Experimental Physics I
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Lecture – 37
Theory regarding compound pendulum has been discussed

So, today I will discuss about the compound pendulum say using compound pendulum how you can find out the; acceleration due to gravity and also the radius of gyration.

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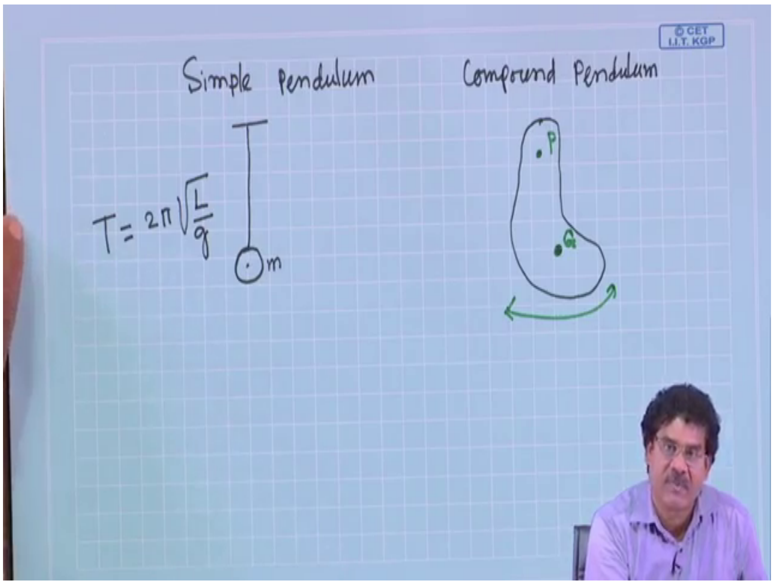

So, this simple pendulum from a support with a thread mass less thread; just hang a mass m right. Then its time period is basically you know this T tend to 2 pi square root of L by g right.

Now compound pendulum say any body if you; so, irregular body can be irregular body or regular body. So, it has center of mass; it has center of mass of this body same is the center of mass is; this all is the center of mass g and it is a suspension through a point say here point P. So, axis passing through this point P, it is a horizontal axis; horizontal axis passing through this point P; this is the point of suspension of the body.

So, with respect to this center with respect to this axis horizontal axis passing through this P, so this body can oscillate; this body can oscillate, this body can oscillate then this is the simple compound pendulum right. So, when it will oscillate; when it will oscillate then there will be restoring force and due to this resting force basically it will oscillate.

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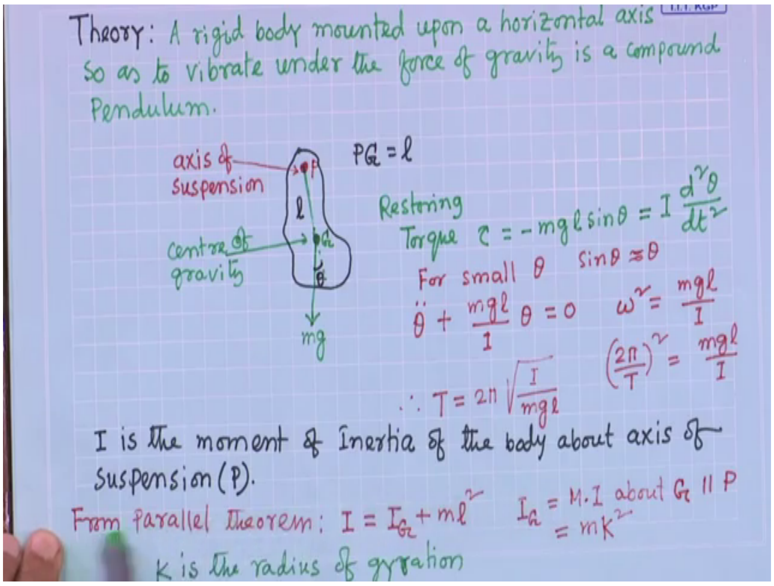

So, this theory of this compound pendulum; theory of this compound pendulum is; so what is compound pendulum? A rigid body mounted upon a horizontal axis; so as to hide it under the force of gravity is a compound pendulum right.

So, this as I told this P point P is the axis of suspension, G is the center of gravity ok. Now this gravitational force will act downwards this mg, if m is the mass of this body. So then this if you so, this if you just cut it this 2 P and G; so this length is say l; small l. And now this is the see if we extend it. So, whatever the angle with this gravitational force if that is theta; then you can reserve this force along this P G. So, that is basically it will be balance the it is the weight along this direction of force along this direction. So, that is mg cos theta and another force perpendicular to this P G. So, that is that will be mg sin theta ok.

So, that mg sin theta is the restoring force. So, we have this is the force and this is the distance with respect to this P that is distance l. So, torque will be mg sin theta into l. So, torque will be mg l sin theta and this negative sin besides a restoring force is working in this direction against the displacement, against the displacement of yes change of theta.

So, this torque is we can equate with this I d 2 theta by dt square like m d 2 x by d dt square. So, m is replaced by I as I mentioned earlier; so this it is basically moment of inertia, it is the moment of inertia of this body with respect to the axis passing through P or horizontal axis passing through P, so that is the moment of inertia. So, for small angle theta one can write sin theta is approximately equal to theta.

So, this equation of motion is theta double dot plus mgl by I by I is sin theta if theta equal to 0. So, this is the equation of motion this is the simple harmonic equation differential from all this equal harmonic motion. So, this part basically theta double dot plus omega square theta. So, omega square is mgl by I omega is 2 pi by 2 right. So, 2 pi by T whole square equal to mgl by I. So, from here from this relation one can write T equal to 2 pi square root of I mgl ok.

So, this is the time period of this compound pendulum when it is oscillating; when it is oscillating with respect to the axis; horizontal axis passing through point P. This is the axis of point of suspension and distance between the this point of suspension and the center of gravity is l and I is the moment of inertia of this body with respect to the axis passing through the point P.

So, from parallel theorem I one can write moment of inertia of center moment of inertia about the axis passing through the center of gravity G which is parallel to the axis passing through the; passing through the to the point of suspension ok. So, axis here and axis here they are parallel ok.

So, the moment of inertia with respect to this axis; axis of suspension is I is equal to the moment of inertia above this axis passing through the center of gravity. So, that is I G plus m mass of this body m and this distance square. So, this is the come from parallel plane of moment of inertia and then I G; I G is basically is the mk square; I G is mk square and this k is called the; it is called the radius of gyration k is the distance.

Moment of inertia of G, about G axis passing through the G which is parallel to the axis passing through the P ok; so, this is equal to the mk square this k is the radius of gyration. So, that; so from this experiment our aim is to find out the acceleration due to; acceleration due to due to gravity basically g and also the radius of gyration k.

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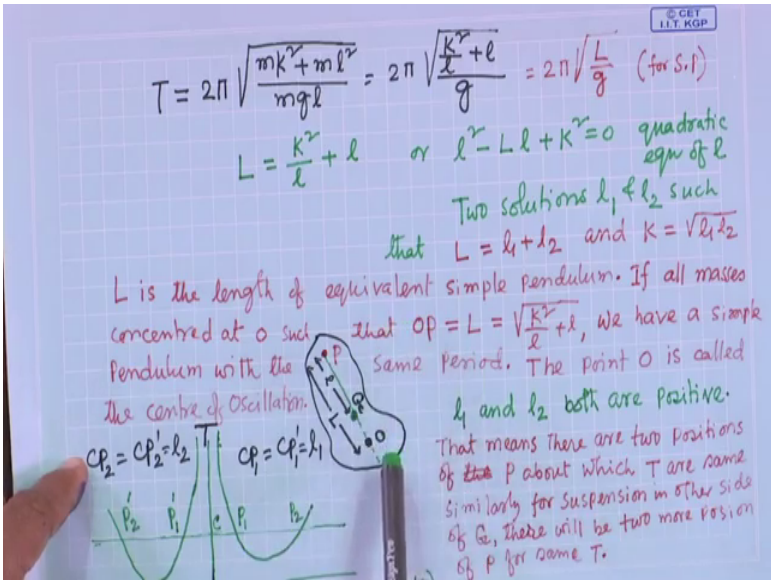

So, we know that T equal to now T equal to 2 pi that I is replaced by mk square plus ml square divided by mgl. So, from here you can write 2 pi square root of k square by l plus l divided by g ok. So, as I showed you that time period for a simple pendulum is 2 pi square root of L by g. So, if you compare this two; so as if L is this L equal to k square by small l plus l. So, when L is when L is this then; that means, this L is different from the small l smallest l is basically distance between the axis of suspension and the center of gravity ok; this small l. Now this L is a new L; this L is a new L it depends on l, as well as it depends on radius of gyration.

So, this is a quadratic equation of l, we can write this l square minus L capital L small l plus k square equal to 0; quadratic equation of l. So, it will have two solutions say l 1 and l 2 such that this L; capital L will be has to be equal to the l 1 plus l 2 and k has to be equal to the l 1 plus l 2 and k has to be equal to the square root of l 1, l 2. So, this is the quadratic equation and solution is has to be such that it has to follow this one ok.

So from here basically this equivalent; so this equivalence, this L is called basically length of equivalent into pendulum length of equivalent into pendulum. And if all masses what is the meaning of this equivalent; length of equivalent simple pendulum means? If all masses centered at a particular point P, the these distance from this from this P; not P; this O centered at O is distance is capital L from the point of suspension ok, then this then this will be equivalent plus simple pendulum. All mass concentrate at this point P distance is capital L from and this L is L is basically square root of k square by l plus l; small l. So, so then it will behave like a simple pendulum it will oscillate with the same period of simple pendulum ok.

So, so that will be the basically equivalent this is reduced to the equivalence of the; of the simple pendulum. So, what from here, so what we have. So, what we are getting basically comparing to the comparing with the simple pendulum. So, this O will tell this is the center is called the center of oscillation ok; this O is called the center of oscillation, G is center of gravity, P is the point of suspension. So, this distance from P to G that is small l; distance from P to O center of oscillation that is capital L ok.

So, so; so now this from this relation L equal to l 1 plus l 2 and k is a square root of l 1 l 2. So, L and l 1 and l 2 both has to be positive; both has to be positive both has to be positive; that means, that means there are; that means, there are two positions; what is l l l? Is basically l 1 and l 2. So, there are two positions there are two positions of P about which T are same; about which T are same because two solution two solution of l and l is basically distance; distance of point of suspension from the from the G; center of gravity.

So, there will be two positions of P; there will be two positions of P ok; there will be two positions of P P about which the time period will be same ok. So, so that means, there are two position; one will be from G 1 will be a 1 and another will be l 2. And similarly if you; if you go other side to the center of gravity, if point of suspension in other side. So, then your there will be two other points, there will be two other points of suspension; again you will get basically two value of l; l 1 and l 2 ok. So, there also time period will be will be same.

So, basically with respect to the center of mass point of suspension from this left side 2 points and also right side 2 points this 4 points we have the same time period.

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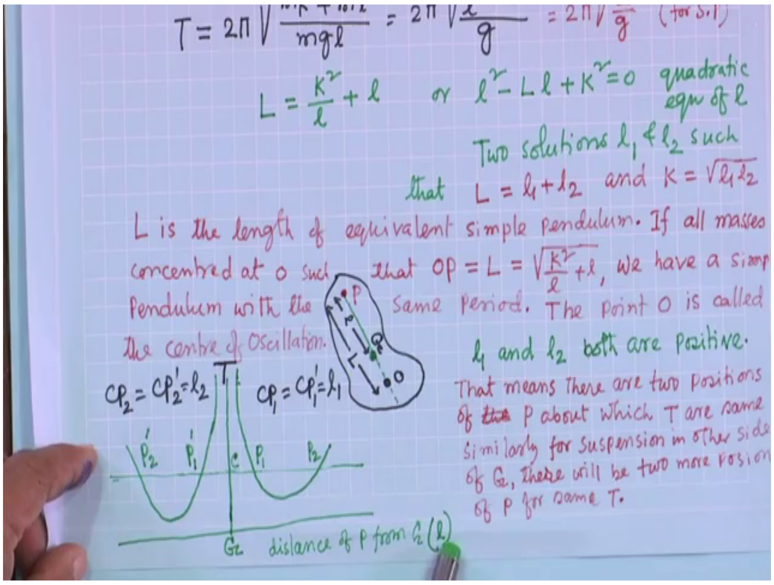

So, if you; so if you plot this time period; if you measure time period for different point of suspension for different point of suspension from both side; from one side of the center of gravity. So, if you plot this time period as a function of distance or as a function of distance of point of suspension from the center of gravity. So, distance of point P from G; so that that is basically we are telling l. So, distance; so it will this graph you will get like this the graph you will get like this. Similarly, on the other side of G; if you have point of suspension if you take if you vary. So, you will get symmetric curve from the other side ok.

So, this is the time period this axis is time period T and this is the distance of point P from G; this side you know one side and this side; the other side point of suspension in the other side. So, here if you just a draw a line parallel to this; to this distance of P from G; so it cuts this graph is 4 points ok. So, this is basically center of the C center of gravity.

So, point P 1, point P 2 in one side. So, this CP 1 that is basically l 1; on the other side CP 1 dash CP 1 dash that is also l 1 and CP 2 will be equal to CP 2 dash that will be l 2 ok. So, basically for this 4 distance; time period will be same time period will be same this is the time period. So, for a particular time period we will get 4 point of suspension 4 point of suspension; 2 is one side of the center of gravity and 2 on the other side of the center of gravity.

So, so if you plot graph then from the graph you will get basically you will get l 1 and l 2 value. So, if you get l 1 and l 2 then you will find out capital L; you will find out capital L l 1 plus l 2 and you will find out radius of gyration k within the square root of l 1 l 2. So, our experimental task is to measure time period T time period T for each; for each distance for each distance for each distance of the point of suspension; in one side of the center of gravity and as well as on the other side of the center of gravity right.

Then when we will plot the graph from there you will find out; you will find out l 1 and l 2 ok. And then you will find out capital L and from capital L you will get basically g; you will get basically g value because time I am measuring time period T and you are measuring you are measuring basically also you are getting capital L. So, from there you will find out the value of g and k directly you are getting from this relation l 1 square root of l 1 and square l 2 ok.

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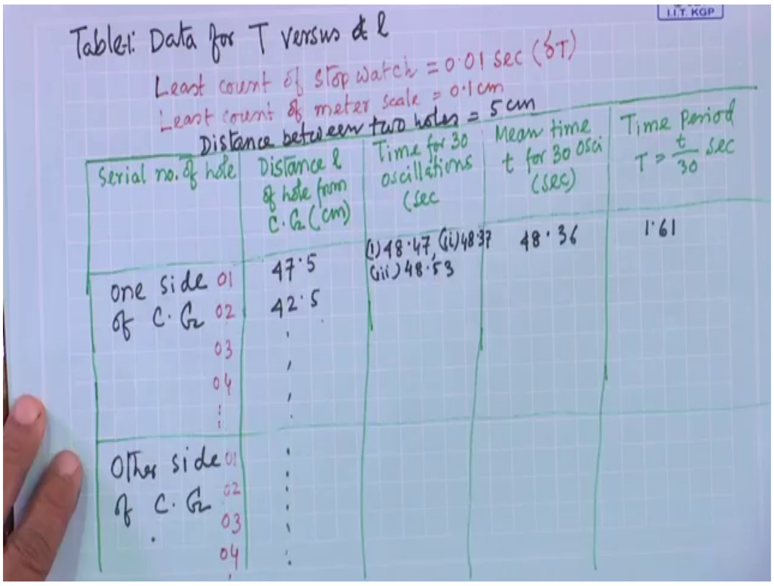

So, what you have to measure? So, then let me show you this table. So, table 1 will be data for T versus distance l from the center of gravity of the suspension point P. So, for this; so you have to note down the least count of stop watch you will use and then least count of meter scale you will use for measuring distance. And in our a bar simple compound pendulum.

So, we have some made some holes on this bar. So, distance between two holes equal to 5 centimeter. So, this whatever the important thing you should note down on the top of the table then serial number of holes; so holes 1, 2, 3, 4 from the top ok. So, note down the note down the of distance l of hole from center of gravity; so that is say 47.5 say.

Then next one just you are reducing you are; so you started from the top end and now coming down towards the center of gravity; so this its difference is 5 you know. Then next one will be 37.5; so this is a you go till up to close to the center of gravity and then other side of the center of gravity again you should you should do the experiment taking starting from the say top end of the other side. So, again that will be around 47.5, then 42.5 etcetera ok.

So, you should note down the varying the distance of point of suspension from the center of gravity. So, this from one side and this is the; from other side and for each length l; each length l, find out the time for say 30 oscillation ok. So, and then 3 times you should do and then take average of it, so from there you will find out the time. So, you are finding out basically time period T and corresponding that distance from the center of gravity of the point of suspension that is l. So, that is what you need for drawing the graph.

So, now then you should you should draw the graph as I showed you; you should draw the graph you should draw the graph ok. You should plot time versus this distance of P from the center of gravity. So, if you plot; so from plot you have to find out these 4 points, you have to find out 4 points then you can calculate the g value and radius of gyration k value ok.

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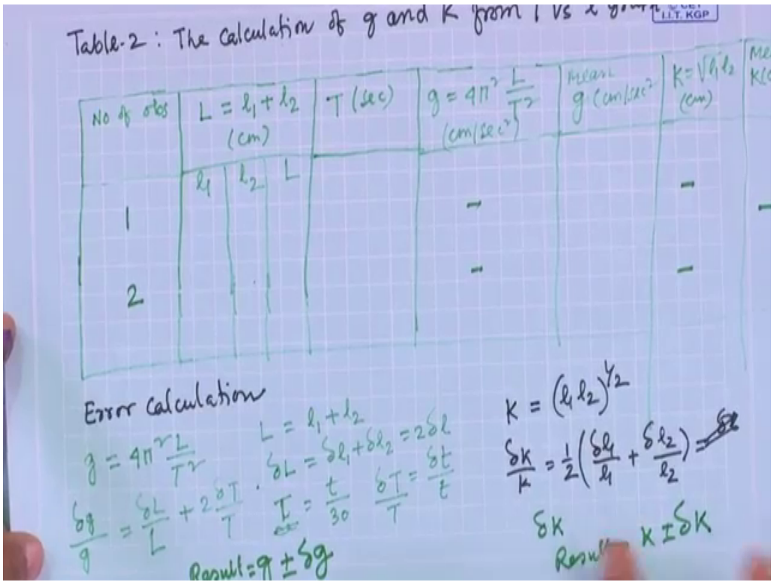

So, then next I will solve for calculation of g value and a value from T versus l graph ok. So, here; so basically you have to find out this from this graph we have to find out l 1 and then l 2, then from there you calculate L. So, which one l 1 and which one l 2 that already I have I have told you I have told you right. So, from center C yes that these 2 points you will get. So, take the distance of this 2 points from C also you take the distance on the other side; CP 1 dash and CP 2 dash. So, say that is a that I have here retained this one side you take calculate and take the other side l 1 l 2 you will get or one this way also one take this P 1 2 P 2 P 1 to P 1 dash that distance divided by 2 that will be l 1 and P 2; P 2 dash that distance will be basically that distance divided by 2 that will be l 2 that way also one can consider.

So, whatever the way you want to follow. So, just I think write this as a number of oscillation; number of observation; so 1 and 2. So, in one case CP 1 and CP 2 that will be l 1 l 2 and other case CP 1 dash and CP of CP 2 dash. So, that will be l 1 l 2 from there you find out L and then corresponding time period already we have. So, write it then you can get calculate the g value right and so 2 g value you will get; so find out the new g value.

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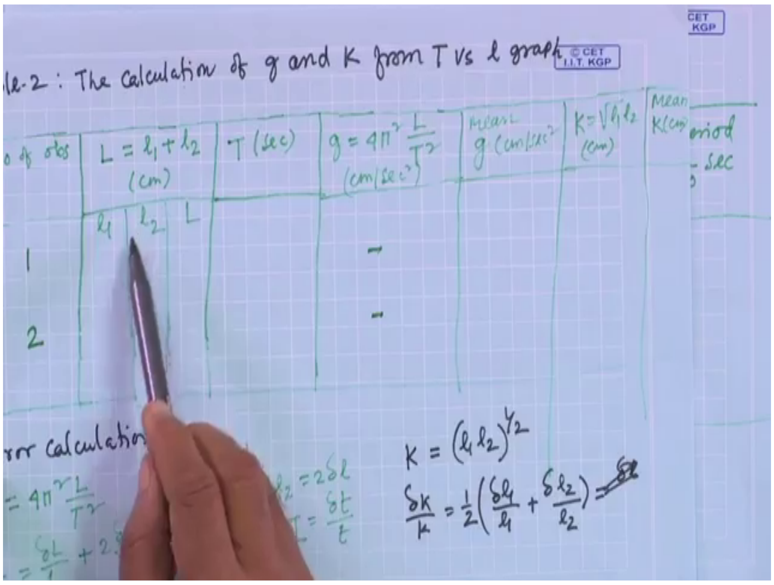

And of course, l 1 l 2 is known to you. So, you will get k equal to square root of l 1 l 2; so again 2 value you will get and then you get mean of these 2 value. So, that will be the basically mean k value unit is centimeter ok. So, I have retained all the unit and you should this is very important you should write it. So, then if you will get value of g and k and error calculation ggg equal to 4 pi square L by T square. So, delta delta g by g equal to delta L by L plus 2 del T by delta T by T. So, you know how to how this relation has come I have discussed many times since very simple.

Now, L directly you are not measuring L; L you are measuring from l 1 and l 2. So, we have to L equal to l 1 and l 2; so delta L basically delta l 1 plus delta l 2 both are same; so it is equal to 2 delta l right. Similarly, T you are not measuring directly T you are measuring T by 30 oscillation. So, one can write in general n, but we have taken thirty in our experiment. So, that is why; so delta T by T will be basically delta small t by t ok.

So, that is will be the error on this g value; we have to find out delta g value. So, then result will be g plus minus delta g. So, that will be your result ok. So, similarly on k; k is this l 1 l 2 square root of l 1 l 2. So, delta k by k is will be half delta l by l plus delta l 1 by l 1 plus delta l 2 by l 2. Delta l 1 equal to delta l 2 ok; so same value in our case with a 0.1; so l 1 l 2 or particular l 1 and l 2 you will find out delta k by k.

So, from there you will find out delta k. So, your result will be; your result will be k plus minus delta k ok. So, that is the way you should write a result ok. So, I think this theory one has to understand then only you can perform the experiment perfectly, nicely and calculate the results the g value and basically k value radius of gyration.

So, thank you for your attention.