Experimental Physics - II
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Lecture – 41
Theory of Diffraction (Contd.)

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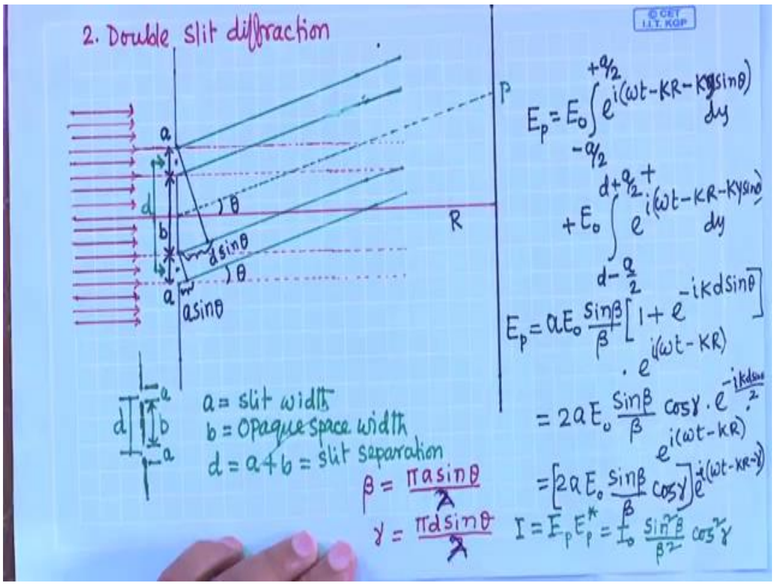

in last class I have discussed about the single slit diffraction, now I will the discuss the double slit diffraction. it is a similar treatment double slit diffraction. So now, the single slit is replaced by double slit; this is the slit with one slit, and this is the another slit. both are having the same width a and there is an opaque space between these two slit that is length is b. here slit width is slit width is a, opaque space opaque width opaque space width is b and slit separation. slit separation if a middle of this slit to the middle of another slit that is d.

that is called slit separation. here separately I have shown this 2 slit of width a and their opaque's width is b and theirs slit separation is this d Now, same way this plane waves are coming parallel rays are coming falling on this double slit and then there will be diffraction, diffractive ways from both slit Now, they will interface these are the secondary wavelets coming from these books slit.

And there will be the interference among the secondary wavelets and we want to find out the intensive distribution on the screen placed here same way from the middle of the slit if I draw a normal and this screen placed normal on this line and if we consider a setup parallel rays; this angle is theta. same so here this whatever this light is coming that is E 0 e to the power i omega t i omega t minus KX. that is here minus KR minus KR minus KY sin theta dy.

if we choose same way in single slit, we have chosen a point along the say if it is y axis. it is a y point y and there we have considered very small width that is dy. your amplitude will be E 0 dy and this your e to the power i omega t minus KX. in that that KX that X are plus additional path that is del. that del is KY sin theta ok, now here you see here what we are doing this is 1 ray.

this ray and this ray what are the path difference between this 2 ray? See if we draw a normal here; this path is same and additional path here. that is the this is a form single slit it will be a sin theta a sin theta. that is why we have between Y sin theta and integration we are doing for this part minus a by 2 to plus a by 2 for this part first Now, for this slit so we can integrate that d for this slit width. d is this what will get d? Slit separation this b plus half a by 2 plus half a by 2 plus half a by 2 plus half a by 2

if you this column this from this part there is no contribution, except it is introducing the first introducing the path difference otherwise no light is coming from this. that is why here from this d plus half means this end and minus half means this end this limit is for this other slit from this slit this part plus from other slit with respect to this with respect to this see this actually we have to; we have to get the from this place this way, what will be the; what will be the path difference? that will be d sin theta that will d sin theta.

And for this part it will be a sin theta if you consider that from here whatever; you are getting the path difference additional path for this one. path is increasing, increasing for this one d sin theta and for this one you are getting additional paths. a sin theta a sin theta plus d sin theta one can write this we have made for this part this integration part minus a by 2 to plus a by 2 and for this one, for this part we are putting this integration limit d minus a by 2 to d plus a by 2

And so, these are coming from the contribution from the 2 slit and we will get at p this result at no 1 amplitude E p. if you proceed if you just proceed same way as we have done this; obviously, this part is same as the single slit part. this will give sin beta by beta, a is 0 sin beta by beta this part and this other part it will have also the same sin beta by beta because plus a by 2 minus a by 2 is there and in both the limit d is there. that d is common part it will be here this part into this part will be for 2nd term.

if you take this part will be common 1 plus this e to the power i omega t minus KR. this part will give cos gamma e to the power i K d sin theta by 2. what is gamma? gamma is pi d sin theta by 2 and beta; obviously, pi a sin theta by 2 that is the thing in this path difference d sin theta. that is why it is a K d sin theta divided by 2. 2 pi by lambda. Where is lambda? K has let me just check whether any lambda is missing K d sin theta, for beta I have written beta should contain is the e to the power I am getting cos gamma

Additional term is this additional path that is K d. I think one lambda will be there K d a sin theta K a sin theta K is 2 pi by lambda; 2 pi by lambda. But where that lambda has gone? I think beta yes of course it is not 2 it is the it has to be lambda; it has to be lambda because these two 2 pi so these 2 will go. pi will be there and by lambda will be there. this is not 2 it is the lambda. pi a sin theta by lambda pi d sin theta by there is a gamma. ultimately, we are getting this term 2 a E 0 2 a E 0 here you can notice for double slit, for single slit it was a E 0, for double slit it is 2 a E 0.

E 0 is we are telling that is a its the amplitude per unit length. this is the strength of the amplitude of the wavelets per unit length. if it E 0 opening is a plus a 2 a. that is why this 2 a E 0 that is the amplitude, that is the amplitude and this now in amplitude after that is modified with these two terms sin beta by beta and cos gamma. your intensity will be E p E p star equal to I 0 sin square beta by beta square cos square gamma.

this sin square beta by beta square is; obviously, for the is the single slit effect, diffraction effect and this cos square gamma you remember there for interference. this is the term for interference we are getting two term: one is for diffraction another for interference, diffraction for overall this is a diffraction. But, one term is contribution from contribution of single slit diffraction and another term is the interference between the wavelets coming from the 2 slits.

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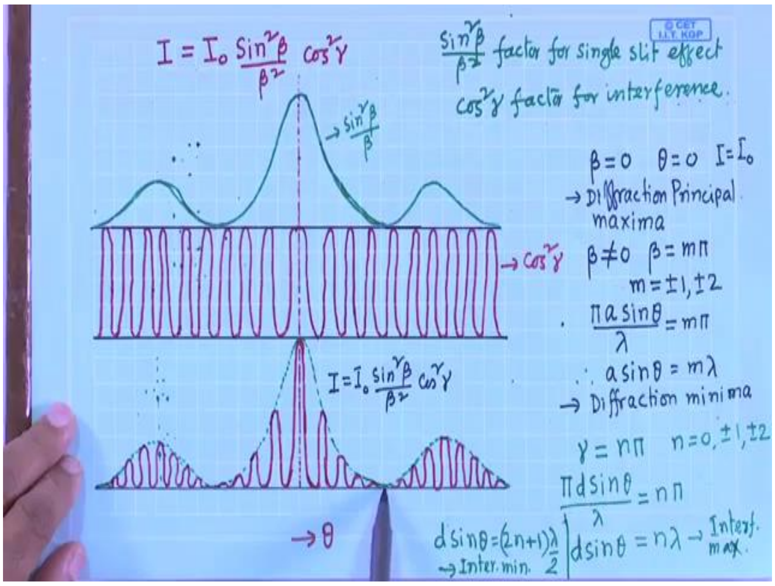

if plot if I plot this, I equal to I 0 sin square beta by beta square cos square gamma. sin square beta by beta square we have seen; this is the principal maxima, diffraction principal maxima and then diffraction minima then secondary maxima. there are more and this term cos square gamma; as a function of gamma. both are as a function of theta here that is why I have written the function of theta.

if you plot you will get this type of wave cos square gamma cos square gamma. this the it is a cos square; this graph is for cos square gamma and this graph is sin square beta by beta this two will overlap. And, what you will get? You get this the resultant diffraction pattern from the double slit

as if these are the interference maxima, minima and maxima minima. difference and this in double slit diffraction we are getting interference, but this interference now its intensity is not constant for all. It is the drastically modulated with the single slit diffraction effect you will get this type of variation. here at theta equal to 0 you will get maximum intensity then increasing decreasing. So now, here one thing is that this it is a varying this maxima minima. it may happen that they are supposed to be a maxima interference maxima here at the same place of diffraction minima.

in that case that diffraction interference maxima will not be able to see because, there is no light in that direction, in that particular theta direction that is why you are grating diffraction minima. that we tell this that order is missing that interference order is missing; I will discuss that one. for diffraction already we have discussed this what is the condition for diffraction minima a sin theta equal to m lambda and for diffraction interference maxima. gamma equal to n pi this is the gamma equal to n pi.

this is the cos term; whatever this it was sin term n pi whatever for sin it will give minima. that n pi or n pi for cos term it will get maxima. this is the condition gamma equal to n pi that is the condition for the interference maxima. for interference maxima d sin theta equal to n lambda ok; d sin theta equal to n lambda.

that is the condition for interference maxima; obviously, for interference minima that will be d sin theta it will 2 n plus 1 lambda by 2 here important is diffraction minima a sin theta equal to m lambda and interference maxima d sin theta equal to n lambda; that is m lambda here n lambda.

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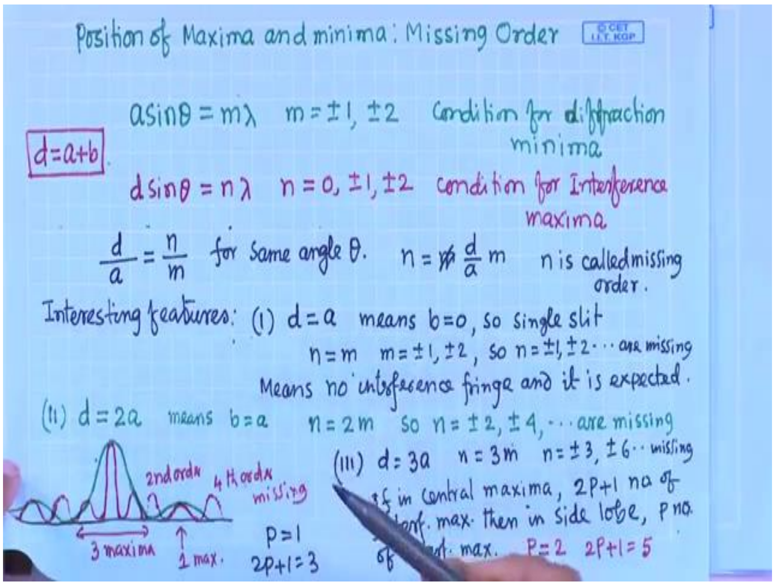

position of maxima minima, if you consider we can get the see concept of missing order. a sin theta equal to m lambda condition for diffraction minima, d sin theta equal to n lambda condition for the interference maxima. For same angle we can write d by a equal to m by n by m n equal to d by a m. n is called missing order for mth diffraction minima. n will be the missing order of interference maxima that m and n that will be decided by this value of d and a.

interesting feature we can discuss considering d and a value, if d equal to a if d equal a then n equal to m. m is 1 2 3 n is 1 2 3 so; that means, all interference maxima are missing: first order, 2nd order, 3rd order. there will not be any interference fringe interference effect that is expected because d equal to b means d is a plus b; that means, that b will be equal to 0 spacing between the 2 slit will be 0, that may actually it is single slit. that is why we will see only single slit effect

when d equal to 2 a means b equal to a n d by a will be 2 n equal to m. missing order will be 2nd order, 4th order, 6th order they will be the missing order. that interference maxima, the position will be exactly at the position of the first diffraction minima, 2nd diffraction, 3rd diffraction minima; that is why they will be the missing. other case if d equal to 3 a then 3rd order, 6th order, 9th order interference maxima will be missing.

here you can see that this is for 2nd order missing, 4th order missing means d equal to 2 a. 2nd order missing this is the; this is the central one at theta equal to 0. this is the central maxima n equal to 0, then n equal to 1 1st order, then 2nd order missing, 3rd order, then 4th order missing then other side minus n equal to minus 1 minus 2.

you can see this in center in principal maxima diffraction maxima, when we are getting 3 interference maxima then in lobes side lobes, we are getting 1 interference maxima because, 4th-one will be missing. that is how we tell when that order if 3 numbers of a principal maxima in the central fringe, in the central fringe central in the central interference in the central diffraction maxima if 3 interference maxima's are there; then inside there will be 1 there will be 1.

when p equal to 1 if in central maxima 2 P plus 1 number of interference maxima, then inside lobes P number of interference maxima when d equal to 3 a 3rd order is missing, 4th order is missing, 6th order is missing. in that case in side lobes you will get P equal to 2 whereas, in central lobe you will get this total 5 2 P plus 1 this 5; that means, if you find; if you find the interference maxima in the central lobe in the central diffraction maxima and in the side secondary maxima. how many are there?

you can tell about the missing order which orders are missing and from there we can find out basically, if you can find out this dn a ratio d and a ratio 3rd order will be missing. whatever this we are getting this how many in central? plus 1 divided by 2 that will be the 3rd order in this case this is case this 3 plus 1 divided by 2 2nd order.

that order 2nd that order and this multiple of them will be missing, integer multiple of them will be missing in 2nd 1 into 2nd 2 into 2nd means 4 3 into 2nd 2 means 6. that order will be missing and other case this 3rd order and then 2 into 3 6 order 3 into 3 9th order will be missing ok; these are the missing order.

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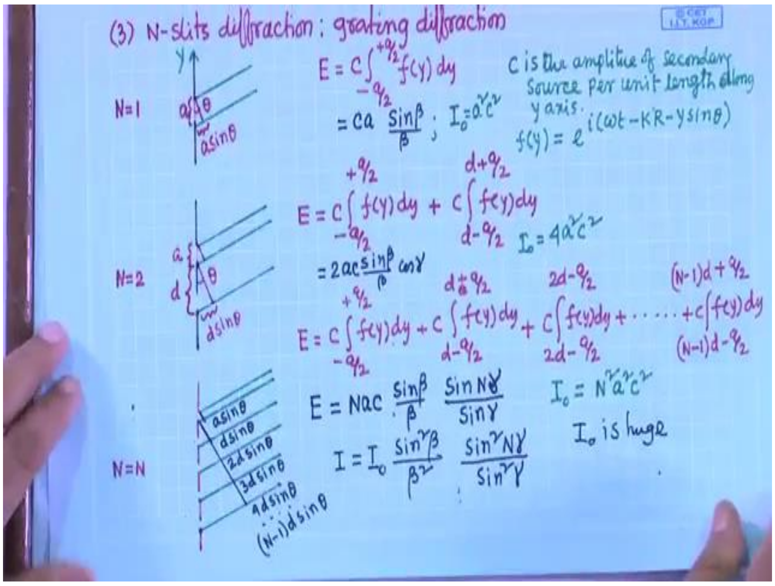

Now, if you considered the N slit diffraction is the same way you can proceed N slit diffraction that is the grating diffraction, grating diffraction. here you have N number of slits. we have to integrate we have to integrate. for single slit E equal to (Refer Time: 23:28) E 0 instead of E 0 I have written C, C is the amplitude of secondary source per unit length along y axis. as I mentioned earlier, I wrote E 0 so now, I have written C.

for single slit this way we proceed, for double slit this two term will proceed. similarly for N slit will proceed this from the single slit and then that this is for 2nd slit ok, d plus a by 2 d minus a by 2 then for 3rd slit that distance 2 d and then plus minus a by 2 2 d minus a by 2 plus a by 2, I think it will be plus for N slit Nth slit that will be N minus 1 this is the 3rd slit 2 d. it is a for Nth slit N minus 1 d plus a by 2 N minus 1 d minus a by 2 these are introducing path difference with this with the light coming from different slits.

if you proceed if you proceed you will get here. Nac a to a for double slit you got 2 a So now, N number of Na and that E 0 instead of E 0 I have taken this C this is amplitude you will get; amplitude we will get modulated with the sin beta by beta And, another term has come instead of cos square a cos gamma here sin N gamma by sin gamma, this term has come just if you proceed it is a easy to do it; you will get this here I 0 is N square a square c square, generally N value is a for N number of slit N values is very high very large. that is why I 0 become very large.

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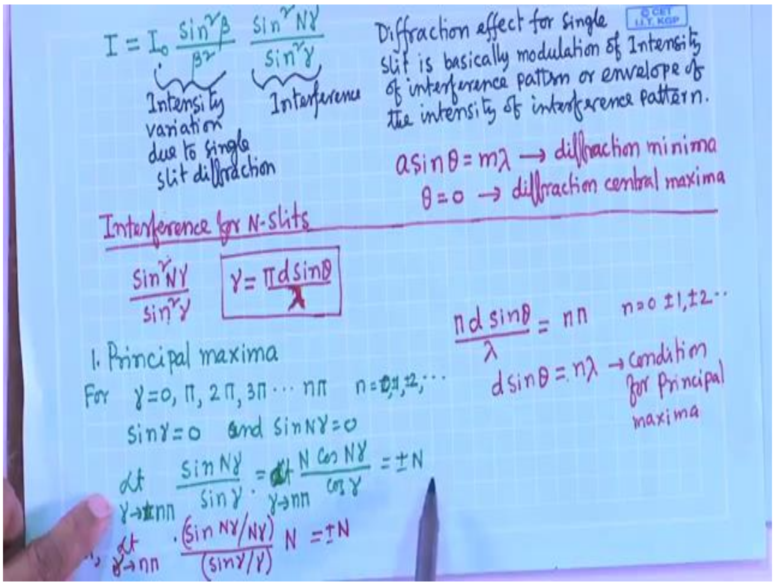

Now, this is the intensity expression I equal to I 0 and then this part. it is a this will give the intensity variation due to single slit and this will give the; this will give the for due to interference. if you proceed this the for diffraction that is fine now for interference for N slits this term now if gamma equal to 0 is gamma equal to 0. What is gamma? Pi d sin theta by lambda same. gamma equal to 0 or pi or 2 pi or 3 pi n pi. then sin gamma will be equal to 0, also sin n gamma will be 0 both on like sin beta by beta similar situation here.

limit gamma tends to n pi this term will give plus minus N. how it you know sin theta by theta; theta tends to 0 equal to 1. this sin N gamma by N gamma thereby sin gamma by gamma I have to multiply 1 N this limit if you put limit sin theta by theta, theta tends to 0. that way both term will be 1. you will get plus minus 1 or just not 1 N or N. condition for pins whatever we are getting this maximum intensity, when we are getting? when gamma equal to n pi when gamma equal to n pi and we are getting for that for this condition.

gamma equal to n pi means gamma equal to n pi. this is giving d sin theta equal to n lambda ok; this condition is called the condition for principal maxima. is an exact same condition as for interference we have interference maximum, we have seen for double slit for N slit this is a is not it is not the interference maxima here it is maxima, but there are other maxima. that is why this we will tell this as a principal maxima are the condition for principal maxima.

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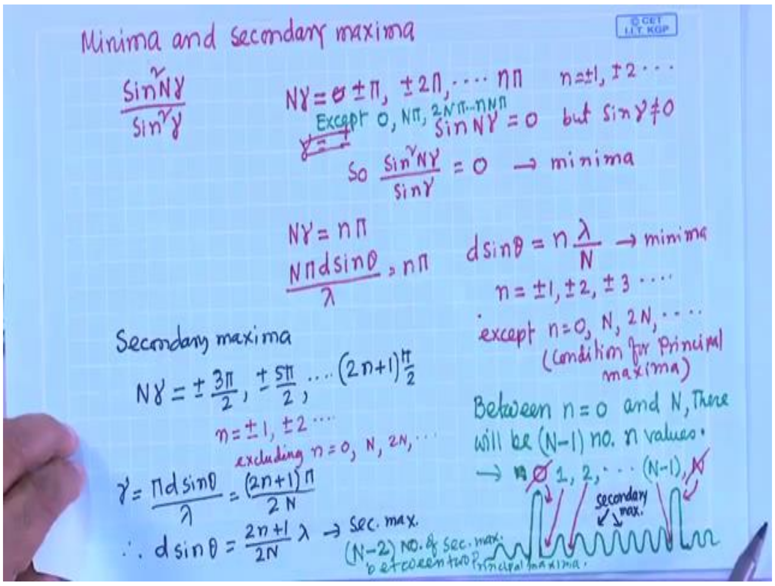

And, if you proceed you will get minima you have maxima. there will be minima and then also there are maxima's this called secondary maxima this is the term. N gamma equal to plus minus pi N gamma equal to plus minus pi plus minus 2 pi it is the n pi If then this part will be 0 not the sin square gamma will not be 0 because, that case gamma equal to n pi by capital N this will not be 0; that there that will be the condition for minima.

here this we have written except 0. pi n pi but n it cannot be 0, it cannot be capital N capital 2, this 2 capital N etcetera because this condition for that case this sin gamma will be 0. that is the condition for principal maxima; this is the condition for this is the minima interference minima this is the when it will be 0 this is the condition for minima and that condition d sin theta equal to n.

if you from here you will get n lambda d sin theta equal to n lambda that principal maxima divided by capital N. that is the that is giving the condition for interference minima ok, where n equal to plus minus 1 plus minus except this is the 0 capital N 2 N, that is the condition for principal maxima and secondary maxima secondary maxima in between here I have shown this is the these are called the principal maxima; this is the interference. principal interference principal maxima and you are getting the pi by n 2 pi by n capital N. these are the up to N minus 1; these are the condition these are the minima interference minima. in between there must be maxima and that is called that there must be maxima, that is called the secondary maxima. And, condition for secondary maxima is the secondary maxima is d sin theta equal to 2 n plus 1 by 2 capital N lambda because, you will get this parts N.

this when you will get this one? When you will get the N gamma N gamma will be when a pi by 2 3 pi by 2, but pi by 2 is not allowed, pi by 2 is not allowed. why not allowed? Yes, but it starts from 3 plus minus 3 pi by 2 5 pi by 2. Why it is not allowed, that I can say because the here I here pi by n equal to 2 n plus 1. n; n 0 is not allowed ok, n equal to 0 that is not allowed for no cases is the 0 is allowed, because n equal to 0 and n equal to N.

that are not allowed n equal to N 2 N is the that are not allowed, there are principal maxima that is why for this case n equal to 1 plus minus 1; it start from 3 5 7 etcetera. these are the condition from if you put this condition putting gamma value, we will get condition d sin theta equal to 2 n plus 1 by 2 N lambda. secondary this is the condition for secondary maxima, and you can check that when you will get between these two principal maxima between these two principal maxima, if there are N minus 1 minima. then secondary maxima will be N minus 2 secondary maxima will be N minus 2.

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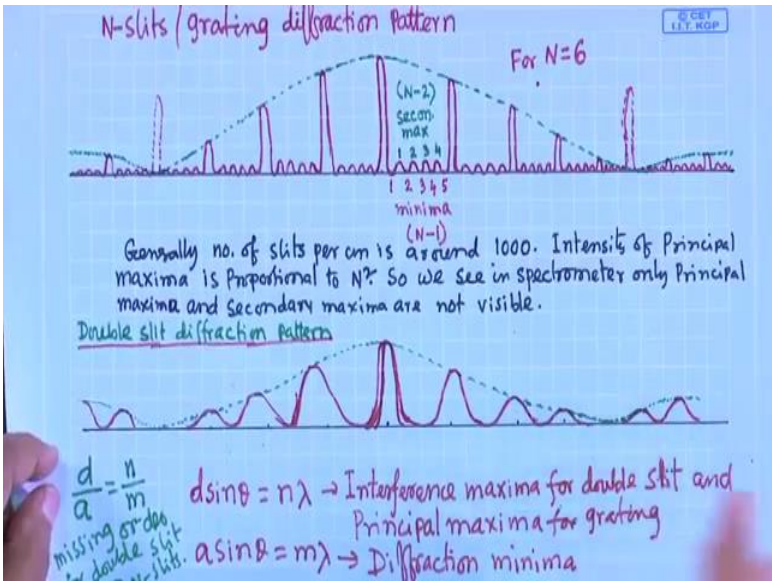

for N slit diffraction grating you will get this principal maxima and in between you will get minima, you will get minima maxima minima maxima. these are the secondary maxima and this whole things are modulated with this single slit diffraction effect. if slits this number of slit per centimeter or per inch if this generally, it is very high if it is around 1000 you know this, I have drawn for N equal to 6. if it is 1000 so now, imagine within this so many principal maxima you will get.

And, this most of the intensity will be occupied this principal maxima and this in between this secondary maxima. We comparatively intensity compare to this intensity of principal maxima these are so weak and if slit number are higher. this will be even weaker we will not be able to see, generally we this type of understood is the similar to the double slit experiment.

The pattern we have seen for double slit, similar pattern we will see only difference is that here these are the principal maxima. There interference maxima minima were there, here we are grating principal maxima and in between there are many maxima and minima.

that may not be able to see. and this missing order in this case the missing of the principal maxima and condition for principal maxima. the same way for double slit whatever d by a equal to n lambda. we can find out the missing order for the principal maxima in case of grating

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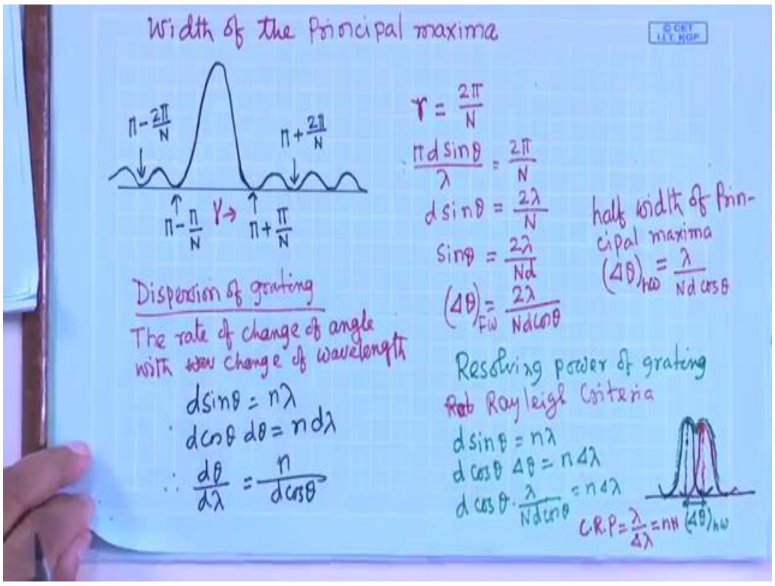

the width of the principal maxima; width of the principal maxima. this diffraction this principal maxima which is a pi plus pi by N. this is the minima. next minima are pi plus 2 pi by N. why pi? This we have to see the condition from that condition we can see; this is the principal maxima principal maxima we get at N equal to pi. If it is pi, then if it is pi then this one will be pi plus pi by N 2 pi by N 3 pi by N. Similar other side minus pi by N pi minus 2 pi by N; here gamma equal to this width is width is 2 pi by N

gamma is this from here sin theta equal to 2 lambda by Nd. is the delta theta full width; full width; full width you can get is just differentiate it cos theta del theta. this full width is 2 lambda by Nd cos theta. half width will be half width will be lambda by half of it will be lambda by Nd cos theta. half width of the principal maxima is this.

this is required for discussing this other thing. now dispersion of the getting dispersive power of the grating. What is dispersion power? The rate of change of angle with the change of the wavelength Diffraction angle rate of change of the diffraction angle with the change of the wavelength. If wavelength is changed then what will be the change of the angle of the diffraction for same order?

we know this for diffraction principal maxima that condition d sin theta equal to n lambda, d cos theta d theta equal to n d lambda If there is a change of lambda what will be the change of theta for a particular order n? d theta by d lambda equal to n by d cos theta this is the I could tell the dispersive power dispersion of the grating. it depends on the order and it depends on the d value and theta value.

and resolving power of the grating is nothing, but this if for a lambda we will get the we will get the principal maxima and for another lambda, lambda plus del lambda we will get another set of principal maxima. Now, these two principal maxima of same order, say first order for wavelength lambda and wavelength lambda plus del lambda. the principal maxima for these two wavelength for a particular order whether we can whether this two peak coincide with each other or whether this two peak we can see separately If you can see separately then you can tell this is a resolved.

two fix for corresponding to a wavelength so; that means, if you tell that this wavelength is can be resolved. that is the whether this grating can resolve it or not. it depends on what? that is the resolving power of the grating. Rayleigh criteria for that Rayleigh criteria for that this is one principal maxima for a particular wavelength, and this is red one is for another wavelength. now this for same order this two peak if peak of one is at the minima of the other one and vice versa. For that angular separation, for that angular separation we can resolve we can see these two separately like this green one

If this separation is less than that then we cannot see; we cannot see them separately, it will be it looks one broaden peak and of course, if it is separation is greater than that; obviously, it will be seen easily. minima we need this separation. maxima of one principal maxima, when it will coincide with the minima of that one other one or vice versa then it can be resolved

that is a call the Rayleigh criteria ok, according to this criteria d sin theta equal to n lambda. for different wavelength if we differentiate this cos theta del theta equal to n del lambda del theta what the minimum separation, we need that is the Rayleigh criteria. this is the this this is nothing, but the half width of the principal maxima.

half width of the principal maxima, we found that lambda by capital N d cos theta; see if I put lambda by capital N d cos theta equal to n del lambda. from here you are getting lambda by del lambda equal to n small n into capital N ok; small n into capital N you are getting. this is called chromatic resolving power. This resolving power in terms of wavelength, resolving power also defined in terms of the separation of the two object whether we can resolve or not.

but here in terms of wavelength that is why it is called chromatic resolving power. that is that depends on the order of the principal maxima as well as it depends on the number of lines, number of slits in the grating capital N. that is what about the one of our diffraction. single slit, double slit and gratings I discuss, we will demonstrate the experiment in the laboratory.

Thank you for your attention.