Experimental Physics I
Prof. Amal Kumar Das
Department of Physics
Indian Institute of Technology Kharagpur

Lecture – 35
Coupled Pendulum

So, again we are in first year laboratory of physics of department of IIT Kharagpur. So, today I will demonstrate the Coupled Pendulum. So, this is the setup for this Coupled Pendulum.

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So, this is a simple pendulum this is one and this is another simple pendulum right.

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So, pendulum for simple pendulum you know this there is a mass is hanged with a jelly thread, but here hanged with a light rod; it is a light rod and this mass is heavy compared to this rod, so this is see this a simple pendulum. So, we can find out the time period of a simple pendulum how to find out that we try, just I will displace it; I will displace it and I will I just use stopwatch.

(Refer Slide Time: 01:23)

So, we have stopwatch so I have to just start 1 2 3, so this way I will count the number of oscillation and collect time and then I can find out the time period for this simple pendulum. Similarly, I will find out the time period for the another simple pendulum similar it is a same mass same length, so it is expected that time to it will be more or less same ok. But we have to find out the time period for this one also so we will find out the time period of this. So, now this is the two individual simple pendulum; we want to couple them and then we want to study about this couple pendulum. So, let us so this will couple this two pendulum using this spring.

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So, we will do the experiment for 2 spring 1 and spring 2, so I have 2 spring so we will repeat basically the expression for the second spring. So, we will do the experiment for first spring and repeat the experiment for second spring. So, let me tell first what we are going to study in this coupled pendulum. That means we should know the theory about this coupled pendulum and what will be the work formula for this experiment ok.

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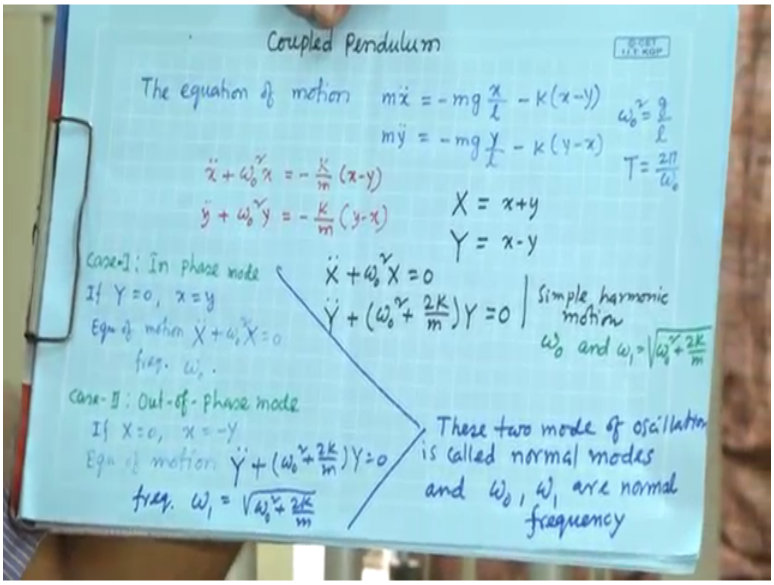

So, coupled pendulum equation of motion I have 2 pendulum, so displacement of this one if it is X; this displacement of other one we are taking that is Y then the since they are coupled. So, we have to displace the; we will assume that both are displaced one is displaced by X another is displaced by Y. So, then equation of motion of this pendulum ok, so this displacement of this one is X; displacement of this one is X, but it is coupled with the second pendulum. So, that effect will also come here so thus it is coupled.

So, equation of motion will be m x double dot d 2 x by d T square right equal to if it is just 1 pendulum not coupled then what it will? So, m g x by l m g this is restoring force m g x by l this restoring force how this term is coming m g.

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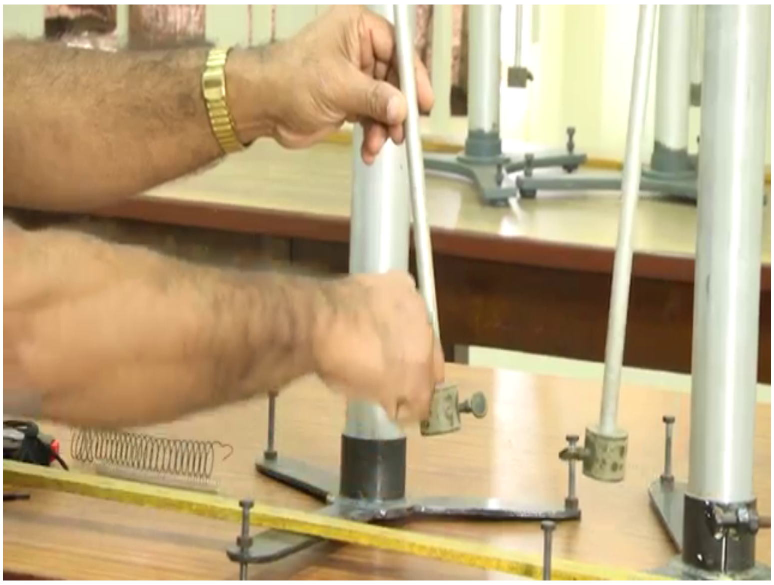

So, this the so when I displaced so this m g is acting just vertically down, now restoring force is basically just this component. So, you can take two component one is perpendicular to this another component will be perpendicular of this component. So, what I want to been so if this is the vertical m g, so this now in this angle is theta; angle is theta with this vertical what are the original position vertical.

Now, force is parallel now with this angle is theta so m g cos theta this acting this way and m g sine theta will act this. So, m g sine theta is basically restoring force and its direction is opposite to the displacement right opposite to the displacement. So, that is why this negative sign is there; negative sign is there m g sine theta, sine theta for small theta you can take as a tan theta or theta.

So, tan theta or just theta will be this by this if angle this angle is theta, so the tan theta is this x by this l. So, that is why here is the x by l x by l m g x by l and minus sign this restoring force is working opposite right. So, similarly for Y also this one also minus m g y by l. So if they are not coupled, so this could be the equation for this 2 individual now since they are coupled this two additional terms has come.

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Now this two term how it is come so now they are coupled I think this spring I have to attach this spring I have to attach this spring ok, I have attached now this is the position. So, this spring I have kept in a such way this one just originally it is just natural length it has natural length it is not extended or compressed so it is the natural length.

Now if I displaced if I displaced this by x and this by y now what will happen this spring if I displaced both by x equal amount, so this spring will be in the same length it will be it will not be neither; it will not be neither either compressed or extended ok, it will have the normal length. So, spring constant basically constant force per unit length it is restoring force per unit length if spring does that compressed or extended. So, it will not apply any force so if it is equally displaced. So, there will not be any force restoring force.

If it is displaced this by x and this by y so that one is extended by x minus y right x minus y or y minus x then is multiplied with spring constant spring a constant is force per unit length. So, x minus y or y minus x that is the ultimate compression or extension right. So, that is why this x minus y times k, so that will be restoring put that will be restoring force due to this spring. So, that is why these two terms has come how this sign has come you can find out. So, if x is greater than y so this x minus y is positive ok.

So, this spring means compressed now at this displaced position this spring is compressed ok. So, now I am starting this one taking equation of this one so due to this extension of the spring ok, so it will force it will apply force restoring force in this direction ok. So, that is why this opposite to the displacement this force rhetorically opposite to the displacement that is why this negative sign will come. So, similarly here also so this equation for that one I have to think about this one so you will get this. So, this the equation of motion for coupled pendulum right from here so omega 0 here omega 0 g by l omega 0 g by l omega 0 square time equal to naturally it is 2 pi by omega 0.

Now here you see I cannot solve this equation just like this because it has 2 2 variable x and y I have to decouple them. So, there is a procedure to decouple them, so this just I have written in terms of omega 0 in terms of omega 0 taking the g by l omega 0 square and equal to this K by m this part. Now if I redefine the variable capital X equal to small x plus small y capital Y equal to small x minus small y. If I define these two variables and put here then I will get this equation.

Now, we can see this equation only this X capital X is involved this equation is only capital Y is involved and these equations now is decoupled. So, it is a like a these two equation just it is a equation of simple harmonic motion. Now I have basically for this coupled pendulum I have 2 motions one variable is capital X and another variable is capital Y. And so this frequency of them is one is omega 0 and another is omega 1 square root of these omega 1 square root of these I have written somewhere yes omega 1 square root of this. So, we will have 2 frequency omega 0 and omega 1 this system this coupled pendulum will have these 2 frequency.

Now, these 2 frequency whether I can realize individually one of them, so it will oscillate with one frequency or one of them of course and then whether they will it will oscillate and it will have this both component. It will have this both component of omega 0 and omega 1, so that is why there are 3 cases case 1 it tells about the in phase mode. Now imagine that if Y equal to 0; Y equal to 0 means x equal to y right Y is we have defined x minus y if Y equal to 0 then x equal to y, then this equation these equation x equal to y y equal to 0 naturally this equation will not be there. So, this now this it this converted to a single equation only this equation.

So, solution of this equation or frequency of this so these are equation of simple harmonic motion and its frequency is omega 0. So, these compound coupled pendulum will oscillate with this frequency omega 0 ok. So, condition is that then will tell it is a in phase mode in phase oscillation and for condition of this one to get this in phase oscillation condition is we have to displace both in same direction with equal displacement because x equal to y ok. So, we will study this condition x equal to y and it will be displaced in same direction.

Second condition is out of phase ok, if X equal to 0 then X equal to 0 means x equal to minus y x equal to minus y right. So, if so when x equal to 0 this equation will go only this equation will be there and for these equation so frequency so these are simple harmonic motion frequency is. So, the second case is out of phase mode if X equal to 0 then x equal to minus y X equal to minus y right x equal to 0 then X equal to minus y. So, then these equation will go x equal to 0 so only this equation will be there. So, that means this system this coupled pendulum will oscillate with only with this frequency omega one with this frequency omega 1 right.

So, this is a out of phase mode so this also we will study. So, when will get this mode x equal to minus y means when this x this will be displaced in this direction it has to be displaced in opposite direction and this magnitude means value of x and y has to be same x equal to y, but it is opposites negative sign is there means in opposite direction ok. If we displace like this then this equation is telling will get out of phase mode and it is frequency will be this. Now this two these two modes of oscillation are called normal modes and omega 0 and omega 1 are called normal frequency, so this will measure will study.

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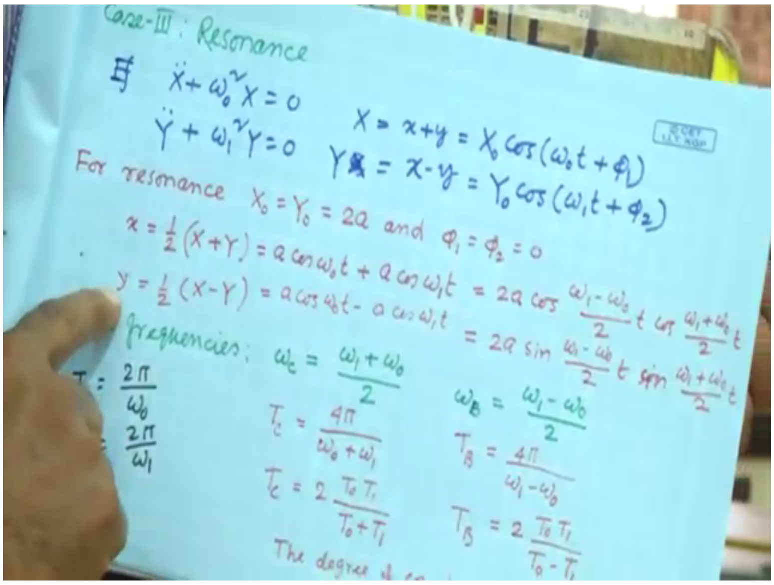

And then another case is resonance another case is resonance. So, now this was our equation X double dot plus omega 0 square x equal to 0, Y double dot plus omega 1 square y equal to 0. So, this was our equation right where X is x plus y and Y is x minus y and solution of the general solution of the this one can write X 0 cos omega 0 T plus phi 1 and Y equal to Y 0 cos omega 1 t plus phi 2. So, these the general solution where phi 1 phi 2 are phase omega 0 and omega 1 they are frequencies and X 0 and Y 0 this the amplitude of oscillation.

Now, if X 0 equal to Y 0 equal to say 2 a some amplitude I have given means this both ok. So, let me first explain the if I considers X 0 equal to Y 0 equal to 2 a some amplitude and phi 0 phi 1 equal to phi 2 equal to 0. If this is the condition then if I apply here apply here, so basically I will find out x small x equal to basically from here half x plus y equal to I will get this and from here you will get 2 a cos; 2 a cos omega 1 minus omega 0 by 2 t cos omega 1 plus omega 0 by 2 t ok.

Similarly, y small y will get half X minus Y equal to this. So, what does it mean I got small x equal to this, means displacement of first pendulum is coupled with the second one. Of course so displacement of first one will vary like this, displacement the other one at a time see it will vary like this it is cos function and it is sine function right. What does it mean when this will come this cos function when it is a for a particular time for a particular time say at a particular time this is 0, this is 0 then this will be maximum for that angle because you know cos 0 is 1 and cos sine 0 is 0 right cos 90 degree is 0 sine 90 degree is 1 ok.

So that means, when x will be maximum for a particular angle this omega t, so other will be 0. So, other will other one will be 0 if it is maximum or vice versa. So, this scenario we should see here, so that means when one is at 0 position other the other will have the maximum displacement and vice versa or we will get this here will repeated manner.

If one is disturb other one is at rest so that means this which one was disturbed that will come at rest after sometime and in the meantime other should have the maximum displacement right. So, that is why it is telling equation is telling and the another part you see here 2 cos term is there. So, one term this term we can take amplitude and this the cosine term this the amplitude and this the sin term ok. So, this so in amplitude this cos term and sin term is there that means amplitude will vary also periodically.

So, it will have two frequency; one is omega 1 minus omega 0 by 2 that is basically it is a frequency of the variation of the amplitude variation of the amplitude. So, these basically this frequency called beat frequency beat and this other one is basically is the frequency of the couple pendulum. So, individual frequency so individually it will as if it has it has frequency that is for couple pendulum and amplitude variation periodic variation will be there and that is basically it is a beat frequency beat will observe beat also.

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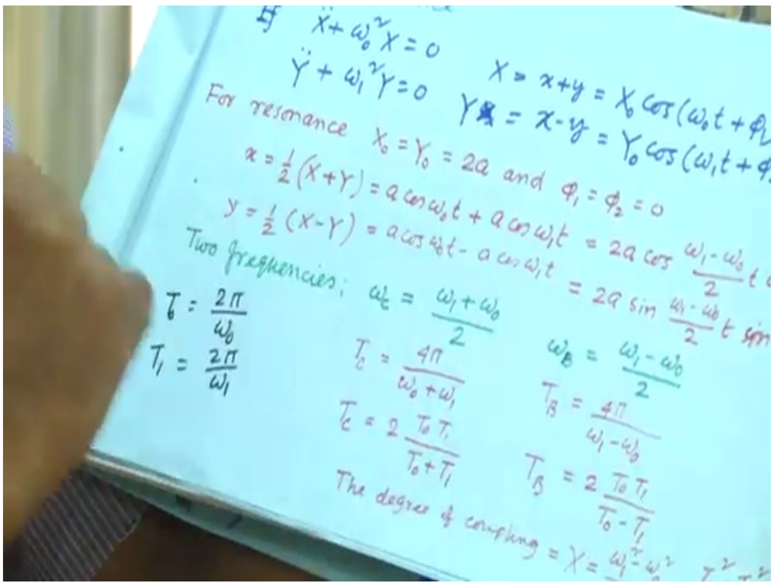

So, here that is why omega c we have written equal to omega 1 plus omega 0 by 2 omega B we have written that is equal to beat frequency that is equal to omega 1 minus omega 0 divided by 2 ok. So, omega you can convert to in terms of T time (Refer Time: 22:09). So, that is why T0 and T1 that here I have shown the relation.

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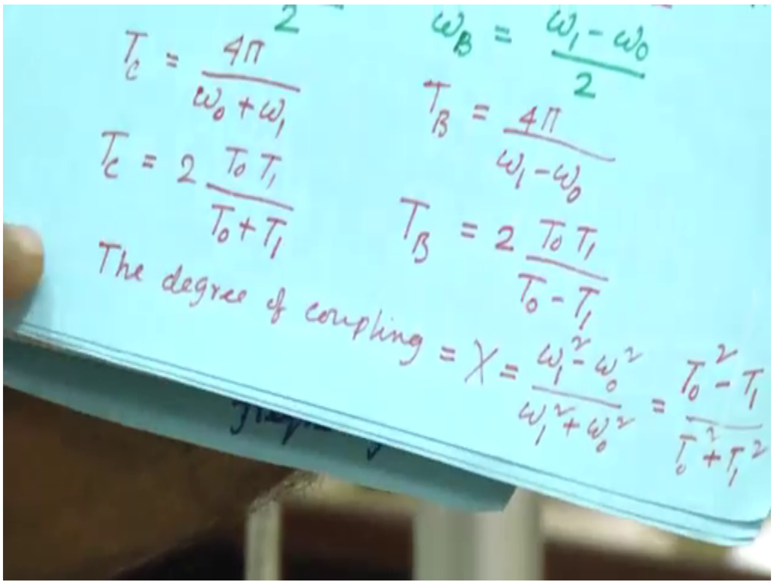

What will be T c equal to 2 T 0 T 1 by T 0 plus T one T B T 0 T 1 into 2 divide by T 0 minus T 1 so and degree of coupling is defined by like this. So, this chi is T 0 square minus T 1 square divided by T 0 square plus T T 1 square. So, here you can see we have to just basically measure the time period T 0 and T 1, then all other things we can calculate time period of these what is the beat time period what is the couple time period for the couple pendulum ok, so that is the experiment will perform.

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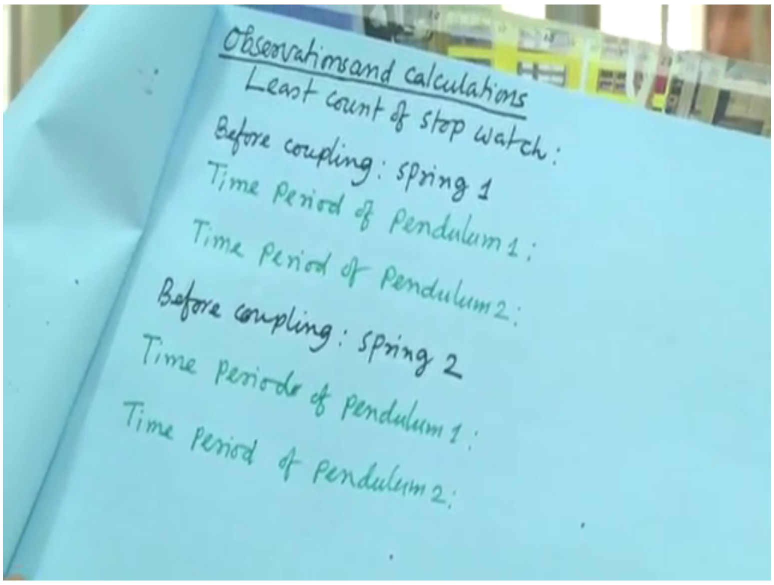

So, here basically here what we have to do observation and calculation we will use this stopwatch what is the least count of the stopwatch that only. I have to note down least count of the stopwatch is in this case 0.0 in this case 0.01 0.01 second so that will note down. Now a before coupling before coupling spring 1 time period of the pendulum 1 as I showed you, so that we have to note down time period of the pendulum 2 that we have to find out before connecting this one so I will note down. And then I will do the for coupling condition I will do the experiment that I will show you and for spring 2 again I will take out this 1 spring 1 then again I will note down the time period of pendulum 1 time period of pendulum 2 before coupling.

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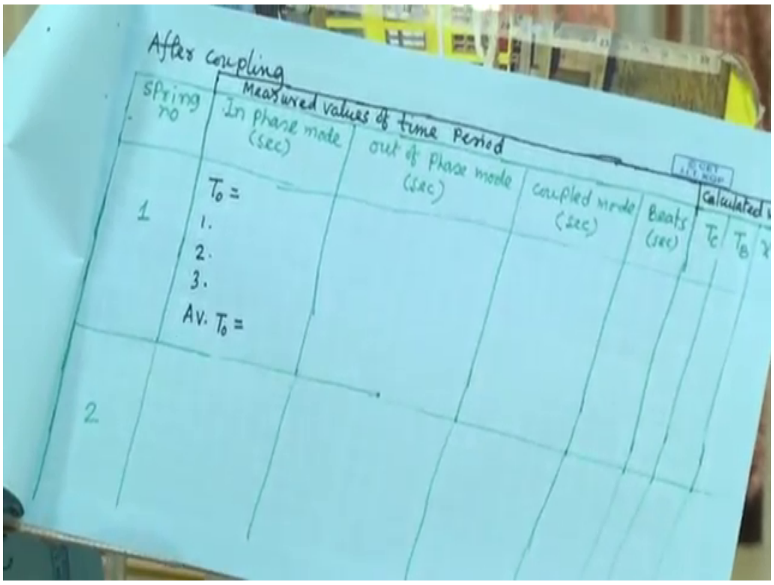

So, now we should go for after coupling, so I will just show for spring 1 for spring 2 you can repeat the experiment what I have to find out actually I have to find out T 0 and T 1 right this there. So, T 0 what is this in phase mode in phase mode I can find out the frequency that is the T 0 and out of phase mode out of phase mode. So, what about the frequency I will find out, so that is the T 1 so that is what I will find out.

So, for each one I will take T reading and then average of that T 0 average of that here out of phase T 1, so then basically I can calculate these couple mode that is T C yes T C I can I can calculate ok. So, I will calculate if I know that T 0 and T 1 I can calculate T C T B and chi. So, that is there another experimentally I can find out this experimentally I can find out this is basically T C experimentally T C and T B also I can find out from this experiment ok. So, let me show you so in phase let me show you in phase means x equal to y that is what the theory tells.

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So, here I have put scale because x equal to y and in same direction ok, for that I have to approximately I have to displaced equally. So, let me just so it is I will take it is a round 37.5 and it is around ok. So, I will displaced y 7.5. So, if I do that this so I have to; I have to this ok. So, I displaced both of them equally x equal to y and in same direction.

Now, I have to take time period for this one so I know this is in phase mode and spring will not be compressed or extended. So, I will collect so I have to make it 0 and then start 1 2 3 4 5, so this way I have to collect 50 oscillation and this for this what is the time and then you note down this time and this number of oscillation and find out T 0.

Second out of phase; out of phase what was that x equal to minus y, so I have to go in opposite direction but equal amount of displacement. So, again I will go this I think I will take this way and this other way ok in opposite direction I have taken in opposite direction I have to find out the time period T 1. So, again let this reset so now I will start that 1 2 3 4. So, count it number of oscillation you take 40 50 and this note down the time ok. So, this way you will find out the T 1 so T 0 and T 1 if we have measured.

Now, let me show you the resonance condition; let me show you resonance condition very interesting resonance condition what was the condition. So, condition was what was the condition I have to that was 2 a x 0 equal to y 0 equal to 2 a and phi 1 equal to phi 2. So, basically what I will do I will keep one fixed other one I displace by 2 a ok.

So, as I told it will be switched over it is now at stunning condition, now other one is maximum displacement ok. If I start you see it will be stopped and it will have the maximum displacement then just opposite. So, energy is transferred from the this one to this one now you see it is a stunning condition. So, here 2 time period; one is individual one you know individual one and another is that beat one amplitude variation as I told ok.

So, these time period I have to calculate, so same way you will calculate this time period ok. So, this one and another this stopping for beats for beats I have to find out time period. So, when it will stop I will start; when will stop I will start; when it stop I will start ok, I think I have to stop is stopped ok.

If I start from there I will start time now so when it is perfectly stopped that we start. So, then time to come to the rest this pendulum and then it will come to the this rest the other pendulum ok. So, how many they are coming in rest so that number you have to note down and for that what is the time ok. So, now from there you will find out the time period for beats ok.

So, this is the experimentally we calculate we find out this time period for T C and T B and theoretically also you can calculate and then compare of them ok. This is the very interesting that one is stopped another is have maximum displacement or maximum amplitude of the displacement of the oscillation, so this way it will be going on it will be going on.

This the phenomena of this couple pendulum basically and very simple, but interesting and basically what at all equation we solve that we verified and from that equation you got the time period etcetera and with this simple setup we can verify them ok. So, I think I will stop here.

Thank you.