

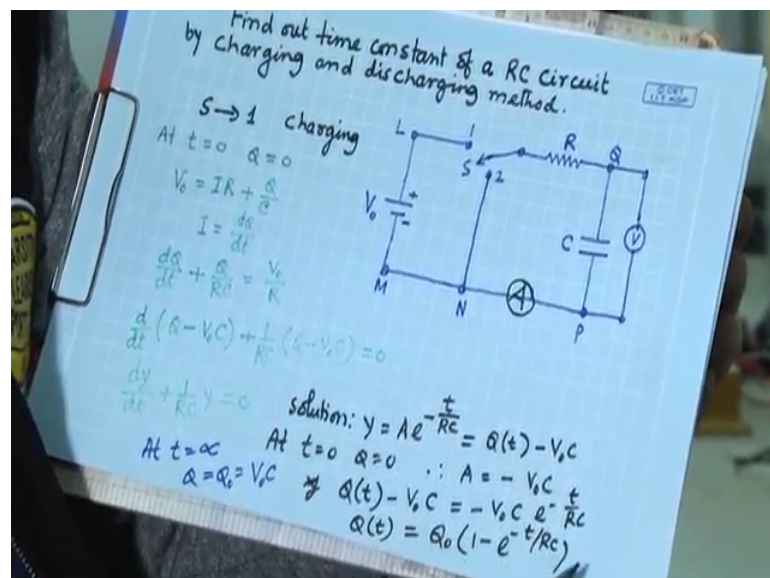
Experimental Physics I
Prof. Amal Kumar Das
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture - 51
Theory on RC Circuit

So, today, we will discuss about the RC circuit electrical circuit. So, generally we are quite familiar with the electrical circuit where we just use resistance. Now, resistance resistor capacitor and inductor this three are very important component electrical component. So, now, all the time in a circuit resistance is will be there. Now, if we add capacitor in that circuit then what happens that is what we want to study. So, basically we tell this in RC circuits, this charging and discharging of the capacitor. So, that is the interesting interesting variation of voltage or current in the circuit across the capacitors across the capacitor.

So, we will see that with time. So, this current and voltage across the capacitor is not constant; it varies with time ok. So, how it varies with time? So, that, we would like to see. Also this in case of say charging in case of discharging also similar things, this how current and voltage varies with time in the circuit. So, that, we would like to see. Now, in these circuit this time constant is very important factor. So, that also will try to find out the time constant of CR circuit. So, let us discuss theory first of this experiment.

(Refer Slide Time: 02:40)



So, actually we will try to find out the time constant of a R C circuit by charging and discharging method right. So, this is a CR circuit.

If you look at this part, so in a circuit of course, you need DC power supply. So, we can use battery, but here will not use battery. We will use constant voltage source, we can vary the voltage, we can choose the voltage as you wish, but in case of battery. So, you cannot change the voltage, but there are voltage divided circuit voltage divider circuit so that you have to use to vary the voltage. So, you have a voltage source. So, it will give you constant voltage. Now, this the circuit R C circuit. So, this R C circuit, R C circuit, so one end of the circuit one end of the circuit is connected to the power supply say it is connected with negative of power supply and this other end.

So, this L V C R circuit, now other end here we have two option. One option this s switch. So, if it is connected with one then circuit it is going to the to the power supply say this positive terminal of the power supply. So, then when the switch is at position 1, so then it is a CR circuit and power supply is connected with this circuit. So, current should flow through this through the circuit. Other option is this if you put this switch at position 2, then circuit complete circuit will be this complete circuit will be this.

So, this is also CR circuit. Now, without, without power supply, without applied voltage, so then what will happen in the circuit? So, this we tell discharging. So, when this switch will be at position 1, then it is a charging of the circuit CR circuit. And when the position of the switch is at 2, then it will be discharging of the of the circuit CR ok. So, in both cases, in both cases charging and discharging, so here we will have connected voltmeter. So, across this capacitor, how this voltage will change, how this voltage will change that we can find out. And also, we can connect we can connect here we can connect here 1 ammeter, 1 ammeter.

So, in both cases, again this during charging and during discharging, how current behave in the circuit. That we can find out from this ammeter. So, ammeter and voltmeter ammeter is basically will give current variation in the circuit during discharge and charging and discharging time. Also we will see the variation of voltage across this capacitor during charging and discharging; so let us see theory. So, during, so when S is connected to the 1, then this is charging ok. So, at time t equal to 0, at time t equal to 0,

so charge on the capacitor is 0, Q equal to 0 right, so now, just at time t equal to 0, just it is connected or we are going to connect ok.

So; that means, is a charge this in capacitor is 0. So, that is why this is the boundary condition, initial condition. Now if you connect it, if you connect it, so what will happen? So, this the circuit and in this circuit, so you can write what will be the voltage drop in the circuit? So, voltage drop across the across the resistance if current in the circuit is I .

So, voltage drop across the resistance is $I R$ and voltage drop across the capacitor. So, that will be Q by C that will be Q by C charge is accumulated in this capacitor that is Q . And if C is the capacitance, so Q by C that will be the voltage drop across this across this capacitor. So, this voltage drop across the resistance and across the capacitance; so this these two will be equal to the voltage to the source.

So, that will be equal to the V_0 right. So, current I is basically dQ by dt charge change of charge with time dQ by dt . So, now this I , I can replace with dQ by dt . So, then equation I am getting dQ by dt plus Q by divide by R . So, R and C is there, Q divide by $R C$ equal to V_0 by R ok, so divided by R and replace I with dQ by dt ok. So, now, this just I can arrange. So, V_0 by R I can take this side and one by $R C$. So, this I can write Q minus $V_0 C$ right q . So, these two term will give me Q minus $V_0 C$. And here this is the differentiation with respect to the time. So, dQ by dt , I can write d by dt of Q minus $V_0 C$ because V_0 is constant, then C also constant with respect to time.

So, d by dt of Q minus $V_0 C$ is equal to dQ by dt basically. So, I have taken in this form, then this is just like dy by dt plus 1 by $R C$ y equal to 0 this is the first order homogeneous equation. And its solution is you know this one can write general solution y equal to $A e$ to the power minus e to the power minus t by $R C$ heres. So, this y equal to whatever this $m \times m$ t Q is a function of t ; so, t is there. So, this general solution of this will be, so this e to the power minus this whatever this m is there here.

So, m t ok, so m is 1 by $R c$, so this, so y equal to Q minus $V_0 C$ Q minus $V_0 C$ Q is a function of time basically. So, if you put this boundary condition at t equal to 0, Q equal to 0. So, you will get basically value of a value of a this is a arbitrary constant equal to minus $V_0 C$ ok. So, ultimately you are getting just replacing a by minus $V_0 C$, you are getting Q t Q is a function of time equal to Q_0 Q_0 is $V_0 C$. So, this V you know Q

equal to $C V_0$, right. So, it is the Q_0 because this when it is fully charged V_0 . So, this that that fully charged means then it is a it is a voltage across this capacitor will be V_0 .

So, then it is the full charge in this in the capacitor that will be Q_0 . So, $Q_0 (1 - e^{-t/RC})$ to the power minus t by RC , so charge in the circuit or this in capacitor will vary with time. So, variation of charge in the capacitor or in this circuit will be will be like this. So, Q equal to $Q_0 (1 - e^{-t/RC})$. Now, if you know this one, so if you divided by C in both side; so, this will be variation of voltage ok.

(Refer Slide Time: 13:03)

$$Q(t) = Q_0 (1 - e^{-t/RC})$$

$$V(t) = V_0 (1 - e^{-t/RC})$$

$$I(t) = \frac{dQ(t)}{dt} = -Q_0 \left(-\frac{1}{RC}\right) e^{-t/RC}$$

$$I(t) = I_0 e^{-t/RC}$$

$$S \rightarrow 2 \text{ discharging}$$

$$IR + \frac{Q}{C} = 0$$

$$\frac{dQ}{dt} + \frac{Q}{RC} = 0 \quad Q = A e^{-\frac{t}{RC}}$$

At $t=0$ $Q = Q_0 = CV_0 \therefore A = CV_0$
 $Q = CV_0 e^{-t/RC} \rightarrow V(t) = V_0 e^{-t/RC}$

$$I(t) = \frac{dQ}{dt} = -\frac{CV_0}{RC} e^{-\frac{t}{RC}} = -I_0 e^{-\frac{t}{RC}}$$

A graph is shown with current $I(t)$ and voltage $V(t)$ on the y-axis and time t on the x-axis. The voltage curve $V(t)$ starts at V_0 and decays exponentially towards zero. The current curve $I(t)$ starts at I_0 and decays exponentially towards zero.

So, you will get $V(t)$, you will get $V(t)$ you will get $V(t)$ equal to $V_0 (1 - e^{-t/RC})$ and this you can find out the variation of current. So, $I(t)$, I is a function of t will be dQ/dt .

So, if you differentiate it, so ultimately you are getting $I(t)$ equal to $I_0 e^{-t/RC}$ ok. So, from now, now you have you have expression for variation of charge in the circuit or across the capacitor you have variation of charge Q , variation of voltage V , variation of current I . So, all are varying with time t and how they are varying. So, this expression is in your hand. So, now, if you plot, so this equation is telling that this current in the circuit will vary will decay with time, will decay with time from at t equal to 0 it will be I_0 at t equal to 0 it will be I_0 . So, current will vary during charging.

So, starting from I_0 is the time t ok. So, variation is a is exponentially it will decay right. So, with time current in the circuit will change like this. And voltage, voltage here 1 minus this part ok, so at t equal to 0 this voltage is it is this will be 1 . So, is a is V will be 0 , V will be 0 ok. So, if I plot voltage if I plot voltage ok. So, initially at t equal to 0 , it will be now then this term is decreasing like this decreasing like this. So, this term is decreasing with time. So, then 1 minus this, so this factor will increase ok. So, which time this variation will be exponentially it will increase.

So, this variation will be like this ok. So, this will be here I can I can draw this the ok. So, this value will be basically V_0 , this value will be V_0 this is I_0 and it will decay exponentially. So, yes. So, this basically I have plotted I t and this I have plotted V t , all right. So, now, here it is a decreasing either current or voltage or charge ok. So, basically measuring charge is difficult than the measuring voltage and voltage and current, ok.

So, that is why I am not plotting discharge so, but in principle if you have this plot you can just find out the variation of charge. So, multiplying with we multiply or divided by divided by C and R . So, now interesting thing is that if you see that if t equal to RC , if t equal to RC , so this factor is always this factor is, so this 1 t by RC when t equal to RC , this is 1 . So, this factor will be 1 by e , this factor will be 1 by e ok. So, in this case, 1 by e times I_0 . In this case, 1 minus 1 by e times V_0 .

In case of charge also, we can find out, in case of charge also this factor is here if t equal to RC . So, this factor is 1 by e , this factor is 1 by e . So, here also whatever the way voltage is varying; so the same way this charge is varying ok; so this variation of charge variation of charge with time, so this charge is as voltage is increasing. So, also charge will increase same way and it will go towards the Q_0 value. So, here when, so t equal to RC what does it mean; t equal to RC . So, that is basically RC is time. So, at that time, at that time, what will be the value of charge in the circuit?

So, charge will be during charging here during charging it is here it is showing the charge will be 1 minus 1 by e times of Q_0 or 1 minus 1 by e times of V_0 V_0 or current will be 1 by e times 1 by e times I_0 right. So, whatever the case either voltage or charge or current, so 1 by e times or 1 minus 1 by e times, so this, so this 1 by e times is very important it seems, right. So, let me come just after discussing this discharge. Now when, so this s this switch is connected to 2 , this switch is connected to 2 at this point.

So, this is the there is no battery in the circuit power supply in the circuit. So, this we tell. So, already this it is charged after fully charge of this capacitor. So, if I connect this one. So, now, we are telling these are discharging of the in the circuit. So, discharging then you can write because this, then $I R$ plus Q by C equal to not V_0 is 0 because no power supply. So, in case of discharging, so this will be the equation and if you just proceed as we did in charging case. So, you will get Q equal to $Q_0 e^{-t/R C}$ means $Q_0 e^{-t/R C}$ to the power minus t by $R C$.

So, just here this form is same as the in case of current for charging, in case of current for charging. So, this is the this just in case of discharging the charge follows the same form of this current in the charging case. So, this equation it is the variation will be. So, like this; that means, charge will decay exponentially and corresponding voltage expression is this as I told the charge and voltage variation would be in same way, variation will be same way. So, this basically it will vary just decay with time.

So, charge and voltage ok, it will decay with time on the other hand this current you can find out $I = dQ/dt$. So, this a minus $I_0 e^{-t/R C}$. So, whatever the, so in case of in case of discharging, so initially current basically 0 ok, so now this current is also exponentially it will decay. But it is a negative, it is a negative sign ok. So, basically here it is telling current will flow in opposite direction. During the charging, during charging whatever the direction of the current, so during discharging, current will flow in opposite direction and variation will follow this.

So, from actually yes, so this is the variation exponential variation, but it is a with negative sign. So, here also when t equal to, when t equal to $R C$, so again $1/e$ times of I_0 or $1/e$ times of V_0 or $1/e$ times of Q_0 ok. So, so what does it mean? So, these we tell this t equal to $R C$, this we tell the time constant of the circuit, time constant of the circuit. So, it is significant of this time constant is that this from here you can see that what about the during discharging, whatever the charge was there fully charge Q_0 . So, to decay to decay this current to this to decay this charge Q_0 to $1/e$ times of Q_0 decay to this charge equal to $1/e$ times Q_0 to this value, this time it will take the time it will take that is called the time constant and that will be equal to the $R C$.

So, so in case of this on in case of voltage also similar to, so decay from V_0 voltage when it will the time it will take to decay to the $1/e$ times of $1/e$ times of V_0 . So,

that time is called time constant ok. So, during charging also to charge to charge, so, initially it was discharge or voltage were 0. So, to charge the capacitors at the value of $1 - \frac{1}{e}$ times of V_0 fully charge ok, full charge V_0 . So, to charge it up to that that time ok, how much time it takes to charge it of $1 - \frac{1}{e}$ times of full charge or full voltage what is the time it takes. So, that time is called the time constant for this circuit and this time constant shall we tell τ , τ equal to RC .

So, that RC we can measure we want to measure. So, in this experiment actually we want to plot the current as a function of time voltage as a function of time during charging as well as during discharging. And then, from this variation, one can find out the time constant τ ok. So, for that generally we plot log we use log plot. So, if you take log if you take log say here ok. So, you will get $\log I$ equal to $\log I_0 - \frac{t}{RC}$. So, if you plot this $\log I$ as a function of time, so you will get a basically straight line. So, from the slopes because minus $\frac{1}{RC}$ into t . So, it is like here.

So,, so I t equal to I am getting here minus I_0 will be some yeah. So, $\log I_0$ plus not plus in this case it will minus $\frac{1}{RC} t$. So, so it is a it is a kind of y equal to $m x$ plus C right. So, m is slope and that m will be this m will be basically $-\frac{1}{RC}$. So, just inverse of the slope will give you the value of RC . So, will plot logarithm will use logarithmic plot and then will find out this slope and inverse of the slope will be the time constant for the circuit ok. So, so this is the purpose of this experiment. So, I will stop it.

Thank you for your attention.