if wave front will pass through if this distance is b. Now, you take light is coming for this center. that is that when it will reach here. this distance is b you know now light will come from this point if this distance is b plus lambda by 2; lambda by 2 then you can tell the path difference between these to light coming here it will be lambda by 2; that means, the phase difference will be pi. they will be in opposite phase.

if we take distance from P on the this on the on this aperture by taking the distance λb plus λ by 2 b plus 2 into 2 λ by 2 b plus 3 into λ by 2. consecutive that separation of the distance or difference of the distance will be λ by 2 half wavelength. taking these distance if you draw here to draw the circle concentric circle here in such a way that the distance from the this circle; distance from this circle point on the circle to this point on the screen P if that is b plus λ by 2, then next one on the next circle this is b plus λ etcetera.

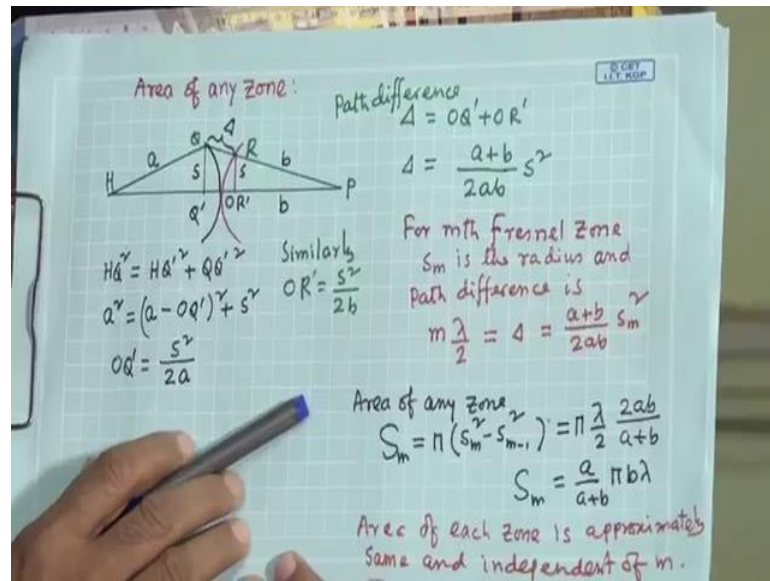
the we can show that; we can show that light will come from the alternate zones will be in phase alternator zones will be. these are called the zones and it is called half period zone Fresnel half period zone, because the light coming from the consecutive planes of, they have phase difference will be λ by 2 or phase difference will be π .

now, from the center if you considered this one it can be is can be spherical it can be spherical wave front, it can be planer wave front also. from center here centre of the aperture o. now, the radius of the radius of the circle concentric circle are is m , m is 1 2 3 4. S_1 , S_2 , S_3 and their corresponding that distance that is screen that is as I told b plus λ by 2 b plus λ etcetera ah.

if then this this is the source from here this is here this aperture circular aperture is there screen there having the circular aperture. light will pass through this wave front will pass through the to the aperture. on the aperture if you considered the this wave front now that wave front you are dividing into concentric circle and this concentric circles are constructed in such a way that this path difference between the consecutive circle light coming from the conductive circle reaching at 0.3 this for difference it will λ by 2 or phase difference will be π .

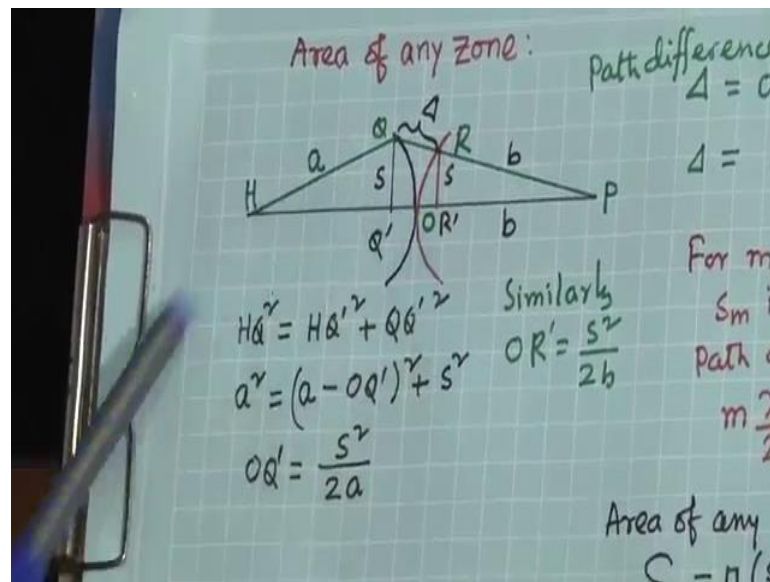
that is the Fresnel's half period zone. that is what I explained you what S is, what is b , what is λ .

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then if then what I can do, I want to find out the area of each Fresnel zone plate each area Fresnel zone not zone plate zone.

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this is the source, and this is the screen and this you're this is the circular aperture it means circular wave front wave front in circular shape. Now here we can see if it is a finite distance sources are the finite distance so; obviously, you will get the spherical wave front.

wave front here this distance if it is a this will be also a; this will be also a. they will be in same phase ok, but if you take an if you take a normal from Q on this H P on this line. this that Q dash is the footprint on this line. here we can see that these when light is reaching here, and you can.

this distance a when light is reaching here, the same time light will reach here and from there when light is reaching here that is b and light also from here it will reach at this distance that is b. total length of the light here you can see this to reach to the screen. that is a plus b here why we have drawn the circle spherical wave front because they are in finite distance if it infinite distance then we put write a planner 1.

now light is passing following this path. this will be a plus b and light is going following this path. this one is a and this one red circle here see this one is b. a plus b an additional path Q R additional path Q R it has switch over light is coming like this and light is going in this way path difference.

path difference or length difference will be between these 2 rays light will be here definitely it is Q R that Q R; Q R one can find out. here I have shown you. this the rectangular triangle you know just I will not just discuss. we can find out easily these are standard.

from here O this the this the centre these the O R dash and O Q dash that is equal to; equal to Q R. that is nothing, but the path difference delta. delta here equal to O Q dash plus O R dash, O Q dash plus O R dash and from the angle triangle ok, you can find out that O Q dash is equal to S^2 by $2a$ and O R dash equal to S^2 by $2b$.

del you are getting here a plus b by $2ab$ S^2 . What is S? S is the; S is the you can see this the is diameter of the circle is the diameter of the here wave front, on wave front you have concentric circle that center of the circle is O And from here you have to have concentric circle. the radius of the concentric circle will be S

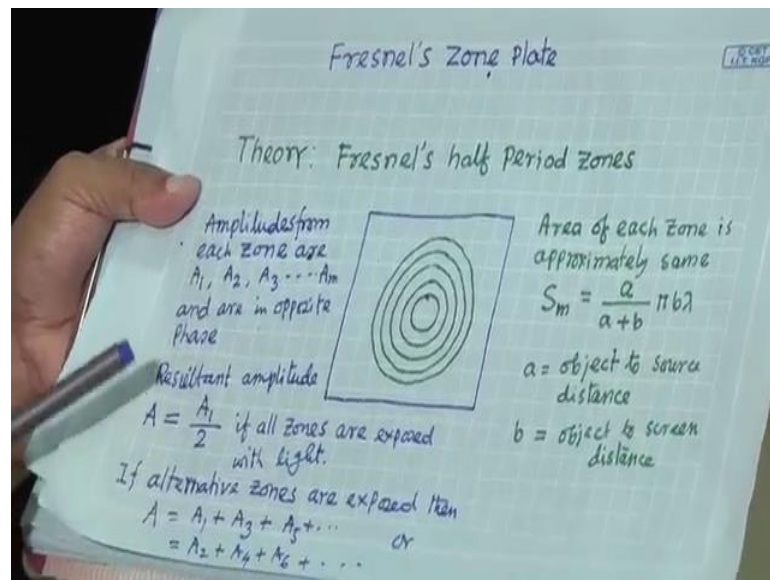
And source distance from this center of the circle is a and screen distance which b whatever here you are getting this path difference it will depend on the source and screen distance as well as it will depend on the radius of the; radius of the concentric circle which we are telling Fresnel half period zone. for mth Fresnel zone S m is the radius and path difference are; path difference is S^2 m square

Now path difference when it is $m\lambda$ by 2 when it is $m\lambda$ by 2 means you will. when this path difference will be a $m\lambda$ by 2 then what will happen, this 2 light having the path difference λ by 2 when m equal to 1 then λ by 2. they will in opposite phase λ by 2 their corresponding phase will π opposite phase you will get dark.

And when λ is 2; λ is 2 then it is λ then you will get b you will get the reason we will get on the screen we will get region of dark b; dark b and you will get in circular form because your object is of circular form. diffractive whatever we see either from reflection or refraction or diffraction or interface that we see the image of the object

Since our object is of circular form show on the screen you will see the image and their image we are getting because of the diffraction in this case diffraction we will see the circle concentric circle and that concentric circle whatever we will see here first order, second order, third order, fourth order will get. alternate order will be the dark and alternate next alternate order will be the b.

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as I told as I showed you, I think you will see; you will see this type of on the screen you will see this type of pattern diffraction pattern. area I got the; I got the; I got the radius; I got the radius of this these c concentric circle. what I got I think I got I can I calculated, and I got it let me see where I kept yes.

that I got. I got here I got this this the radius of the circle of the different order . here you can see this S_m^2 equal to $m \lambda$ by 2 divide by this part now, if you know the radius you can find out the area. Now the area of each zone area of each zone or any zone you can write πr^2 here $S_m^2 - S_{m-1}^2$ that there 2 concentric 1

this a difference of the between these 2 area will give the; give the; give the area of the between these 2 area between these 2 circle that you can calculate. that will be the S_m capital S_m . area of the m th zone that will come a by a plus b pi b lambda. interesting fact is that this area is independent of m so; that means, you can say that area of each zone first order, first zone, second zone, third zone area or set it is independent of m area is same.

this Fresnel zones about the Fresnel zones what we came to know, what you know now that area of each zone is same and light will come from that from each zone to the screen at a point P. this alternate light from alternate zone will be in phase, but light from the. light from the 2 consecutive zone. that will be in there that will be out of phase. now, if we can block the alternate zones means say 1 3 5 7 odd zones if you block it make it block and even zones 2 4 6 if you keep it transparent.

Then from that instead of circular aperture if you use this one whole region is the transparent. zone wise alternate one of transparent other alternate one is obstacle block. then light will come from this zone to the zone plate and these to the screen then what will see, what will happen that the we are going to study; we are going to study.

what will happens if we use this type of zone that is call zone plate Fresnel zone plate what will happen about the amplitude these to the screen at the point P.

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Amplitude at P due to whole wave:

$$A = A_1 - A_2 + A_3 - A_4 + \dots + (-1)^m A_m$$

If m is odd

$$A = \frac{A_1}{2} + \left(\frac{A_1}{2} - A_2 + \frac{A_3}{2}\right) + \left(\frac{A_3}{2} - A_4 + \frac{A_5}{2}\right) + \dots + \frac{A_m}{2}$$

$$= \frac{A_1}{2} + \frac{A_m}{2}$$

If m is even

$$A = \frac{A_1}{2} - \frac{A_m}{2}$$

Due to obliquity factor $\frac{1}{2}(1 + \cos \theta)$
 $A_m < A_1$ for m is very large

$$A = \frac{A_1}{2}$$

Intensity at P = $I = A^2 = \frac{A_1^2}{4}$

amplitude at P due to whole wave front. amplitude from region one because amplitude from each region of the area are same, but due to obliquity factor, but due to obliquity factor the amplitude will not be same. amplitude here the total light coming from a zone; from a zone that is the; that is the.

from first zone it will P amplitude A 1, second zone A 2, A 3, A 4, etcetera and already you know that they will be alternate 1 will be in phase and the even 1 will be in phase and odd 1 will be out of phase with respect to the even 1 for vice versa we can resultant amplitude we can take we can add this total amplitude of add this all individual amplitude A 1.

next if it is plus A 1 if you considered the next will be minus A 2 because there in opposite phase. then plus A 3 minus A 4 plus A 5 etcetera up to m. If m is odd and m is even here, I will not just describe it will take time. here you can show you can if it is odd you see I can write A 1 is A 1 by 2 A 2 is A 1 by 2 plus A 1 by 2 A 2 A 3 A 3 by 2 A 3 by 2 this I put in a bracket.

as if A 1 is of A 1 and A 3 will be equal to the A 2 that are this will be cancel only this and this term will be there A 1 by m A 1 by 2 plus A m by 2, even for m is when m is even this is m is for odd. then you will get same way if you proceed you will get A equal to A 1 by 2 minus A m by 2.

Now, due to obliquity factor I explain it mth zone it is outer. obliquity this factor theta will be higher. this term will be this K theta about this obliquity factor it will be smaller. A_m by 2 will be very small compared to; compared to A_1 by 2. one can neglect the same it will be A_1 by 2. if all zone is open then the amplitude resultant amplitude will get a point P that is half of the amplitude of the first whatever the amplitude in first zone this is very small one small intensity.

it will be A_1 's intensity square of the amplitude. it will be A_1 square by 4 ok that will be the amplitude. Now, if I block alternate one then what will you get then what will get, what will get it is an obvious A will be A_1 plus A_3 plus A_5 plus A_7 etcetera or it can be a will be A_2 plus A_4 plus. all will be added you know all will be added when all are open this when all are open transparent circular aperture is transparent you are getting only A_1 by 2.

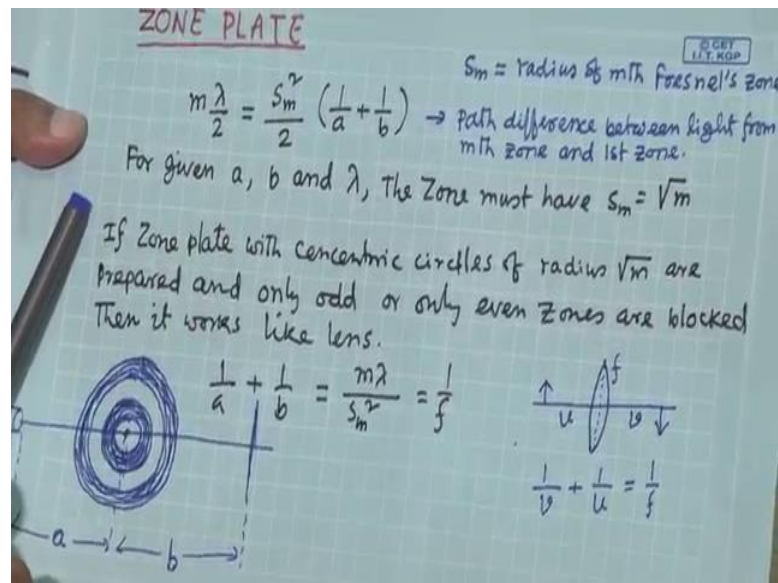
Whereas, if alternate 1 are blocked you are getting it is a just half amplitude is half of a of one zone and on the other hand you are getting summation of amplitude of all alternate zone. you can imagine A_1 plus A_3 plus A_5 plus if it is say m is 1000 or 100. now, intensity is square of that one is 10000 or I do not know 1000 square is 10000 times more than the intensity of when the whole region are open transparent then what are the intensity will get it alternate 1 is blocked you will get form equal to 100 you will get 10000 times intensity more.

it is a you can think that we focuses as if this is using this zone plate one can focus the light at a point P it is nothing, but like a convex lens we use convex lens to focus the light at a point at the focal point, sometimes one can burn the papers also this is the beauty of this zone plate now, I think I have anything else to tell you yes.

I think we will or doing experiment what is the working formula I will use that is from if a just summarize it is. we have seen the area of each zone is approximately same. that is area S_m capital S smallest we have considered for smallest we have considered for the radius of the zone.

that is a by a plus b pi lambda. b I have just here and a is the object to source distance and b is the objects to screen distance And when all zones are open then amplitude will be half of the amplitude of first zone only, if alternate zones are exposed only then it will be amplitude will be A_1 plus A_3 plus A_5 or A_2 plus A_4 plus A_6 etcetera.

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that is what from the discussion we learn and then what we are getting and you have seen that a plus b by a b by to a b into S m square equal to m lambda by 2 that what we have seen , when we derived again should I show yes I think here that is what that is not the relation we got it this relation is we will use for this experiment. now, you have this relation (Refer Time: 49:46) and where is S m is the radius of the m th Fresnel zone.

And this is the path difference between light from m th zone and first zone the path difference of light from the m th zone compare to the first zone if this is a relation we got. for a given a, b and lambda means if you keep constant the source distance the screen distance and the for particular light wavelength is constant. Then the zone must have radius a same equal to square root of m from here we can easily see S m square equal to m because lambda a b are constant

from here you can tell that S m means the radius of the zone must be square root of m, if zone plate with concentric circles of radius square root of m. now it is in your hand how to construct that also I showed you discussed you or prepared and only odd or only even zone are blocked, then it works like a lens then it works like lens because 1 by a plus 1 by b equal to m lambda by S m square radius square this equal to you can think this a 1 by f. this the source and screen distance.

it is just like a source u and screen distance v image distance. equal to 1 by f this relation is or convex lens. from here you are getting this the relation it is a similar relation like

convex lens you are getting where important is this part $\frac{1}{b} + \frac{1}{u} = \frac{1}{f}$ where $\frac{1}{f}$ is $\frac{m \lambda}{S m^2}$ here you can see for a particular wave length. now, f focal length will depend on order on m

for different m you will get you will get different focal length it is not 1 it is not 1 focal length you know it is not 1 focal length it is the m number of focal length you will for a particular zone plate. that is why we tell is a multi (Refer time: 53:04) convex lens.

this relation is in your hand now what you want to do here, from here you can see if I can find it if I can find out the focal length; focal length for experiment what will do from this relation you see that if a is the source distance if now source if a if I take parallel rays if I take parallel rays source that light falling on the zone plate if it is parallel

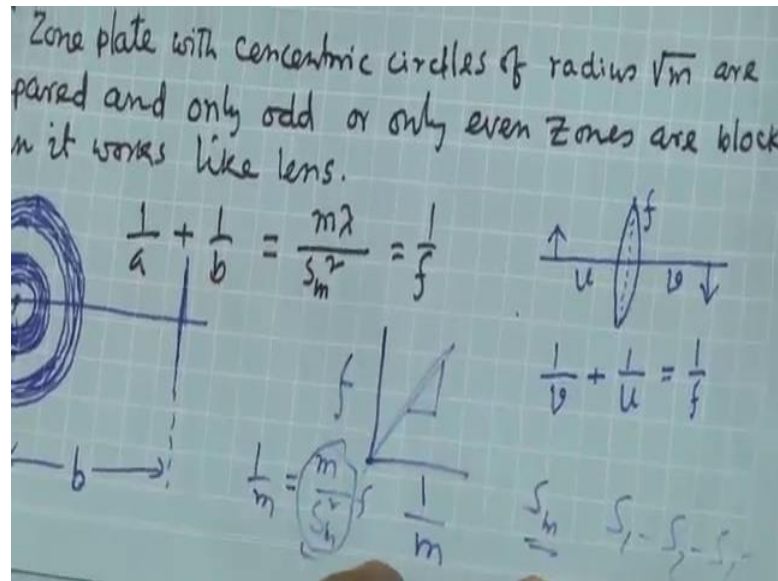
then we can take this is infinity object distance is at infinity. $\frac{1}{a}$ it will be 0. will design the experiment such a way this part we can make it 0. now, $\frac{1}{b}$ $\frac{1}{b}$ is nothing, but the $\frac{1}{b}$ is nothing, but the distance of the screen distance of the scene

this $\frac{1}{b}$ is will be equal to $\frac{1}{f}$ f will be $\frac{1}{f}$ will be b experimentally we can find out the object distance we have to find out the sorry we have to find out the image distance from the zone plate ok, object distance is have infinity if you can find out the image distance from the zone plate then we know that is nothing, but the focal length.

Now, as I told this it is it has multi it will have it will be multi (Refer Time: 55:19) for m equal to 1 we get f_1 or b_1 . f_2 will get for m equal to 2 ok will get for different order. for, so what we have to do, we have to find out the position of the we have to; we have to; we have to find out the.

if light focus if light is focused it will be focused at different distance. if you can find out this different distance. that will be if and m th that will be different f_1, f_2, f_3 and that will be for m equal to 1 2 3 etcetera. will experimentally will find out the b or f b or f m for different f for different m now we will plot say $\frac{1}{m}$ and f ; f is b it will plot. will get a straight line; will get a straight line that is the slope.

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that is slope will be λ by S_m^2 1 m per 1 m and then equal to $m S_m^2$ f
 this will be the slope this will be the slope and so will experimentally we will find the
 data and plot this graph and from graph we will find out the slope and if you find the
 slope then you can able to find out the S_m S_m we will get; S_m will get. for S_1, S_2, S_3
 etcetera you will get and then experimentally that radius you're finding out. then you
 compare with this relation is S_m should be square root of m .

this the theoretical value now, experimentally you can find out and compare it whether it
 is following this relation or not. that is the experiment will demonstrate. I think I will
 stop here now and this in next class I will just demonstrate this experiment.