

Experimental Physics - II
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Lecture – 32
Interference phenomena (Contd.)

in last class we are discussing out the superposition of waves. we have seen that depending on the wavelength frequency amplitude and direction of the light, if we considered to two waves depending on their direction of the motion. there are three phenomena observed; one is interference, second is standing wave and third is beats. then we will continue the discussion on the interference.

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Interference

Let us come back to Interference

$$Y_1(x,t) = A \sin(kx - \omega t + \phi_1)$$
$$Y_2(x,t) = A \sin(kx - \omega t + \phi_2)$$
$$Y(x,t) = A \sin(kx - \omega t + \phi_1) + A \sin(kx - \omega t + \phi_2)$$
$$= 2A \sin\left(kx - \omega t + \frac{\phi_1 + \phi_2}{2}\right) \cos \frac{\phi_1 - \phi_2}{2}$$
$$= 2A \cos \frac{\phi_1 - \phi_2}{2} \sin\left(kx - \omega t + \frac{\phi_1 + \phi_2}{2}\right)$$

Amplitude depends on phase difference of the two waves.

Square of the amplitude is the intensity

$$I = (2A \cos \phi)^2 \quad \phi = \frac{\phi_1 - \phi_2}{2}$$

Interference fringes
↑ bright bright

in last class we have we have seen that this; these two wave Y_1 and Y_2 , now due to superposition of these 2 wave. this displacement Y as a function of x t comes like this. this is a travelling wave, but its amplitude depends on the phase difference of the two waves. square of this amplitude is the intensity of light.

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$$\begin{aligned}
 Y(x,t) &= A \sin(kx - \omega t + \phi_1) + A \sin(kx - \omega t + \phi_2) \\
 &= 2A \sin\left(kx - \omega t + \frac{\phi_1 + \phi_2}{2}\right) \cos \frac{\phi_1 - \phi_2}{2} \\
 &= 2A \cos \frac{\phi_1 - \phi_2}{2} \sin\left(kx - \omega t + \frac{\phi_1 + \phi_2}{2}\right)
 \end{aligned}$$

Amplitude depends on phase difference of the two waves.

Square of the amplitude is the intensity

$$\begin{aligned}
 I &= (2A \cos \phi)^2 & \phi &= \frac{\phi_1 - \phi_2}{2} \\
 &= 4A^2 \cos^2 \phi
 \end{aligned}$$

Interference fringe

intensity of light depends on the $\cos^2 \phi$; ϕ is the phase difference between the two waves at the point where they are interfering; they are meeting with each other; and these ϕ has to be constant for all time it has to be constant for all time at that meeting point.

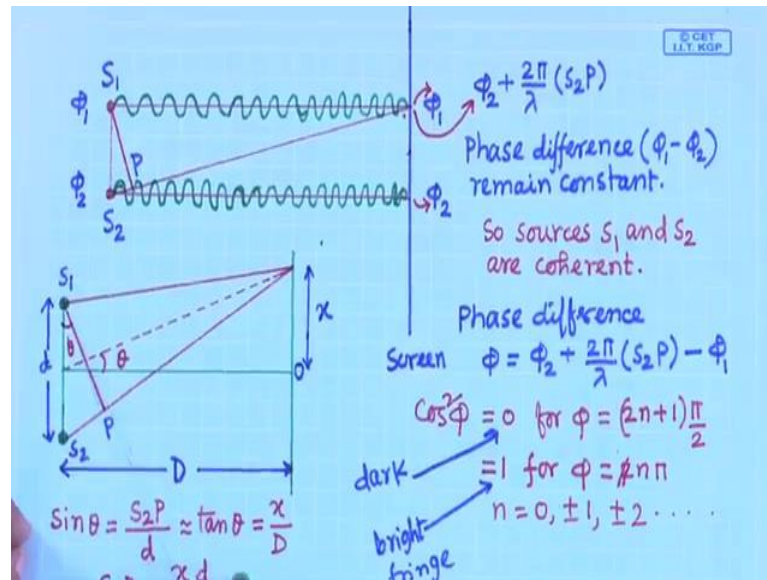
if you plot ϕ as a function of that is intensity as a function of ϕ , we will get this type of variation of the intensity. b dark, b dark this kind of contrast we will see and that is called interference fringe. this interference fringe whatever you are seeing that depends on the phase difference between the two waves at the point where they are interfering.

b dark that contrast you are getting; their intensity is 4 times of maximum intensity 4 times of square of amplitude and this dark there it is 0. if this amplitude of these both waves are equal; then contrast is better means we take highest intensity as well as lowest intensity that is 0, if A and A is not same amplitude is not same for both say it is A 1 and A 2. then it will be this b 1 it will be $A_1 + A_2$ whole square and this put times it will be; there will be some other parts of course, $A_1 + A_2$ whole square this maximum intensity and $A_1 - A_2$ square that will be the minimum intensity.

if you take equal. contrast will be better otherwise contrast will be poor if the difference of A 1 and A 2 is large. minimum will not be if difference is large; that means, this A 1 minus A 2 that will be large. we will not get zero intensity we will it will not looks like dark, only we will see some weak intensity and higher intensity. contrast will not be

better; that is, we tell in terms of visibility. visibility will not be better. for better visibility if this amplitude is equal that is the best.

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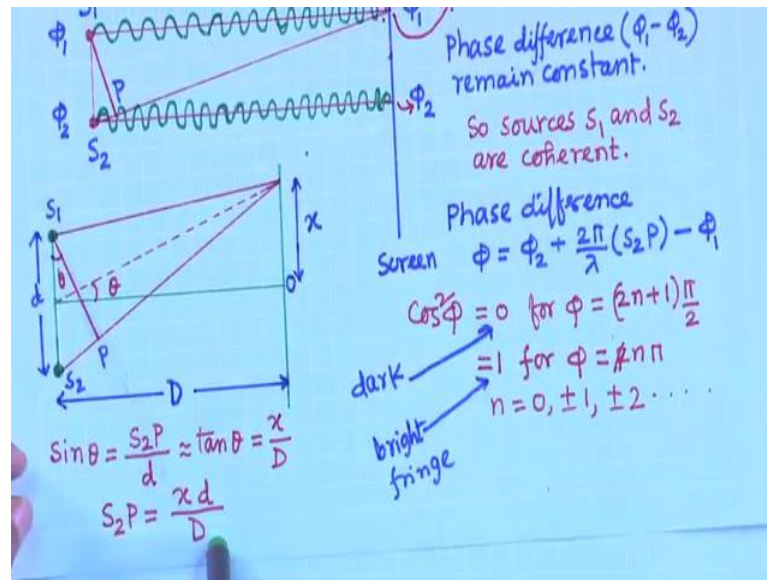
from two source; two waves are coming and from two source 1 wave is this another wave is this. these two interfere at this point. path difference or phase difference between these two is for one case it is phi 1 whatever at the source it was there and for another case it is initial phase was phi 2; phi 2 plus these path difference. this is the normal. this side path is same additional path travelled by this wave is S 2 p. corresponding phase different 2 pi by lambda S 2 p.

these phase difference phi is phi 2 plus 2 pi by lambda S 2 P minus phi 1. condition for b and dark was this cos square phi equal to 0; that is a condition for dark. when we will get 0, when phi is 2 n plus 1 pi by 2. And this is the condition we will get dark if phase difference between these two waves at the interfering point or at the point where they are meeting superposition of these two waves at this point. depending on their phase difference, if phase difference is this, they will get dark; if phase difference is this phi equal to n pi, we will get b; we will get b fringe ok this for both cases this is n equal to 0, plus minus 1, plus minus 2 etcetera,

since it is square is there; minus value also square of if it will be plus. what is S 2 P, how to calculate? that if this angle is comes this is the normal distance between the source and screen if it is capital D and this source distance is small d and that their meeting point is

at from the centre it is x distance if it makes angle θ . this will be also θ this is normal and here this is normal on this these two angle will be same. if; if angle is small, we can write $\sin \theta$ equal to $S_2 P$ by d equal to $\tan \theta$ that is x by capital D ,

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From there $S_2 P$ equal to $x d$ by d . path difference is the distance of the point where we are observing the path difference or phase difference or interference effect. if that distance is x ; x into source distance, distance between 2 source divide by D , capital D that is the distance between the source and screen,

this is very common a relation for this type of all experiment, this relation is valid whenever two source are there, their distance is known; distance of the screen from the source is known. Then that x will vary at different points what will be the phase difference depending on that we see b dark b dark this type of fringe that is interference fringe.

in this relation here is this phase difference; there $S_2 P$ that actually we found it is this. this can be replaced here, this can be replaced here and then in terms of this parameter x small d capital D we will get the phase difference,

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For nth bright fringe $S_2P = \frac{x_n d}{D} = n\lambda$ assuming $\phi_1 = \phi_2 = 0$

For (n+1)th bright fringe: $\frac{x_{n+1} d}{D} = (n+1)\lambda$ Bright fringe width (β) $x_{n+1} - x_n = \frac{D}{d} \lambda$

For nth dark fringe $S_2P = \frac{x_n d}{D} = (2n+1)\frac{\lambda}{2}$ Dark fringe width (β) $x_{n+1} - x_n = \frac{D}{d} \lambda$

For (n+1)th dark fringe $\frac{x_{n+1} d}{D} = (2n+3)\frac{\lambda}{2}$

For interference fringe, the width of bright fringe and dark

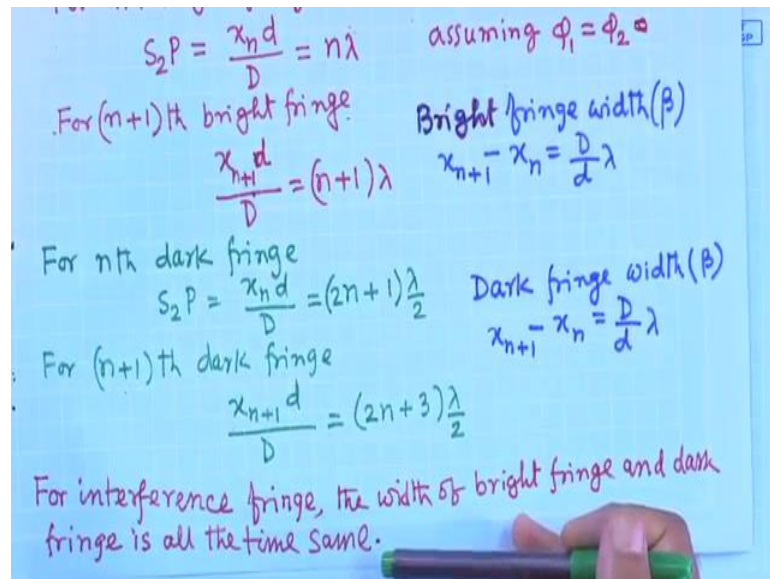
conditional for dark and b fringe at x if it is n number; nth number of fringe for nth b fringe; x we are writing $x_n d$ by D that the path difference. we are assuming say $\phi_1 = \phi_2 = 0$ or may not be 0, but this initially at the source the initial phase is equal. $\phi_1 - \phi_2$ it is then 0. only in that term there will be path difference S_2P and phase difference 2π by λS_2P .

now then you can write that this path difference will depend on for a particular source and scheme distance this x_n . nth this is a general term x_n nth b fringe, say n minus 1, n minus 2, n minus 3 etcetera you can get from this relation. that will be when it is nth path difference will be n lambda.

for n plus 1 b fringe, so it will be at distance x_{n+1} . path when their path difference will be n plus 1 lambda. then distance between these two n plus 1 and nth b fringe. distance between these two is the width of the b fringe. if it is beta; beta will be equal to from here you can find out D capital D by small d into lambda. this is the fringe width.

Similarly, for dark fringe, so this relation is $2n + 1$ lambda by 2 or n plus half lambda. for nth dark fringe, this difference nth b fringe and nth dark fringe this is the path difference will be lambda by 2. we will get nth b fringe if this is the path difference and n plus 1 dark fringe for this n equal to n plus 1 if we put this will be the path difference. width of this dark fringe will be capital D by small d lambda.

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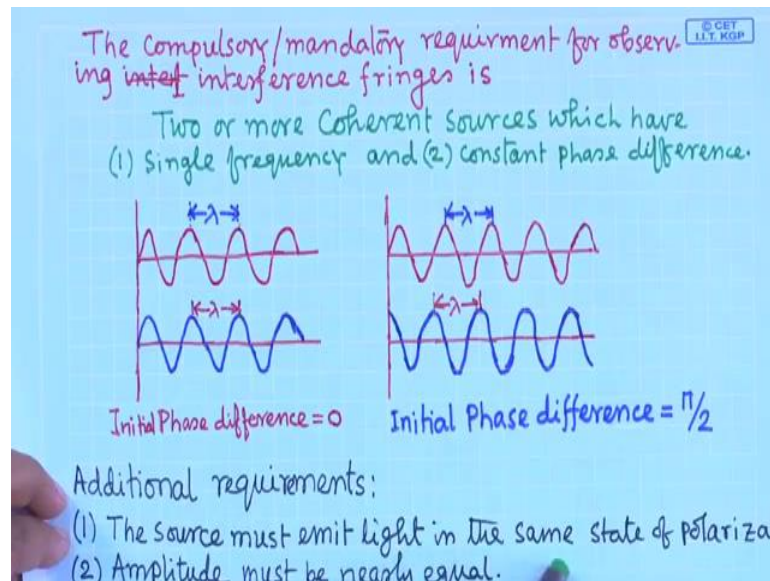


from the here you can tell you can see that the fringe width in general we can tell it can be difference between two successive b fringe or it can be difference between two successive dark fringe so; that means, width between dark and b fringe will be half of it; will be half of it. for interference fringe the width of b fringe and dark fringe is all the time same.

and this is the condition we have to remember this fringe width capital D by small d into lambda. From here what is the wavelength if you want to find out; it is small d by capital D into fringe width.

now, here one term I use that these two source has to be coherent; these two source has to be coherent. their phase difference; their phase difference will remain constant over the time. let us discuss this very important fact for seeing the interference effect the source; the two source we are using that has to be coherent.

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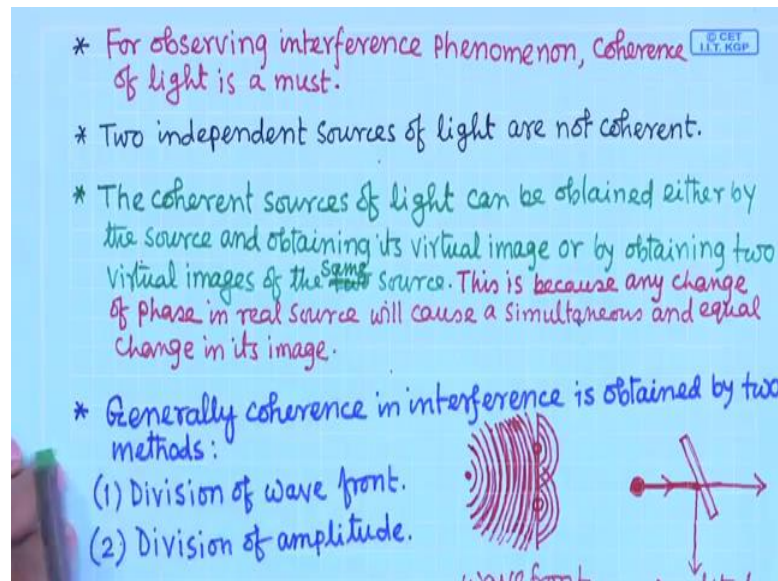


mandatory or compulsory requirement for observing interference fringe is the two or more coherent source which have single frequency and constant phase difference. two or more coherent sources and those coherent source should have single frequency and constant phase difference. if you just like these two waves they are wavelength are same, frequency is same; in this case their initial phase difference is 0 and other case you can see ok; here this is it is a amplitude is minimum 0 and here amplitude is maximum.

this one case is sin function and other is cos function. initial phase between these two is π by 2. this initial phase difference has to be constant in general wherever also at the meeting point where they will interfere at that point also total phase difference including the initial phase difference has to be constant with time. then we tell the source are coherent.

Additional requirements source must emit light in the same state of polarisation, this is the additional requirement, so if lights are polarized. the source must emit light in the same state of polarisation. Sigma polarisation π polarisation, so these two will not interfere. interference will happen only between π or between sigma and amplitude must be nearly equal, as I told may not be equal, but it is a nearly should be nearly equal. these are additional condition to see the better visibility of the interference pattern.

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for observing the interference phenomena, coherence of light is must. two independent source; how to get two coherent source? How to get 2 coherent source that is the question. two independent, if you take two independent source of light, they never be coherent; they never be coherent. the coherent source of light can be obtained either by the source and obtaining it is virtual image or by obtaining two virtual image of the same source

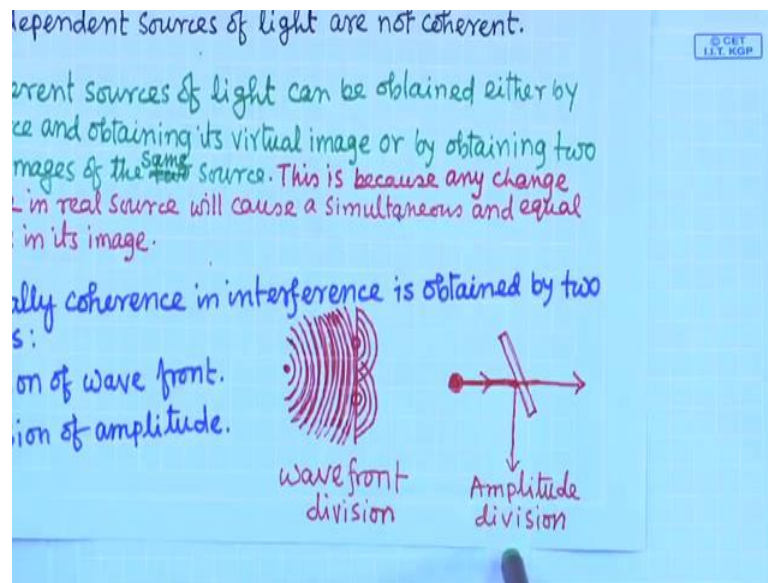
we have to we cannot take two different source; we have to from same source, we have to generate another source. that we tell that is the image of the source virtual image of the source either original source or another is it is virtual image of that source. these two can be coherent or from the original source if we generate two virtual source and that two virtual source we have taking as for our experiment for observing the interference effects. then these two virtual source are also coherent.

this is because any change of phase in real source will cause a simultaneous and equal change in it is image. that is why from same source we have to generate another source, or we have to generate two more source from the same source then they will be coherent.

generally, coherence in interference is obtained by 2 methods from same source, we can get the two or more coherent source by two method. that is the one is division of wave front and another is division of amplitude. wave front you know this is the spherical wave front,

now here if we have two pinhole, this wave front is falling on this screen where two pinholes are there. this from each pinhole again; these fronts are dividing into two part. one is; one part is entering this way another is entering this way. this is the division of wave front.

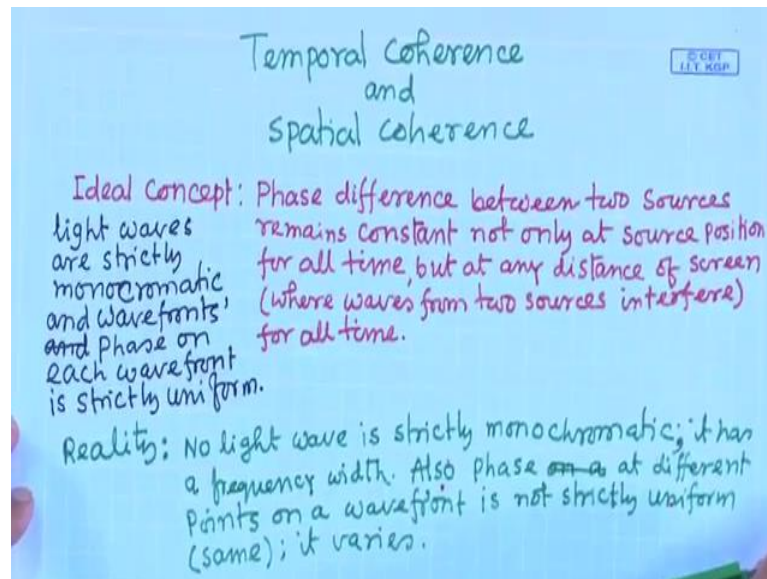
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And division of amplitude is this is the source light is coming. here you have a beam splitter or half silver glass plate; it can reflect as well as it can (Refer Time: 23:20). from this glass plate one part is reflected and another part transmitted, we are generating two part ok; these two can be used for interference. we are generating two waves by this method. this method is based on the division of the amplitude. whatever amplitude there it is a divided into two parts, this called division of amplitude

now, important fact is that the, this to get the coherent source we are using as I mentioned that two methods either division of wave front or division of amplitude. it is and we are telling that this phase difference should be independent of time between these two waves which will interfere, now coherence slightly difficult subject and let me slightly discuss more about the coherent.

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there are two types of coherent, this is called temporal coherence and spatial coherence. what is these two types of coherence? this for getting the interference effect, so we are telling that source has to be coherent means or the two waves which will interfere at some point. they are at that point their path difference or phase difference has to be independent of time, that is the condition for to be coherent.

Now, this condition remains the phase constant phase difference constant, to get this condition there are other factors are there or how we can maintain this condition. to understand or to realize that one; that we express in terms of these two types of coherence; one is temporal another spatial.

ideal concept that phase difference between two sources remain constant not only at source position for all time, but at distance of screen where waves from the two source interfere for all time. these phase difference constant not only at the source level source position, but it has to be at a point on screen at any distance for all the time.

reality is that no light; before that let me tell you that light waves. to achieve this one condition is that light wave has to be strictly monochromatic and the phase wave front is which one; what is wave front? Wave front is the phase on a surface; phase on a surface is uniform is constant, so then, that surface is called the wave front

this is the definition of wave front and if it is strictly uniform as well as we tell this for interference, we need monochromatic light, means for a particular wavelength. you can

see for one wavelength will not interfere with another wavelength because as I as we told this interference only possible of same frequency and wavelength.

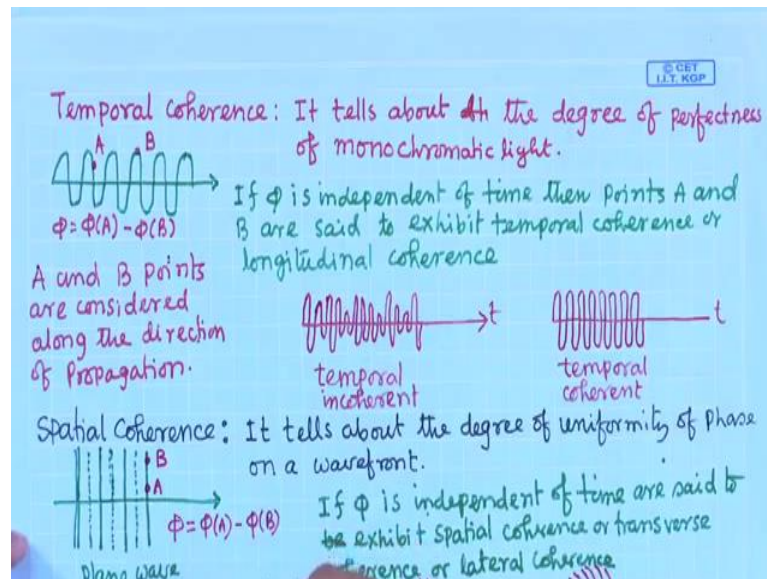
if you have two source; if you have two source their frequency or wavelengths are different, then there will not be interference that is clear. But if you take source of wavelength λ no source have this single wavelength λ , always every source have a natural width bandwidth, whatever the definition we give for monochromatic light definition we give for wave front if that is strictly followed then there is no then this phase difference will remain constant at any time at any distant.

but in reality, it is not possible; it is not possible, what is not possible, to get strictly monochromatic light whatever light we will use. that has bandwidth; bandwidth maybe very small, but still it will be the. no light wave is strictly monochromatic it has a frequency width; also phase at different points on a wave front is not strictly uniform it varies.

this is the ideal concept of monochromatic light and a uniform wave front. phase will remain constant at all points on a wave front. this is the ideal concept, but in reality, we deviate from this concept; we deviate from this ideal concept so; that means, we are admitting that. whatever light we will use, it will have bandwidth it is not single frequency single wavelength as well as whatever light will emit to wave front. those wave fronts are not perfectly uniform means, they are phase on the wave fronts are not same at everywhere; it will vary.

it is looks it is a defect from ideal concept it is deviate there is a defect on that ideal concept, whatever the condition we are telling for getting coherent source, so there is a limitation on that, for any time it will not happen, for any distance it will not happen ok there is a limitation on it. that is what this temporal coherence and spatial coherence these two terms have come up. we will discuss more on this issue.

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what is temporal coherence, it tells about the degree of perfectness of monochromatic light as I discuss. other way if phi; what is phi? this is the (Refer Time: 33:09) this the wave. if you take two point, A and B in the direction of propagation on the wave, if phase at point A it is phi A and point B it is phi B. phase difference between these two point A and B is phi A minus phi B. If phi is independent of time, then point A and point B are set to exhibit temporal coherence or longitudinal coherence.

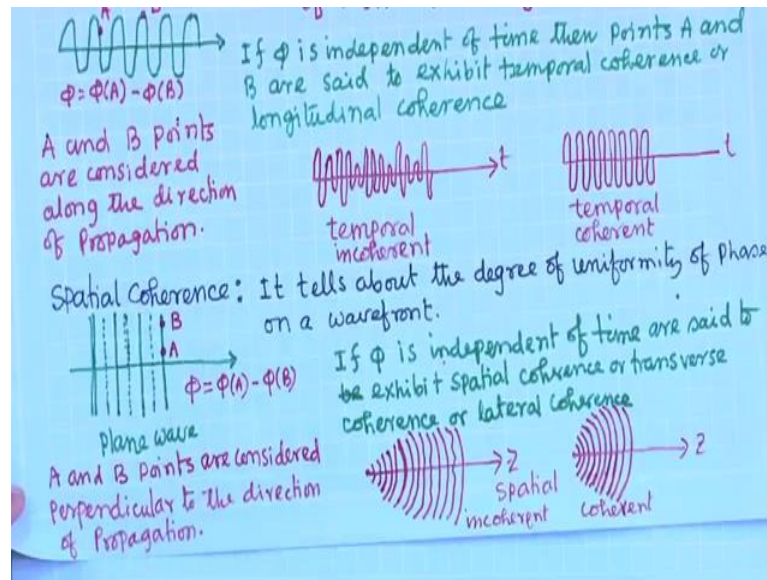
what is temporal coherence? between two points; between two points on the wave at the direction of propagation, if their phase difference remains constant if it is independent of time then we tell that the source. This source is temporarily coherent, if it varies then it is not coherent, it is incoherent.

this is the example of incoherence; it is it varies along the width time it varies it is a time axis. width time it varies; width time it varies phase varies at different points. or at a particular point say A or B, so width time it is varying like this then we tell width time the phase at a point or between difference between two points are will not remain constant. this source; this source is not coherent because it; which type so that now we classify, why it is not coherent? Because time part. In time part it is incoherent. that is, we are telling temporal coherence or temporal incoherence

if time width time it varies at a particular point if it is varies like this. it is all the time; it is fine periodically it will come. phase difference between two points will remain constant with time. then we tell this temporal coherent. this is the different way people

tell about the to understand that coherence temporal coherence; this is one way to understand the temporal coherence. And the similar way this spatial coherence it tells out the degree of uniformity of phase on a wave front.

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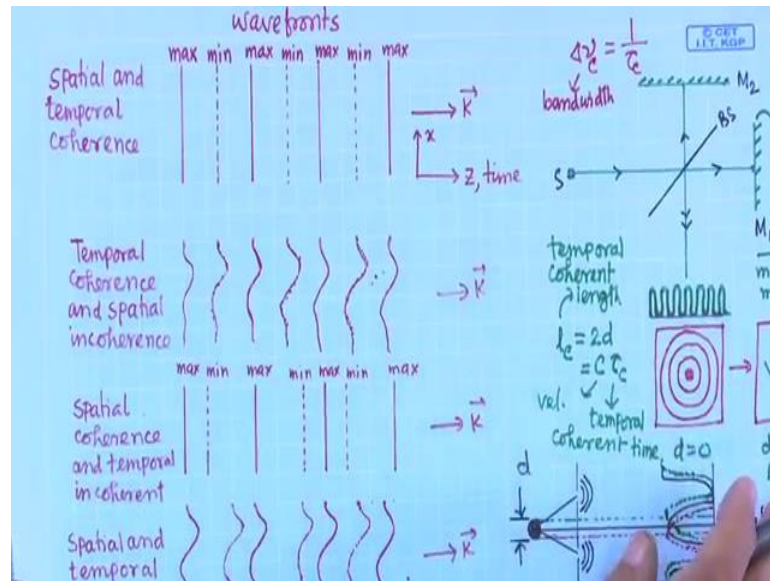
this is the wave front plane wave front, this not spherical wave, front plane wave front. now, spatial coherence we on a plane on a wave front, so if you take two point A and B and if their phase difference remains constant with time then we tell them that they are spatially coherent; they are spatially coherent,

if this phi is independent of time, if this phi is independent of time then it is said to exhibit spatial coherent or transverse coherence or lateral coherence. from this discussion it is clear that when the wave is travelling in this direction in transverse direction if you take two point it can be on same; it can be on same wave front or it can be on different, no it has to be on same wave front. in case of spherical; that means, we have to take point will not be on the same perpendicular line, but it will be A here and another B here on spherical surface,

phase should be same supposed to be same at this both point if it is not same, if there is a difference and if difference remain constant over the time then we tell them it can be 0 or it can have some value. if it is independent of time all the time then we tell this is the, this source is spatially coherent,

this is the example of spatial coherent is the spherical or this the example of and, but if it is like this. if you take two points on wave front, so this phase is not uniform, then we tell this spatially incoherent to observe interference source has to be coherent; means, source has to be temporarily coherent as well as spatially coherent, it has to satisfy both condition.

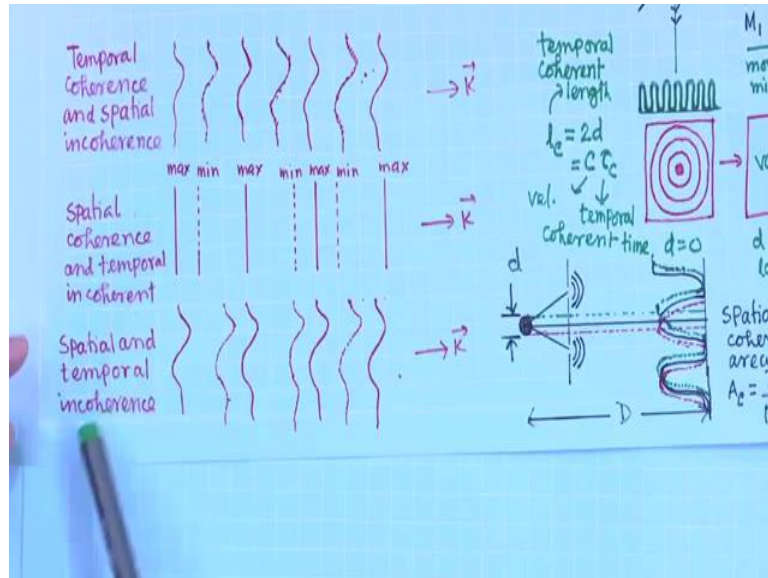
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if I discuss slightly more, so this way also we can understand this. This is wave from plane wave front we have taken. here if you it is propagating these direction; propagating these direction is the time axis as well as propagation this as well as space axis Z as well as time, over time if you look at two point along this direction. you can see this they will remain constant it is propagating, so they will remain constant; their phase will remain constant and over in transverse direction also if you consider two points. there phase uniform on this wave front.

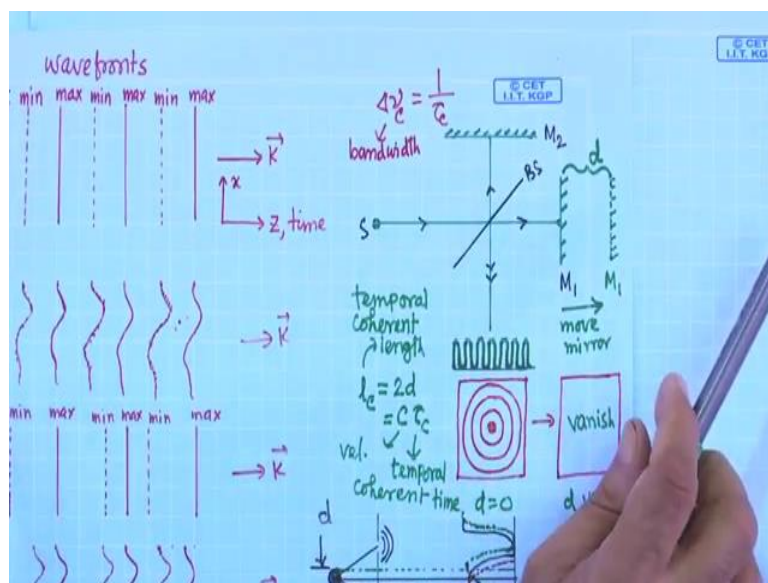
this type of source wave front waves we tell spatial and temporal coherence, if it is like this; if it is like this. it is a vertical direction this phase is changing with time, but along this direction; along this direction you can see this their distance; it is a moving, phase difference between these in longitudinal direction it will remain constant. this type of source we can tell this it is a temporal coherence, but spatial incoherence.

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Another; next one it is showing that this phase uniform in longitudinal direction transverse direction, but in this direction, they are not; this they will they are not like this, phase difference between two points at the direction of propagation it will not remain constant. this we tell this is this spatial coherent, but temporal incoherent. And then just in both way there is a phase difference along the transverse direction as well as along the longitudinal direction of propagation. then we tell this source is spatial and temporal incoherent. this way we try to understand.

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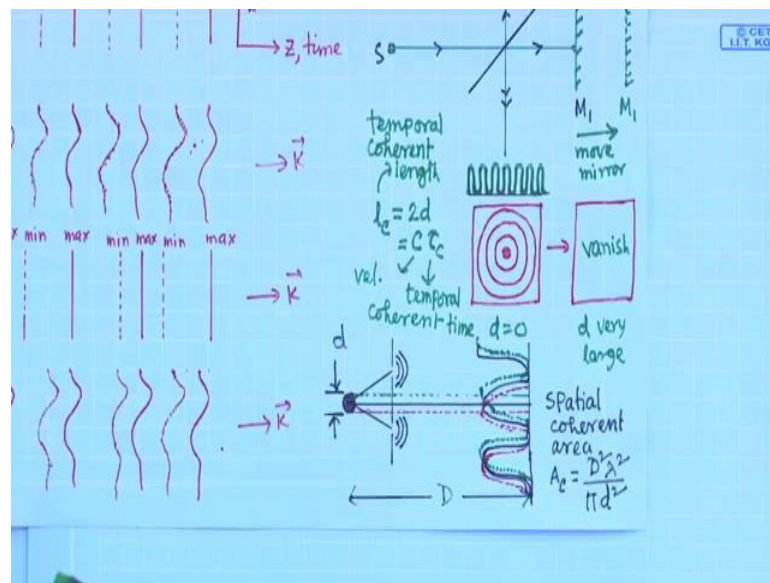


one can measure the temporal coherence time or temporal coherence length, what is temporal coherence time and temporal coherence length? Actually, the if source is not

strictly monochromatic if it has bandwidth, if $\Delta \nu$ bandwidth. then it is actually we do not get wave long wave we get wave in bracket form; it is like pulse form. this coherent time that depends' coherent time that depends on the bandwidth of this light; it is the inversely proportional.

coherent time will be infinity if this $\Delta \nu$ is 0 if there is no bandwidth, all the time it will be coherent. it is a time it is infinity, if it is higher this value is higher, then this time coherent time will be lower and over the length with velocity C if it moves.

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over the length l_c ; l_c equal to $c \tau_c$ coherent time; this is called coherent length. over this length it is temporal coherent along the direction of propagation. if this type of arrangement is there where path difference between two light which will interfere, I will discuss more about this is a Michelson interferometer geometry, I will discuss more about this. if you change the distance d ; change the distance d initial for 0 distance you will get nice fringe pattern, now you are changing the distance and after when d will be very large then we will see this it will vanish,

what is that distance after which it vanishes that value will tell the temporal coherent length. And if you know the temporal coherent length, you can find out the temporal coherent time, and for longitudinal this is the longitudinal for transverse or a spatial coherent if your source dimension size is bigger. If it is point source it will be spatially coherent, but if size is increase, there is a critical size after that you will lose the

interference fringe pattern it will vanish. if point source, if really point source then it will be spatially coherent; it will be spatially coherent.

Now, if source size is d it is not source separation; size is d . there will be many point source on this. from middle this is the light is coming from middle from that source point source we will get interference pattern. b one you will get that the centre; b one you will get at the centre and if you take green one here black line and if you take another point source at the extreme end two points source at the extreme two end. their fringe pattern you will get like this from this because this will be centre of this fringe. you will get this wave (Refer Time: 47:05).

what you are seeing this at as if the it is the botany of the interference fringe and then there will be overlapping of minima of another one with the maxima of other one. As if this is happening due to the many point source means due to the larger size of the point; we will lose the point. coherent; spatial coherent depends on this size of the source as well as it depends distance between these two source if it is smaller, then better there are some factors, but size of the source is important fact,

here whatever the source, here two source if their size if is not pin hole, if it is size is bigger you will lose the visibility of the interference effect. the size generally is depends on size as well as depends on distance of the screen also. it is not unique relation, but some theorem tells that critical area over which this we will get spatial coherence; so that is, that equal to it depends on this relation is $d^2 \lambda^2 / \pi d^2$ square

here it is clear that these if this size of the source is smaller this area will be higher. Means, over the higher area we will get the it will remain the spatial coherent and if distance from the source is higher then also area will be this spatial coherent area will be higher.

screen distance is higher is better, higher wavelength also it is better for spatial coherent and smaller source size is better for spatial coherent. that means, whatever coherent we are talking about. we have to be careful that it has to be spatial coherent spatially coherent as well as it has to be temporal coherent. both condition has to satisfy. I will stop here.

Thank you for your attention.