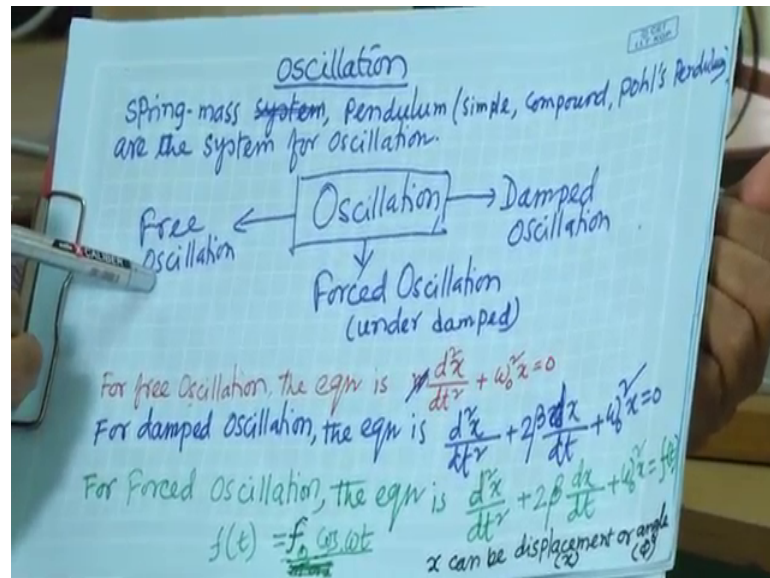


Experimental Physics I
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Lecture - 34
Forced Oscillations – Pohl's pendulum

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We are in first year laboratory of physics department of IIT, Kharagpur. So, today I will demonstrate one experiment that is basically oscillation and we will start with the oscillation. So, oscillation you know this spring mass system oscillate, then pendulum, simple pendulum, compound pendulum oscillate right.

So, this oscillation basically see free oscillation, then damped oscillation, then forced oscillation under damped. So, this mainly this three types of these three types of oscillation ok. So, what is the free oscillation just without any damping, without any external force if spring mass system or pendulum oscillates, then tell it is a free oscillation.

And now under damping also if it oscillates, then we tell this is the damped oscillation, damped oscillation without external force. And with external force, then we tell this forced oscillation ok. So, today I will demonstrate this three types of oscillation in a single instrument, this call basically Pohl's pendulum. This Pohl's pendulum so this the experimental setup of Pohl's pendulums, so I will come to this set up.

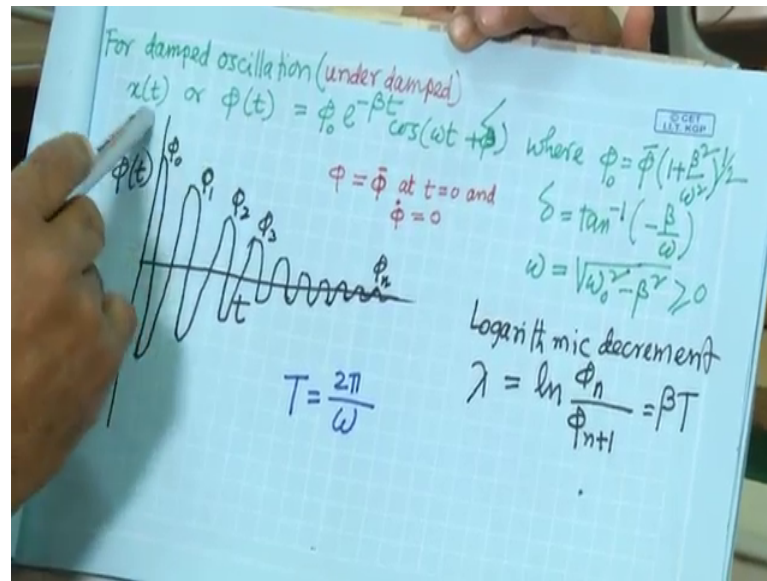
So, before that let me tell you about the theory of this experiment. So, you know this for free oscillation equation of motion is $\frac{d^2 x}{dt^2} + \omega_0^2 x = 0$. So, this is free oscillation, there is no damping term here.

Now, for damped oscillation, this additional term is here. So, this damp is basically proportional to the velocity, so and this β is basically a damping constant ok. So, $2\beta \frac{dx}{dt}$ is added with this one, so then equal to 0. So, then it is free oscillation, forced oscillation sorry damped oscillation without any external force that is 0.

If any external force is applied, then this external force this basically f as a function of function of t basically ok, so then this is the equation of motion for forced oscillation ok. So, this x can be displacement or it can be angle ϕ ok. So, in these experiment this basically change of angle, we will see this we will change the angle not it is the linear displacement it is a angle.

So, x can be x or this ϕ , and this external force is a periodic force is given. So, this $f(t)$ is equal to $f_0 \cos \omega t$, so this is the form of external applied force ok. So, this is the standard equation of motion of oscillators free oscillator, then under damping, then under external force right. Then under external force this three types of oscillation, we express by three equation of motion. So, for free oscillation from these equation you can see this frequencies ω_0^2 , frequencies ω_0^2 ok, so ω_0 is the basically natural frequency of this of the oscillator and yes.

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So, in case of damped oscillation what is the frequency of the oscillator, so that you can so for damped oscillator oscillation, so this phi t or x t, so here I will prefer to write phi t, because in this present case is the angle change. So, phi t the solution real solution of this equation of motion differential equation of motion is phi as a function of t equal to phi 0 e to the power minus beta t cos omega t plus delta ok, where phi 0 is phi bar 1 plus beta square by omega square ok, this to the power half square root of this basically. And so this is the phi 0, and e to the power minus beta t is the exponential term is there exponential term is there ok.

What is this phi bar phi bar equal to basically initially at t equal to 0, we are we are just displaced, we are just changing the angle initial angle starting angle at time t equal to 0, and that time it is at rest. And then just we will release it, so that is amplitude that is the amplitude is a is phi bar. This delta is basically if this fetch delta is fetch yes, so that is tan inverse minus beta by w, so if you solve this differential equation, you will get.

So, here important thing is this two part. One is under damped condition, one is the what is the frequency of these oscillator damped oscillator. So, this are frequency omega equal to square of angular frequency, of course it is a square square root of omega 0 square minus beta square ok. So, beta is the damping constant, and omega 0 is the natural frequency. So, to get it one has to find out beta ok. If you find out beta, then you can get this omega ah damped frequency of damped oscillator.

And also how amplitude changes with the with time, so in this solution if you see, so this part is basically amplitude. So, amplitude will decay exponentially, so amplitude will decay exponentially, so this is this amplitude ϕ this, we have plotted amplitude, amplitude variation with time. So, it is the cosine function it is a cosine function, so it is amplitude is this is amplitude now, decreasing amplitude is decreasing ok, so that is also we want to study how amplitude is decreasing.

And from this amplitude variation, we can find out the logarithmic decrement that is defined by \ln natural logarithm of ϕ_n by ϕ_{n+1} means this ratio of successive amplitude of the oscillator. So, any two successive amplitude you can take, so logarithmic of this ratio is basically λ . And this λ equal to βT , so that you can find out from this here itself, because this is ϕ_n this is ϕ_{n+1} and yes so from their e to the e is there, so that will come as a log and it will be βT ok.

So, if you can find out ω , then you will get that time to T or if you find out T , then you will find out this ω . And so from this amplitude variation, you can get this value λ value. So, λ you will measure from here you will get from here, and T you will get from T will measure also T will measure time period will measure. So, we can find out this β from this relation.

So, if you find out β yeah from this relation, you can find out ω_0 or if you know the ω_0 and β , then you can find out ω whatever so this is the interesting part of the of this damped oscillation. Here we can see this damping of the oscillation and this damping this is basically it is a exponential decay, so that we want to study.

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For forced oscillation with damping

$$\phi(t) = \phi_a \cos(\omega_a t - \alpha) \text{ where } \phi_a = \frac{F_0}{\sqrt{(\omega_0^2 - \omega_a^2)^2 + 4\beta^2 \omega_a^2}}$$

$$\tan \alpha = \frac{2\beta \omega_a}{\omega_0^2 - \omega_a^2}$$

For a fixed value of F_0 , amplitude ϕ_a exhibits a peak at a frequency, ω_{res} , given by

$$\omega_{res} = \sqrt{\omega_0^2 - 2\beta^2} = \sqrt{\omega^2 - \beta^2}$$

And then another part is a force oscillation. So, differential form of force oscillation you have seen that (Refer Time: 10:50) solution of this force oscillation is $\phi(t) = \phi_a \cos(\omega_a t - \alpha)$, where ω_a is the frequency of the applied force, ω_a is the frequency of the applied force ok, so that force we applied $F(t) = F_0 \cos(\omega_a t)$, I think $\cos(\omega_a t)$.

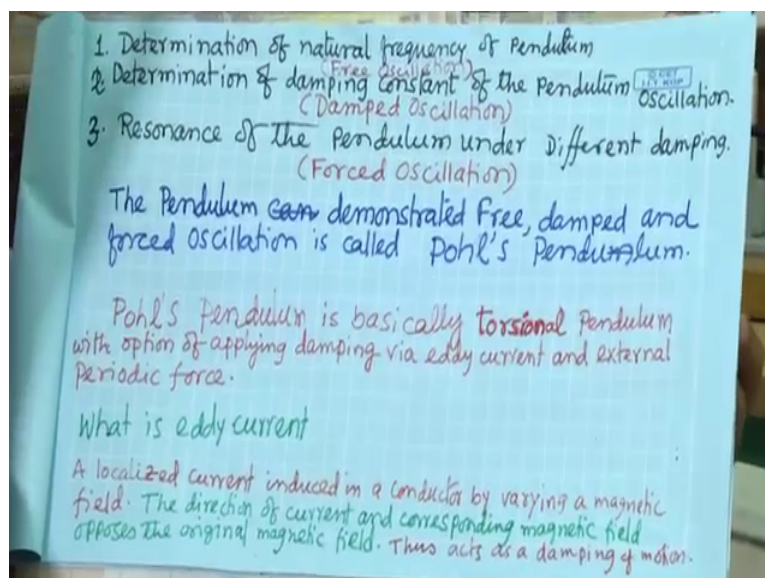
So, the solution is this where this is the applied frequency of the force, and ϕ_a this is the amplitude. So, ϕ_a is $F_0 / \sqrt{(\omega_0^2 - \omega_a^2)^2 + 4\beta^2 \omega_a^2}$, and this $\tan \alpha$ or α equal to \tan^{-1} of this one ok. So, here so basically, here we will study in this case, we will study the resonance because this our damped oscillator, it has it has frequency ω_a .

Now, I have applied force having the frequency ω_a , now if I change the ω_a , so when this ω_a is close to the value of ω_0 this frequency of the damped oscillator. So, then there will be resonance so that resonance frequency will measure. So, this resonance frequency is basically ω_{res} is square root of $\omega_0^2 - 2\beta^2$ that is equal to $\omega^2 - \beta^2$, because ω^2 was ω_0^2 square minus β^2 square ok.

So, basically ω_{res} , we can write square root of $\omega^2 - \beta^2$ [vocalized-nise] ok. So, this is the basically basic theory of the oscillation free

oscillation, then damped oscillation, then force oscillation ok. So, this is the basic I think we have studied in class twelve, this type of oscillation.

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So, let us come to the to study this oscillation, we will use this Pohl's Pohl pendulum as I told, so what is Pohl pendulum what is Pohl pendulum. So, how we are getting, how we are applying, how we are applying damping, how we are applying external force in this in this setup. So, Pohl's pendulum is basically the torsional pendulum with option of applying damping via eddy current and external periodic force ok.

So, this is the arrangement we tell Pohl's pendulum, where we can demonstrate free oscillation, damped oscillation, and forced oscillation ok. So, how we apply damping, so damping we apply via eddy current, and external periodic force we have we have option to apply external periodic force. So, using this one, this pendulum will demonstrate we will basically determine the natural frequency of the pendulum. We will determine damping constant of the pendulum oscillation and we will find out the we will study the resonance of the pendulum under different damping condition ok.

So, I think these term may be new to you this eddy current what is eddy current, eddy current is a basically a localized induced localized current induced in a conductor by varying a magnetic field ok. If a conductor is moving in a magnetic field or magnetic field is changing and there if there any conductor is there, then a current is induced localised current is induced in the conductor ok, so that current is called eddy current.

Now, when eddy current will flow eddy current will flow in the in the conductor, then again it will generate magnetic field. And that magnetic field basically direction will be such that direction of the current induced current will be such that the this magnetic field produced by the eddy current will oppose the original magnetic field ok, so that is that is why that is what faradays law, faradays law of induction right.

So, basically so thus we are opposing the motion of the we are opposing the motion of the conductor in a magnetic field ok. So, when a conductor is in motion in a magnetic field, then there will be induced current which is called eddy current. Due to eddy current there will be magnetic field, which oppose the original magnetic field. So, thus conductor will feel resistance ok, so that is thus it is basically giving the damping to the motion of the conductor ok. So, thus acts as a damping of motion.

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Working formula

Free Oscillation
 $\omega_0 = \frac{2\pi}{T}$ $\beta = 0$
 $\phi(t) = \phi_0 \cos \omega_0 t$

Damped Oscillation
 $\phi(t) = (\phi_0 e^{-\beta t}) \cos(\omega t + \phi)$
 $\omega = \frac{2\pi}{T}$, $\beta = \sqrt{\omega_0^2 - \omega^2}$, $\omega_0 = \sqrt{\omega^2 + \beta^2}$
 Amplitude = $\phi_0 e^{-\beta t}$ $\phi_0 = \phi \sqrt{1 + \frac{\beta^2}{\omega^2}}$
 $(\phi_n) - \beta T$
 $\phi_1 = \phi_0 e^{-\beta T}$, $\phi_2 = \phi_0 e^{-2\beta T}$... $\phi_n = \phi_0 e^{-n\beta T}$

Logarithmic decrement $\lambda = \ln \frac{\phi_n}{\phi_{n+1}} = \beta T$

Forced Oscillation
 $\phi(t) = \frac{f_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2 \omega^2}} \cos(\omega t - \phi)$
 $\omega_{res} = \sqrt{\omega_0^2 - 2\beta^2}$
 $= \sqrt{\omega^2 - \beta^2}$
 $\omega = \sqrt{\omega_0^2 - \beta^2}$
 $\phi_{res} = \frac{f_0}{2\beta \sqrt{\omega_0^2 - \beta^2}}$

So, so let me show you let me show you the before that let me just tell the working formula just in a concise form, what is the working formula for this experiment. So, for free oscillation omega 0 equal to basically 2 pi by T. So, if I measure T, so I have to measure T basically, so in this case beta equal to 0. So, this motion will follow this phi t equal to phi 0 cos omega 0 t ok. So, it is a is the cos variation of the of the phi ok. So, with time this there will not be any change of this amplitude means yes amplitude ok. So, it remains all the time it remains constant same amplitude.

And then for damped oscillation this motion of the oscillator is $\phi = t$ equal to this amplitude part, this you see now amplitude will decay ok. And its frequency is ω , $\omega = 2\pi/T$, so this is a time period we have to find out, then we can find out this ω . And then I know ω_0 , I know ω_0 , I know ω ok, then we can calculate also β from here. Of course, ω_0 you calculate this from here, you can write this relation also.

Now, amplitude variation, we will study; amplitude variation we will study. And from there we will find out log decrement and forced oscillation. So, this is the motion of the forced oscillation. So, in this case, we will find out the resonance frequency $\omega_{\text{resonance}}$ ok. So, experimentally we can find out, we can also calculate ok, we can also calculate. And also we will find out the amplitude and resonance amplitude and resonance. So, its relation is like this. So, you can calculate also experimentally you can find out ok.

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So, let me describe now the our setup ah. So, this is the Pohl pendulum. What is there, here you see just if I is basically torsional pendulum, there is a spring here you know, there is a spring here. So, this spring is basically giving restoring force on this on this wheel. So, it is it is oscillating it is the free oscillation, it is just simply free oscillation you can see this amplitude is in this side it is 14.8 and in this other side it is around 16. So, it is the it is vary, but it is free oscillation it is not completely free, because there are

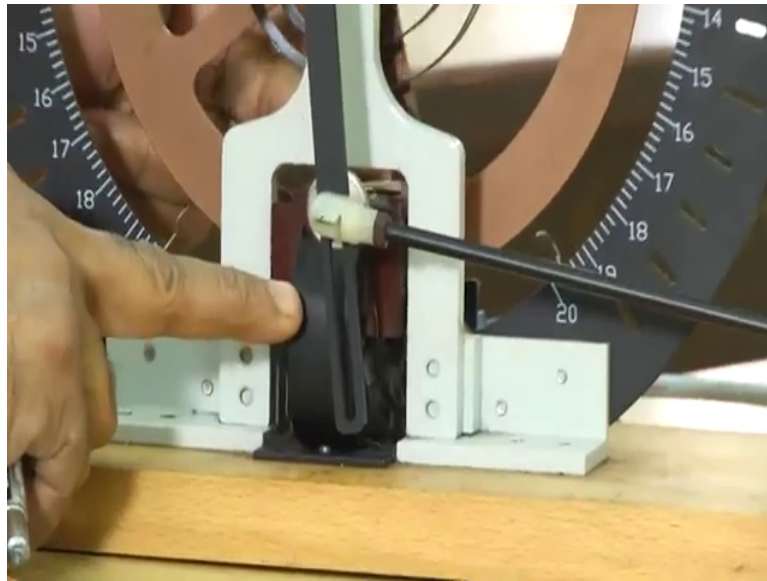
some air resistance when it is in motions of some external resistance is there, but it is a very small. So, this is the free oscillation ah. So, without any applying any damping without any applying, applying any damping, so this I will take as free oscillation, yes, it is free oscillation. So, I can find out the time period for this free oscillation.

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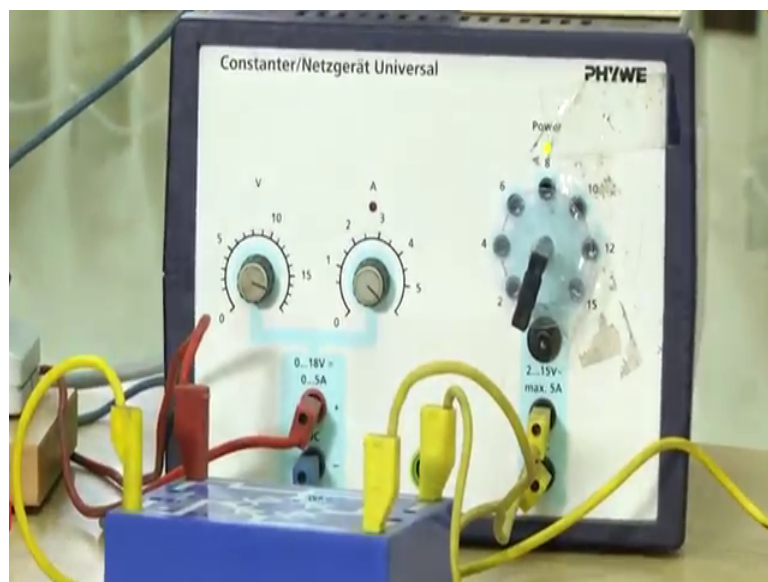
So, I have this ah stopwatch ok. So, I will take time for 20 20 oscillation or 30 oscillation. So, I have to say start 1, 2, 3. So, you know how to take because in case of other experiment like compound pendulum that we have seen how to take this time for few number of oscillation, and then you can calculate so total time divided by this number of oscillation, so that will be time period for this free oscillation ok. So, now we will apply damping. So, we will apply different damping and then for each damping we will find out the time period ok. So, for applying damping here, you can see we have a unit ok.

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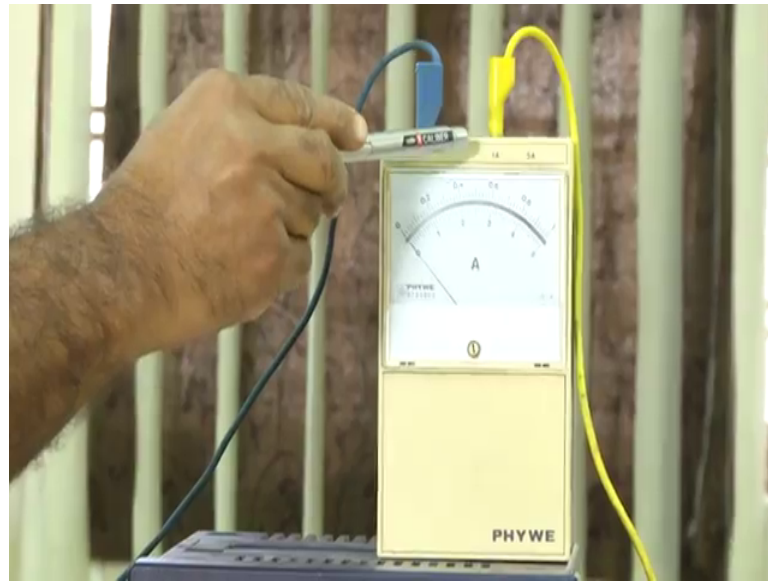
So, let me show you here, here this wheel is in motion between a magnet between a electromagnet. This is the electromagnet ok.

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So, we are passing current to this electromagnet, we are passing current to the electromagnet, from here we are passing current to the electromagnet from here ok. So, this ac current this is the ac current. So, here this is a rectifier. So, from this rectifier from these two end current.

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You see we have connected in emitter, we have connected the emitter. So, current is going current through this is current is going here electromagnet and coming out from the other end red one red part this one ok. So, circuit is closed, circuit is closed. So, how much current we are applying ok. So, magnetic field will be proportional to that one right magnetic field is it proportional we connect whatever a magnetic field have, yes, it is proportional in case of electromagnetic, it is a proportional. So, your current so current will flow through the coil of the electromagnet and then it will produce magnetic field. And in this magnetic field, so between these two pole piece between these two pole piece, this is the wheel is rotating. So, basically this is the metal, metal copper is made off copper metal.

So, now metal conductor, so now, conductor is in motion in the magnetic field. So, there will be eddy current, eddy current on this conductor on this wheel. And due to this eddy current, as I mentioned, so there will be damping, there will be damping. So, this damping that basically we are giving in terms of current here I have option to give this, you see, now I will I put here at 4 means I have applied 4 volt. Here I have option to apply 2 volt, 4, 6, 8 to up to up to I think 15 volt here, up to 15 volt, and maximum current this unit can give 5 ampere.

So, now here you see I will just note down the current, because here damping is related with the current. So, this current I have to note down. This is I think 2.6 around 2.6 not

two point, 0.26, yes, this is 0.1, 0.2, 0.3, 0.4. So, this is 0.26. So, I will note down the damping current is 0.26. And for this damping, again now I have to find out time period as well as I have to find out the amplitude.

So, let us find out first time period. So, I will take again just displays it ok. Now, I will start the watch, now I will start the watch ok. So, start 1, 2, 3, etcetera, etcetera, so you find out the prime period for damping. So, then I will change to the another damping, so I will put at 2 or 6 ok, so for three damping we will do the experiment. So, just I showed you one. So, for you take out this knob and put it as a 2. So, now, you will see current is 0.6 ok. And for this again I have to take time period, and then I can put 4 and then I can put at 6 at 6. So, at 6, this current is 0.4 four 0.4 ok.

So, for dc current damping current, we will do experiment, but I will show just only one that is for this middle one, so that is for middle one ok. So, this is the arrangement for applying damping to this oscillator. Now, under this damping condition we will find out the time period ok. So, when you if you after finding out the time period, so now, then we will study the amplitude variation with time.

So, what I will do amplitude I will displace it initially whatever I displace that is basically ϕ bar as I showed in working formula 5 bar ok, so that I will note down. Now, for each time period t , then zero time, then t then $2t$, t is time period, then $3t$ means if I start from here, so I will take this reading then coming back I will take reading. So, that is at time t equal to capital T , then again I will take $2t$ ok. So, this way first I will take reading.

Then I will take reading again I will do the same way starting here, then I will take the opposite side reading ok. So, that is basically at time t equal to t by 2 then again coming back at time t equal $3t$ by 2, $5t$ by 2. So, I will get, so ah at a time it is difficult to take this reading in both side. So, that is how we break it first we will take one side, then we will take the another side. So, now I have amplitude at time t at time 0, then t by 2, t , $3t$ by 2, $2t$. So, I have data log. So, then we will plot.

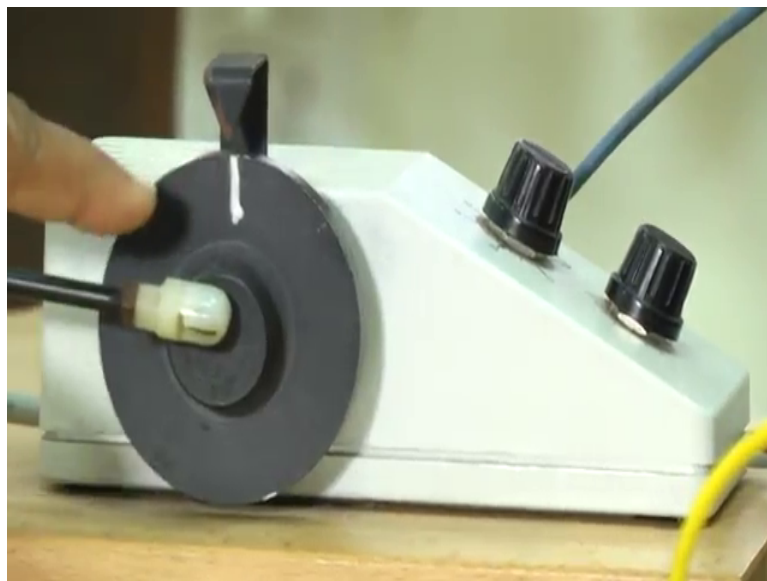
So, then we will find out basically the oscillatory, yes. So, I think if I showed you. So, then I will plot, I will plot, I will plot it. This is a t equal to 0, then t at t by 2, then $2t$ sorry t then $3t$ by 2. So, this way I will plot and then I will just connect it. So, then I will

see how this amplitude is varying ok. And from there I can find out this ϕ_n and ϕ_{n+1} . And yes of course, you can calculate this logarithmic decrement ok.

So, yes, so just let me just demonstrate one. So, starting I have taken at 18, say at 18, so I will note down this reading, then 16, then 14.8. So, this way I have to take (Refer Time: 30:10) that is a decreasing, it is a decreasing. Now, it is 9.3, now it is 7.4, now 6.8 so, this is a decreasing. So, I have to take reading ok. So, this is for 0, t , $2t$.

Now, again I will start at that same position from 18, and I will take reading other side ok. So, at t by 2, this is 19, then 18, then 16.8 ok. So, this basically t by 2, 3 by 2, 5 t by 2, so this way we have to take reading. So, this is the damped oscillation for three damping current we will do this repeat the experiment. Then let us go for force oscillation. So, we have option for force oscillation. Here you see you can see just, let me stop it, let me stop it.

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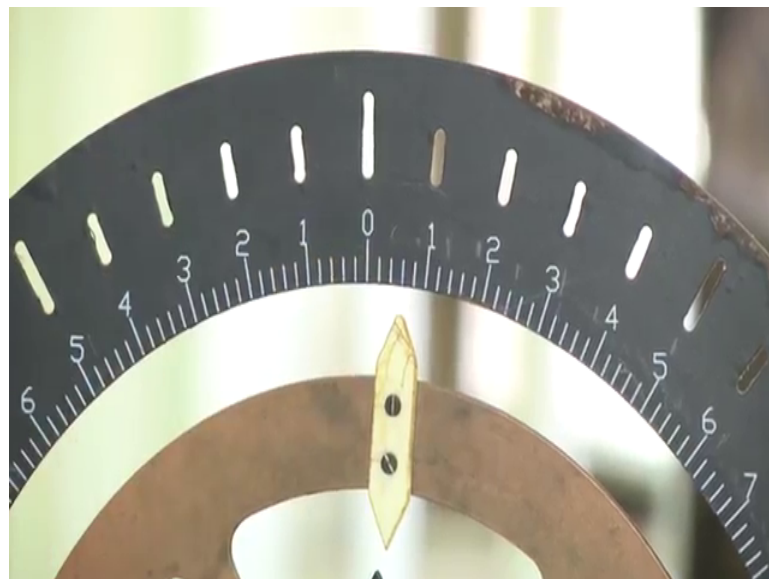


So, I think let me take indicator here. So, just we have one indicator, yes. This marking now coincides with this indicator and it is at 0. So, I have already this damping current is given this is ah yeah for 0.26 at this damping now externally I am applying force external force so to these wheel ok. So, here I have this arrangement. So, this wheel so I will apply here, you see from here, I will apply ac current to the motor this is the motor, I will apply ac current and this frequency of this is a current I have option to change from here, I have option to change from here ok.

So, then this force amplitude of this force is decided by this voltage and current, so that we have kept fixed maximum value have given. Now, only we will vary the frequency of the applied force ok. And this force we have chosen basically it is a cosine right, it is a force this form this in is it is cosine form that I have shown in theory ok.

So, now let us just apply let me on it ok. So, just you see it is rotating. So, this is this, this. So, this force now it is it is forcing this wheel. It is forcing this wheel right. It is it is coming from here, it is coming to this, this wheel. So, this is the externally we are applying force on it ok. Now, I will what I will do at different frequency, at different frequency, I will note down the amplitude, I will note down the amplitude ok.

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So, so left side and right side of the 0, what is the value that I will take and then basically average of that value I will take the amplitude for this frequency. Now, here how I will calculate frequency. So, I will find out the time period. What is the time period you see this is the, so I will use stop watch, I will use stop watch to for a complete one rotation what is the time it taking. So, I will take basically few number of rotation and collect time, and then from there I find out the time period ok.

So, if you start your watch, ah yes, start so one. So, this way I will collect is a very slow, so I will collect time for 5 rotation ok. So, this time divided by 5 that will give me time period for this external force ok. Now, from there you can find out the ω , in theory whatever I have shown ω . So, you can find out ω . So, now, for this what is

the amplitude this side it is a say 1, and the other side it is 0.6. So, average is 0.8. So, I will note down for this time period or frequency, it is the amplitude is.

Now, I will just increase the amplitude ok, I will increase the amplitude. I think I will go slightly. Next step I have done ok. Now, here again I will find out the time period for this rotation and corresponding amplitude I will note down. So, this way I will vary the frequency and note down the amplitude.

So, then frequency versus amplitude, amplitude versus frequency graph, we can plot. Then basically at a particular frequency this amplitude will be maximum. So, that is the resonance condition that I will show you. And after resonance also you will vary the frequency and we will basically we will see a symmetric curve Gaussian like curve, and peak will be the basically at resonance frequency. So, from the peak we can find out the resonance frequency if we plot this amplitude versus frequency ok.

So, let us just continuous, just let me change and find out the I am increasing now, I am increasing the amplitude frequency of these external force, it is increases it is increasing. For each step again I have to take this reading. So, just yeah you have to take and just going towards the resonance frequency I am increasing, I am increasing you see this amplitude also increased.

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So, I have to because this curve is like this you know. So, now slight change will increase amplitude more. You see now, its motion is faster. So, we have to find out time period amplitude also increased. So, let me increase slowly, slowly. We are yes we are going towards the resonance, yes, we are going towards the resonance. Now, it is decreasing. So, I cross the resonance position, resonance frequency. So, I have to go back, I have to go back ok.

Let me again increase. So, we have to wait to get it at equilibrium oscillation ok. So, I think I will get resonance. So, I think this is the resonance condition. So, slightly we have to adjust to get perfectly this I think position for, so it is changing, I stopped it.

So, let me let me again find out the resonance position, slowly, slowly, you have to move and step by step. And each step you should wait. Now, it becoming sensitive to the frequency. Now, very slowly we should rotate change the frequency, wait and see, whether, yes. So, I think I got the resonance position. So, now, what is the time period for this resonance condition that I have to find out.

So, starts let me start 1, 2, 3, 4, 5 so, this I take 20 oscillation rotation, and this time divide by this 20 will give the resonance time period from there you can find out frequency ok, $2\pi/t$ that will be ω , ω resonance. And we have to note down the this amplitude. In this side it is 19.4; in this side, it is 18 or 17.8. So, this divided by 2, this plus this divided by 2 that will be that is the average amplitude at resonance. So, this I will note down. So, I think I will stop this experiment, or let it we oscillate it. So, I have to note down all this data, I have to note down all this data right.

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Table-1: Time period of oscillations with different damping current

Least count of stop watch = ...

Damping current (Amp)	No. of oscillations n	Time t (sec)	Time period $T = \frac{t}{n}$ sec
0			
-			
-			

Table-2: Amplitude versus time for different damping

Damping current (Amp)	Time t sec	Amplitude ϕ	Time t sec	Amplitude ϕ
0	0		$\frac{T}{2}$	
$T =$			$\frac{3T}{2}$	
$2T =$				

Plot ϕ vs t

I have table time period of oscillation with different damping current ok. So, damping current is say first we will take at 0 current, then current two point 0.26, then 0.26 or this I think it was when I put here 2, so what is the value that at that current we will take, then second current is 0.26 and then third current, we showed this 0.44 ok. For this three current including excluding the 0 damping ok, so we will take this reading, we will find out the time period for each case right. So, we will note down, then amplitude versus time for different damping as I showed you for different damping.

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Table-2: Amplitude versus time for different damping

Damping current (Amp)	Time t sec	Amplitude ϕ	Time t sec	Amplitude ϕ
0	0		$\frac{T}{2}$	
$T =$			$\frac{3T}{2}$	
$2T =$				

Plot ϕ vs t graph.

can be displacement

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[illegible]

So, then this table frequency versus amplitude under forced oscillation. So, damping for a particular damping current that I showed you for a particular damping current, and I showed you there. So, here presently 0.26, 0.26 damping. So, what is the time period from here we have to find out and from there we can find out ω that is frequency and corresponding amplitude as I told this for different for a particular damping current for different, so we find out time period corresponding frequency. And for each frequency, we will note down the amplitude. And then we can plot the graph this frequency versus amplitude, and there we will get the basically peak position, from there you can find out ϕ resonance, amplitude and also you can find out ω resonance ok.

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Table-4: Result-

Damping current Amp	λ	β	ω_{res}		ϕ_{res}	Natural freq $\omega_0 = \sqrt{\omega_n^2 + \beta^2}$
			observed	estimated		

And so that is observed means experimentally observed, and another estimated means using the formula you can calculate. So, basically result whatever the data we have taken so, for different damping, so we will calculate lambda log decrement, then damping coefficient beta, then omega resonance for different damping then phi resonance for different damping and you can from this data also, you can find out the natural frequency. Natural frequency omega 0 equal to square root of omega square plus beta square. So, all data are available with you. So, you can find out this one ok. So, this is the very nice experiment, where you can demonstrate, all types of oscillation phi oscillation, damped oscillation, force oscillation, so that is why this pendulum whole pendulum is very interesting and important ok. So, I think, I will stop here.

Thank you for your attention.