

**Experimental Physics I**  
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**Lecture – 27**

**How to calculate the value of moment of inertia of a flywheel from the recorded data**

In last class we have demonstrated the measurement of moment of inertia of a flywheel.

(Refer Slide Time: 00:33)

Measurement of Moment of Inertia of a Flywheel

- \* We have demonstrated the measurement in laboratory in last class.
- Let us now analysis the data and find out ~~very much~~ Moment of Inertia and errors.
- \* The working formula for this experiment is
$$I = \frac{m(2gh - r^2\omega^2)}{1 + \frac{n_1}{n_2}}; \quad \omega = \frac{4\pi n_2}{t} \quad \text{and} \quad h = 2\pi r n_1$$
- \* Measurands (parameters are to be measured) are  $m, h, r$ , ~~and~~  $t$  including counting of  $n_1$  and  $n_2$ .
- \* The experiment can be repeated for three different masses and for each masses Two or Three height can be taken.

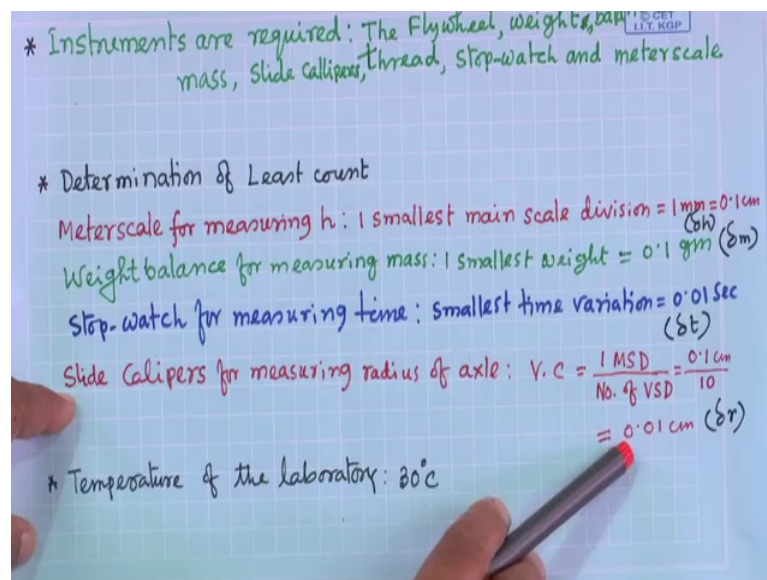
So, in laboratory you have seen the instrument and how to perform the experiment that we have discuss. So, let us now analysis the data and find out moment of inertia and errors on the measurement. So, the working formula for this experiment is  $I$  equal to  $m \frac{2gh - r^2\omega^2}{1 + \frac{n_1}{n_2}}$ .

Omega is basically is a angular velocity of wheel. So, angular velocity of wheel and axle the same velocity so, that omega it can be written as  $\frac{4\pi n_2}{t}$ . So, what is what is  $n_2$  that you know this correspond maximum speed maximum omega. So, how long it takes time to come to zero zero velocity? So, before coming to zero velocity, how many complete rotation revolution. So, that is number is the  $n_2$  and also height can be expressed as  $2\pi r n_1$  because one round of this turn ok.

So, that is that is the if radius is  $r$  then it is a circumference this is  $2\pi r$ . So, for  $n$  1 turn. So, into  $n$  1 so that would be the basically height because here we assume that after  $n$  1 turn, just this mass will touch the floor. So, height one can measures directly or one can one can find out using this relation because we are measuring  $r$  as well as  $n$  1. So, anyway so, here basically our measurands the parameters are to measured basically that is mass, height  $h$  radius  $r$  and time  $t$  including counting of  $n$  1 and  $n$  2.

And this so and the experiment can be repeated for 3 different masses and for each masses 2 or 3 height can be taken ok. So, that is in deriving demonstration I was mentioning most of the things. So, for these experiment, what are the instruments are required?

(Refer Slide Time: 03:55)



So, basically this flywheel we have seen the flywheel and then weight or mass, then balance to measure the mass or here we have been measure weight balance or anyway simply balance. Then mass, then we need slide callipers, thread, stopwatch and meter scale ok. So, these are the instruments we required for this experiment.

Now first as I all the time I mention for the any experimental devices. So, when you are doing experiment. So, you have to first determine the least count of your instrument. So, that is meter scale we are using for measuring height or without measuring height using the metre scale also one can find out using  $2\pi r n$  relation.

So, anyway, but meter scale whatever we have in laboratory. So, we should know what is the smallest main scale division. So, that is the least count. So, this a 1 millimetre that is 0.1 centimetre and then for weight balance for measuring mass this smallest weight that is a it is a 0.1 gram. So, that is the  $\Delta m$  this one  $\Delta h$  and stopwatch whatever we will use. So, or we have already used. So, that is the smallest time variation we can detect. So, there is a 0.01 second. So, that is  $\Delta t$ .

Slide callipers for measuring radius of axle. So, the vernier constant you know this 1 main scale division divide by total number of vernier scale division. So, 0.1 centimetre by 10 this is 0.01 centimetre this is there  $\Delta$ . So, all these things first you have to find out and note it down. Then I think this for this experiment, you do not need this temperature because moment of inertia basically, it will not vary with temperature slide variation of temperature of course. So, but it is always good practice to note down the temperature of the laboratory. So, this in all laboratory is a temperature is around 30 degree centigrade.

(Refer Slide Time: 06:59)

Table-1: Radius of axle by slide calipers  
V.C = 0.01 cm

No. of obs.	Reading of the main scale (cm)	Vernier scale	Total reading of diameter (cm)	Mean radius = $\frac{\text{dia}}{2}$ (cm)
1	-	-	-	1.00
2	-	-	-	
3	-	-	-	

And then let us first I think noted down the radius of the axel, measured by slide callipers. So, make table. So, table 1 these are for measurement of radius. So, we should write down the vernier constant of that one. So, number of the observation the reading of the main scale vernier scale. So, already you know this for other experiment I have I think discussed about this tables.

So, this is the standard table for our slide callipers. So, this total reading of diameter is unity centimetre and then you have to find mean radius. So, that is basically we have to diameter by 2, it is in centimetre. So, just note down note down the reading and then find out the mean radius. So, in our case this is radius mean radius of this axel is 1 centimetre. So, this is the table 1 then.

(Refer Slide Time: 08:19)

Table-2: No. of revolutions ( $n_2$ ) and time ( $t$ ) for different mass ( $m$ ) and height ( $h = 2\pi r n_1$ ). Circumference of wheel = 61 cm

Mass ( $m$ ) (gm)	For Height $[h = 2\pi r n_1]$ Recorded $n_1$ $2\pi r = 61.28 \text{ cm}$		Revolutions ( $n_2$ )			Time $t$ (sec)	
	$n_1$	$h$ (cm)	Complete no. revolution $x$	Fraction of revolution $y = \frac{d}{D}$			
				$d$	$Y$		
59.9	8	50.24	8	38.4	0.63	8.63	
	7	43.90	-	-	-	-	
	6	37.68	-	-	-	-	
75.0	8	-	-	-	-	-	
	7	-	-	-	-	-	
	6	-	-	-	-	-	
90.0	8	-	-	-	-	-	
	7	-	-	-	-	-	
	6	-	-	-	-	-	

Next this main part of the experiment is basically to this table 2.

So, number of revolutions that is  $n_2$  and time  $t$  for different mass and height. So, this is the main part of the experiment. So, we will take a mass and that type that we have seen that is it is hanged, this mass is hanged with thread and then thread is turned around the axel and how many turns that is the  $n_1$ . So, this then we will release the mass and then we will we will start to rotate and finally, after  $n_1$  turns after  $n_1$  rotation, this mass will just touch the floor or detach from the it will detached from the axel ok.

So, by a theory it was attached to the axel round, it will just it will be detached. So, then when just it detach, then we have to start the stopwatch and how much time it is taking to come to the rest and before coming rest how many turns it will complete. So, that is what we have to count. So,  $n_2$  we have to count and we have to measure a time  $t$  time  $t$  that is for a particular mass and particular height ok. So, basically for a particular mass we will take 2-3 height.

We will take reading for 3 height then we will change mass. So, here what are the columns mass? So, we have taken here I have noted down 3 masses. So, 1 is around 60 gram then 75 90 gram ok. So, for each mass, then this for different height so that height  $h$  equal to  $2\pi r n_1$ . So, varying the  $n_1$  so, this if I keep 8 turns or and then next 7 turns and then next 6 turn and for this then corresponding  $h_1$  can find out because  $r$  is known to us.

$R$  is 1 centimetre as we found is. So, this  $2\pi r$  so, this I have noted down  $2\pi r$  here is basically 6.28 centimetre and then 8 turns 7 turns 6 turn. So, corresponding height you can find out. So, for each height now for this mass for each height now you have to find out time as well as number of revolution that is  $n_2$ . So,  $n_2$  to find out  $n_2$  so, it may not this before coming to the waste we may not have the all complete rotation; so, there maybe fraction of revolution.

So, you have to so, there in column. So, complete number of rotation its say  $x$ . So, in our case this for this height is the 8 complete rotation and then after 8 complete rotation, there is a fractional rotation that we have to find out. So, basically this wheel circumference of wheel is 61 centimetre that you can find out just want you take a thread and just round it, and then you measure the length of this that also there is a circumference. So, in our case that is 61 centimetre ok.

So, now, after 8 complete rotation what is the fraction distance? So, this arc you have to measure. So, just again you use thread and find out this length. Basically if a yeah using meter scale, you find out this length of this thread which is this fractional part and that we are telling; so, that fractional revolution for 1 complete rotation so, this 61 centimetre. So, now, this you have this part of that is we are taking this length is  $d$ . So,  $d$  by  $s$  that is  $y$  we are writing. So, that is the basically it will be fractional a fraction of revolution. So, this  $d$  this part of the revolution is not complete 1; 8 complete.

Revolution complete number of revolution and then  $d$  that is the part of that; so, now,  $d$  by  $s$  that is  $y$ . So, then corresponding  $y$  will be 0.63 just you can calculate and then actually 8 plus this one will be the actual number of rotation ok. So, that is  $n_2$  8.63 and from stopwatch this we have noted down the time. So, that is 25.44. Now again you repeat the experiment for the same mass, but now  $n_1$  turn that is 7.

So; that means, height is different. So, so then we have taken 3 different height for a particle of mass and repeat the experiment then change the mass to the 75, another mass and then again for 3 different height you noted down the reading for third mass 90 gram again for 3 these 3 height you repeat the experiment and here you will get the for each case for of each case you will get the time and time t and number of revolutions that is this n 2 ok.

So, that is what is the main part of the experiment, and after measuring after noting down this data now, let us calculate moment of inertia.

(Refer Slide Time: 15:51)

Table-3: Calculation of moment of Inertia  
 $g = 9.8 \text{ m/Sec}^2$  radius (r) = 0.01 m

Mass (m) (kg)	Height (h) (m)	n <sub>1</sub>	n <sub>2</sub>	Time (t) (sec)	$\omega = \frac{4\pi n_2}{t}$ rad/sec	$I = \frac{m(2gh - \omega^2 r^2)}{\omega^2}$ $\frac{1 + \frac{n_1}{n_2}}{\omega^2}$ (kg·m <sup>2</sup> )	Average I kg·m <sup>2</sup>
0.0599 ≈ 0.06	0.50	8	8.63	25.44	4.26	0.0175	
0.060	—	—	—	—	—	—	
0.060	—	—	—	—	—	—	
0.075	—	—	—	—	—	—	
0.075	—	—	—	—	—	—	
0.075	—	—	—	—	—	—	
0.090	—	—	—	—	—	—	
0.090	—	—	—	—	—	—	
0.090	—	—	—	—	—	—	

So, there is a table 3. So, our moment of inertia formula is this one. So, I need to. So, in this table there should there 1 column for mass m in kg here and then one we need h height. So, height h; so, this is the height column and then I need n 1 n 2. So, for n 1 for n 2 this columns are there, then I need basically yes time.

We have noted down time because we have to find an omega, omega is  $4\pi n_2$  by t ok. So, we have noted down here in this column time. So, from here n 2 and t are at the level and using this one. So, this omega in radian per second we have to find out. So, then now g value whatever. So, (Refer Time: 17:06) of we should write other parameters which even not is the constant and other already is we have measured. So, this radius we have noted down here; so, from all this data.

Now, for you see this only I have shown here 1 data I have other data also, but I have not shown just the same way one can. So, this is for 3 mass 3 heights the same mass 0.06 is a just approximately 0.06 kg for this mass what are the other parameters. So, just you note it down and for this data we have calculate I. So, that is moment of inertia is 0.0175 and other also you will get and then you should take average and find out the average I in kg, in kg metre square.

So, this you should write this. So, actually I have all data and if we take average; so, this will be average will be like this anyway forget that one let us say this is the result ok. So, this is the procedure to calculate the moment of inertia of it like will now say this is the this is our result, but actually you have to work with this average one since I do not have a that I do not have shown other data. So, just let us. So, take this is the only value we have.

(Refer Slide Time: 18:59)

\* Error Analysis

$$I = \frac{m \left( \frac{2gh}{\omega^2} - r^2 \right)}{1 + \frac{n_1}{n_2}} ; h = 2\pi r n_1 ; \omega^2 = \frac{(4\pi n_2)^2}{t^2}$$

$$I = m \left[ \frac{2g \times 2\pi n_1 r t^2}{(4\pi n_2)^2} - r^2 \right] / \left( 1 + \frac{n_1}{n_2} \right)$$

$$\text{or } I = \frac{m r}{1 + \frac{n_1}{n_2}} \left( \frac{g t^2 n_1}{4\pi n_2^2} - r \right) \quad \text{since } \frac{g t^2 n_1}{4\pi n_2^2} \gg r$$

$$\therefore I = K m r t^2 \quad \text{where } K = \frac{g}{4\pi n_2 \left( \frac{n_2}{n_1} + 1 \right)} = \text{constant}$$

$$\frac{\delta I}{I} = \frac{\delta m}{m} + \frac{\delta r}{r} + 2 \frac{\delta t}{t}$$

$$\delta I = \left[ \frac{0.1}{60.0} + \frac{0.01}{1.00} + \frac{2 \times 0.01}{25.44} \right] \times I$$

$$= 0.0124 \times 0.0175 \text{ kg} \cdot \text{m}^2$$

$$= 0.000217 = 0.2 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Answer/Result:  $I = (17.5 \pm 0.2) \times 10^{-3}$

So, now error analysis error analysis: so, this; so, of how to how to find out the expression for error that already you are quiet familiar with that. So, here I equal to this is the expression and where h is this and omega square is this all ok. So, here h and omega square; so, this we are not measuring directly r we are measuring directly n 1 n 2 basically that is count we are not using any tools. So, there is no delta n delta n is 0, delta n 1 delta n 2. So, this part we can one can take consider as a constant and of course, mass is the earth.



So, here if I just put the value of  $h$  and  $\omega^2$ ; so, then  $m$  this is  $2g$ , then  $h$  is basically  $2\pi n_1 r$  and by  $\omega^2$  means this. So, it is squares will go up and then  $4\pi n_2$  whole square and this minus  $r$  square divide by this ok. So, just take out this  $m$   $r$   $m$  is there  $r$  is take out and then it will this  $4\pi$ . So, below  $4\pi$  square is there. So, one  $4\pi$  will go. So, another  $4\pi$  will be there and  $n_2$  square and here  $g$   $t$  square and this  $n_1$  and then minus  $r$  ok. So,  $r$  I have taken 1 outside divide by this divide by this;  $1 + n_1$  by  $n_2$  ok.

Now in this expression if we look at this expression. So, here  $r$  is 1 centimetre as I told this ok, but here the  $g$  value  $t$  value  $t = 25$  point something and then  $n_1$  and divide by  $n_2$  square ok. So, these term basically is very very greater than  $r$ ; if it is very very greater than  $r$ , then you can neglect this  $r$  you can neglect this  $r$ . So, then we can basically write this I equal to  $m r t^2$  square  $t$  square and rest of the things are basically constant ok.

Ah Because we have not measured them yes using the instrument;  $n_1$   $n_2$  of course, you have we have this is the counting, but just we are not using any instrument to count it. So, we can we can treat them as a constant because  $\Delta n$  is 0 in this in our case. So, this case basically  $g$  by  $4\pi n_2$  if you just simplify it will be  $n_2$   $n_2$  by  $n_1$  plus 1 just rest of the part is the is the constant.

So, now this is a very simple expression. So, its are in multiplication forms. So, take the; so, this will be if this relative error will be added for each parameter. So,  $\Delta I$  by  $I$  equal to  $\Delta m$  by  $m$  plus  $\Delta r$  by  $r$  plus  $2 \Delta t$  by  $t$  ok. So, this is very simple now, it is very simple expression for here now just we have 1 sets of data just I have put them and yeah multiplied by  $I$   $\Delta I$  equal to this multiplied by  $I$ . So, we have calculated this error is this 1. So, answer or result is  $I$  equal to that was.

We have seen just whatever  $I$  have not average one, but let us take this is average one. So, 0.0175 so, that I can write 17.5 in to 10 to the power minus 3 17.5 into 10 to the power minus 3, so 17.5. So, after decimal is the 1 digit is there. So, I can keep this errors only up to 1 decimal point. So, this I can write this one is basically I can write 0.2 into 10 to the power minus 3.

So, we have to take in the same format ok. So, then basically error is 0.2 into 10 to the power minus 3 right 0.2 into 10 to the power minus 3. So, we write in this for 17 point sorry 17.5 plus minus 0.2 10 into 10 to the power minus 3 kg metre square ok.



(Refer Slide Time: 24:37)

$$I = m \left[ \frac{2g \times 2\pi n_1 r t^2}{(4\pi n_2)^2} - r^2 \right] / \left( 1 + \frac{n_1}{n_2} \right)$$

$$\text{or } I = \frac{m r \left( \frac{g t^2 n_1}{4\pi n_2^2} - r \right)}{1 + \frac{n_1}{n_2}} \quad \text{since } \frac{g t^2 n_1}{4\pi n_2^2} \gg r$$

$$\therefore I = K m r t^2 \quad \text{where } K = \frac{g}{4\pi n_2 \left( \frac{n_2}{n_1} + 1 \right)} = \text{constant}$$

$$\frac{\delta I}{I} = \frac{\delta m}{m} + \frac{\delta r}{r} + 2 \frac{\delta t}{t}$$

$$\delta I = \left[ \frac{0.1}{60.0} + \frac{0.01}{1.00} + \frac{2 \times 0.01}{25.44} \right] \times I$$

$$= 0.0124 \times 0.0175 \text{ kg} \cdot \text{m}^2$$

$$= 0.000217 = 0.2 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Answer/Result:  $I = (17.5 \pm 0.2) \times 10^{-3} \text{ kg} \cdot \text{m}^2$

So, this is the way one has to present the experimental data and one has to calculate the in this case is the moment of inertia and then error on it  $\delta I$ .

And then result should be written like this whatever I got here. So, I cannot write plus minus just this is ok. So, you should write in proper format. So, that is. So, the I have shown how to write. So, here for error calculation we have seen right it without simplification also one can do it directly, but you see it will be a slightly lengthy calculation as well as we have it will look slightly yeah I think bigger form. So, that I have also calculated like this. So, I am just I will show you.

(Refer Slide Time: 25:57)

$$I = \frac{m \left( \frac{2gh}{\omega^2} - r^2 \right)}{1 + \frac{n_1}{n_2}}$$

$$\frac{\delta I}{I} = \frac{\delta m}{m} + \frac{\delta \left( \frac{2gh}{\omega^2} - r^2 \right)}{\left( \frac{2gh}{\omega^2} - r^2 \right)} \cdot \frac{\delta \left( 1 + \frac{n_1}{n_2} \right)}{\left( 1 + \frac{n_1}{n_2} \right)}$$

$$\frac{\delta I}{I} = \frac{\delta m}{m} + \frac{1}{\left( \frac{2gh}{\omega^2} - r^2 \right)} \left[ \frac{4gh}{\omega^2} \frac{\delta t}{t} + \frac{2g}{\omega^2} \frac{\delta r}{r} - 2r \frac{\delta r}{r} \right] + \frac{\delta \left( \frac{2gh}{\omega^2} - r^2 \right)}{\left( \frac{2gh}{\omega^2} - r^2 \right)} \cdot \frac{\delta \left( 1 + \frac{n_1}{n_2} \right)}{\left( 1 + \frac{n_1}{n_2} \right)}$$

$$= \frac{4gh}{\omega^2} \frac{\delta \omega}{\omega} + \frac{2g}{\omega^2} \delta h + 2r \delta r$$

$$\delta \left( 1 + \frac{n_1}{n_2} \right) = \delta(1) + \delta \left( \frac{n_1}{n_2} \right)$$

$$\delta n_1 \text{ \& } \delta n_2 \text{ are zero}$$

$$\frac{\delta I}{I} = \frac{\delta m}{m} + \frac{\delta r}{r} + 2 \frac{\delta t}{t}$$

Alternative way:  
 $I = \frac{m \left( \frac{2gh}{\omega^2} - r^2 \right)}{\left( 1 + \frac{n_1}{n_2} \right)}$   
 $h = 2\pi r n_1$  and  $\omega^2 = \frac{4\pi n_2}{t^2}$   
 $\therefore I = \frac{m \left[ \frac{2g \times 2\pi n_1 r t^2}{(4\pi n_2)^2} - r^2 \right]}{\left( 1 + \frac{n_1}{n_2} \right)}$   
 Since  $\frac{g t^2 n_1}{4\pi n_2^2} \gg r^2$   
 $I = \frac{K m r t^2}{1 + \frac{n_2}{n_1}}$   
 Where  $K = \frac{g}{4\pi n_2 \left( 1 + \frac{n_2}{n_1} \right)}$  is constant  
 $\frac{\delta I}{I} = \frac{\delta m}{m} + \frac{\delta r}{r} + 2 \frac{\delta t}{t}$   
 $\omega = \frac{4\pi n_2}{t}$   
 $\frac{\delta \omega}{\omega} = \frac{4\pi \delta n_2}{4\pi n_2} + \frac{\delta t}{t}$   
 $\frac{\delta \omega}{\omega} = \frac{\delta n_2}{n_2} + \frac{\delta t}{t}$   
 $\delta n_2 = 0$   
 $\frac{\delta \omega}{\omega} = \frac{\delta t}{t}$   
 $h = 2\pi r n_1$   
 $\delta h = 2\pi n_1 \delta r$

If you wish you can do also. So, I equal to this.

So, this is a multiplication form if you take. So, this 3 got 1 2 and this 3. So, delta I by I delta m by m plus delta of this part divide by this plus delta of this part divide by this. So, these will be 0 because is we are taking constant. So, forget this part, this is also fine now you have to work on it ok. We have to work on this part. So, delta now here delta x minus y right; in this case, delta x minus y equal to delta x plus delta y. So, summation and subtraction rules we have seen. So, basically this we can write delta of this plus delta of this and then again this parts delta q.

If q is a function of x and y so, you know del q by del x delta x plus del q by del y delta y. So, that is are same thing here your h and omega square is there; so, just 2 g h 2 g h. So, delta omega minus 2 I have taken up divide by delta omega del omega delta omega similarly on del h ok. So, this way you proceed and of course, here is. So, this delta r square is there. So, again delta r squares; so, delta q. So, del r square by del r delta r. So, just this way 1 should proceed.

And this part as I told this is a0 and omega is there. So, here delta omega is there. So, omega form omega you can find out delta omega by omega. So, it is basically delta omega by omega is coming delta t by t because delta n 2 is 0. Similarly h equal to for because here del h is there. So, h equal to this; so, from there you find out del h ok. So, it

will come in terms of  $\Delta r$ . So, this you put here you put here in this expression all this whatever and then finally, you will get in this form ok.

$\Delta I$  by  $I$  equal to  $\Delta m$  by  $m$  plus 1 by this part is that what is this? This part this part is there here and then yeah 1 by this part and then for this one for this one whatever you have find out using this all. So, that will be  $4gh$  by  $\omega^2 \Delta t$  by  $t$ . So,  $\Delta t$  by  $t$  plus  $\Delta r$  by  $r$  plus  $2r$  square  $\Delta r$  by  $r$ . So, in this form you will you will get the expression for  $\Delta I$  by  $I$ . So, this also one can use, but its looks likely if bigger expression. So, of if you wish you can use this and of what as we all tried to ways I already discussed.

So, is a looks simpler, but you have to take approximation this approximation. So, I think these are all for about the about the moment of inertia of flywheel, whatever we have studied basically theoretical calculation means working formula we have derived and then we have demonstrated in laboratory. And then we have analyse data and find out the error from the measurement and report the final result with error ok. So, I think I will I will stop here so.

Thank you for your attention.