Experimental Physics I
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Lecture - 19
Basic analysis (Contd.)

So, we are discussing about the error analysis for our measurement. So, we have, we have find out found out that this if one physical parameter is related with the some measurement parameters and there is a relation between physical parameters and measured parameters. So, this in this relation, there may be summation of the parameters or subtraction or difference product or division of the parameters or some special cases like if some parameters have power or this some parameters have multiplied with some constants. So, all cases we have discussed and also we have discussed that if the physical parameter is a function of a one parameter, so say q is a function of x, then how to calculate the error. So, this those things we have discussed.

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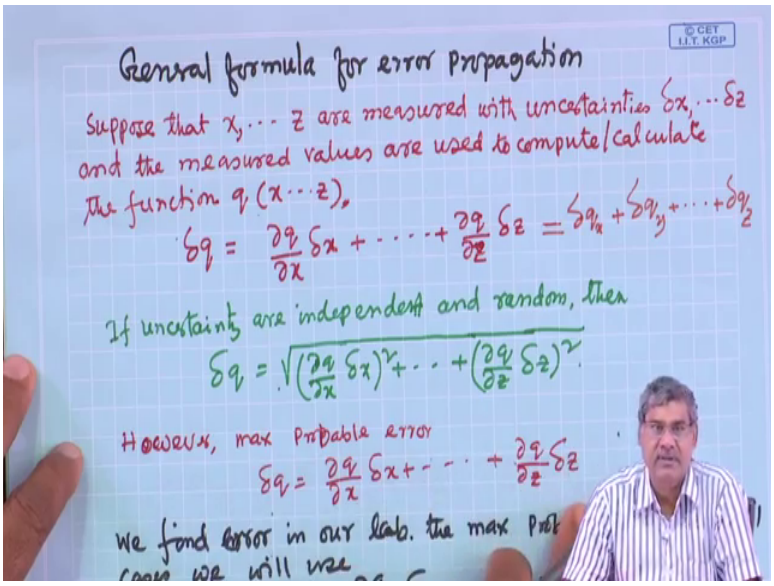

So general formula for error propagation we can tell that say, in case of in if it is function form. So, we can tell that the suppose x, y, z etcetera are the measured parameters with uncertainties del x, del y, del z and the measured values are used to compute or calculate the functions q which is function of x, y, z etcetera, so then, this error over q absolute error over q that is del q equal to del q by del x delta x plus del q by del y delta y etcetera plus del q by del z delta z ok. So, this basically we can we can tell this q x is basically this error del q x from x for x parameter error is del q x. So, this are will be summation of this del q x, del q y and then your del q z right. So, this is the form of error delta q, so if q is given, it is a function of x, y, z. So, just we will take the partial differentiation with respect to x, y, z and this each term will be multiplied with this individual error on the xyz. So, then this basically for each parameter whatever the error delta q x, delta q y, delta q z so, this is final error on q will be the summation over on the on the individual errors for different measured parameters.

And as if mentioned that if uncertainties are independent and random, then this statistical in nature and then also error can be in this is quadrature ok. So, this error also can be expressed as del q equal to square root of the square of the each terms, summation of the square of each terms and then we take square root of it. But, and del q, this del q equal to this is less than this ok. So, this is the overestimated error even if it is statistical in nature. So, if you calculate this way, so, then it is a overestimation but still we prefer to calculate error in this way. So, that is why we tell this error is that is the maximum error we report the maximum error. So, that is called maximum problem most probable error or maximum probable error del q equal to this.

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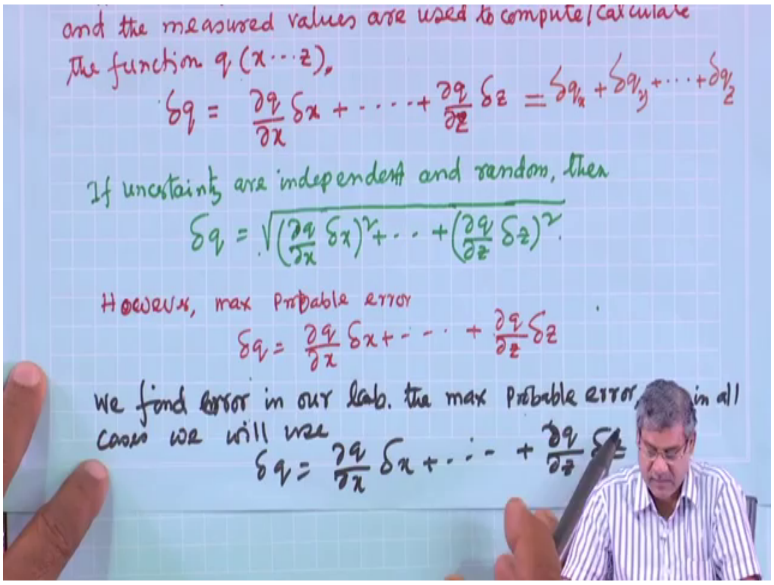

So, we will find error in our laboratory generally that is the maximum probable error and mean all cases, we will use this formal using whatever we have shown here ok.

So let us just discuss some examples for error calculations.

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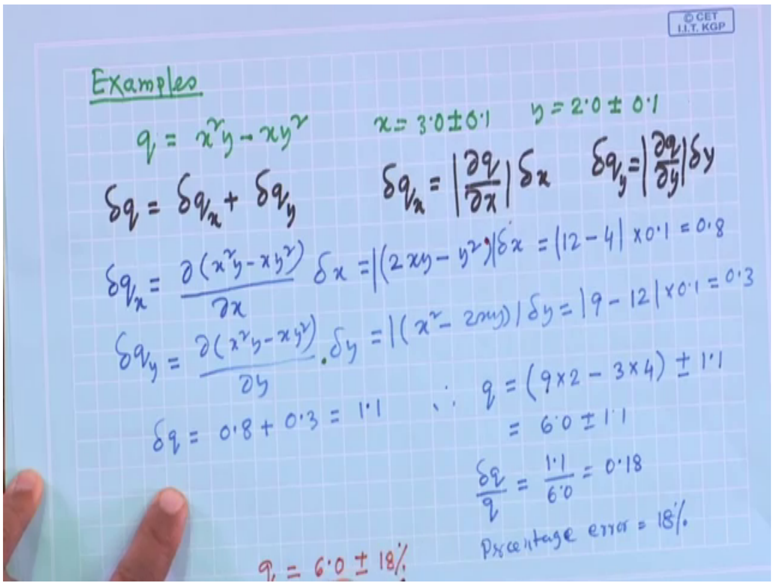

So, here it is the one examples is that if physical parameter q equal to is x square y minus x y square, if this physical parameter is related with the measured parameters or quantities x and y like this, so then how we will find out the error where x is measured parameter? It is given x best is 3.0 and delta x equal to plus minus 0.1, y equal to 2.0 and delta y is plus minus 0.1. So, following the general formula for error propagation, this q equal to delta q equal to del q x plus del q y. What is del q x? del q by del x delta x, del q y is equal to del q by del y delta y, ok. So, now, q is given this. So, just we will take partial differentiation partial differentiation of this with respect to x and with respect to y and multiplied with del x, delta x and delta 0, right.

So, if you differentiate it, so what we will get? 2 x y minus y square into delta x. So, here value are given of x y see if you put this value here, it will be 2 into 3 into 2. So, 12 minus 4 y square y is 2 4 into this delta x is 0.1. So, it will be 8 into 0.1 equal to 0.8. So, this is delta q, x. So, this error due to x this error is coming and due to y similarly we can just calculate for y, this error will be delta q y and that is 0.3 ok. So, total error on q that is del q delta q equal to del q x delta q x plus delta q y. So, this is 0.8 and 0.3 plus 0.3 equal to 1.1, ok.

So, and what is the q value? q value is 3, 9 into 2 minus x is 3 into y square 4 so, 18 minus 12, so basically, 6. So, 6.0, we have to write point 0 because here our measured value is up to one digits after decimal. So, we have to write 6.0 and then this error over this q plus minus 1.1 ok. So, this is the value of this q with the error. So, that is what now our answer we have to report this answer of q with the error like this.

So, if you find out the fractional error, then del q by q del q is 1.1 and q is 6.0. So, from here, one can get 0.18, ok. So, then percentage error is 18 percent. So, result one can write also q equal to, result also one can write q equal to 6.0 plus minus 18 percent ok. So, here one has to use unit sorry ok, here not able to see. So, here one has to write unit 6.0 centimeter or gram or kilogram whatever it is. So, this way also, one can report this error so, either this way or this way 6.0 plus minus 1.1, ok.

So this is the way, we will calculate error over the physical quantity. So, let me show you another calculation.

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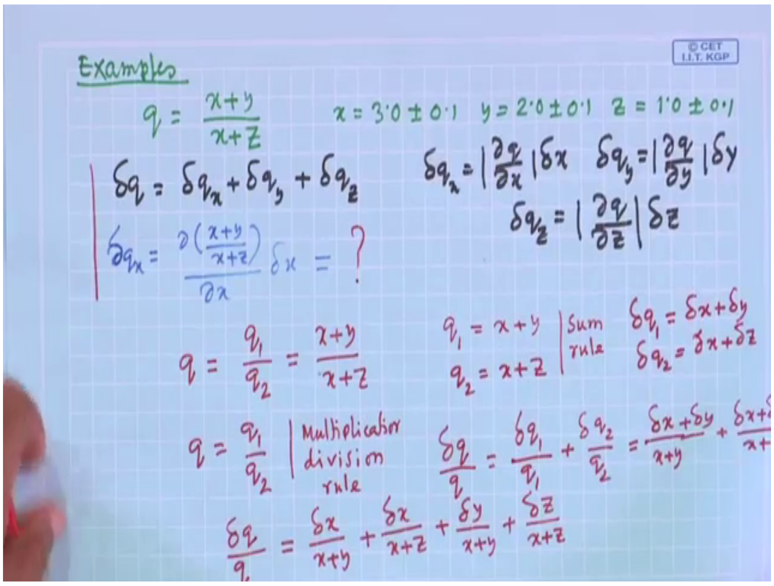

So, if this physical parameter q is have relation with measured parameter x and y like x plus y divided by x plus z. So, x and y and z values is given like this right. So, using the differential form, so error will be del q equal to del q x plus del q y plus del q z; del q x is again you know del q by del x delta x and etcetera delta q y equal to del q by del y delta y and similarly for z right. So, now, to find out delta q x, delta q x equal to then we have to differentiate with respect to x. So, del of this q x plus y by x plus z divided by del x delta x.

Now, here I feel just little bit difficult is to prove to do this partial differentiation. So, it is it can be done, but it can be slightly longer. So, then what I will do? I just left this approach. I approach with other known ways. So, this also I can write that q equal to that q 1 by q 2 equal to x plus y by x plus z. So, q 1 is x plus y and q 2 is x plus z, right. So, applying some rule, we can write delta q 1 equal to del x plus del y. Because for summation or subtraction, just error will be individual error will be summed up for the error on this physical parameter say.

So, delta q 2 will be del x plus del z right so, then, q equal to q 1 by q 2. So, we will apply multiplication or division rule right. So, again in both cases fractional error will be summed up, right. So, delta q by q you will be equal to delta q 1 by q 1 this fractional error over the q 1 plus fractional error over the q 2. So, delta q 1 I know here, del x plus del y and q 1 x plus y plus delta q 2 also I know, del x by plus delta x plus delta z divided by x plus z, right. So, from here just I can arrange that delta q by q equal to just I have added this one, del x by x plus y plus from here del x by delta x by x z plus delta y by x plus y plus delta z by x plus z ok. So, this is the fractional error over this over this over this q, this physical parameter. So, this way it is looks easier to calculate ok. So, so, just I showed you that alternate way also, one can use to find out the error.

So, if you find out want to find out the absolute error delta q. So, you have to multiply with the q. So, if I multiply with the q, yes if I multiply with the q, so, I think I will show you.

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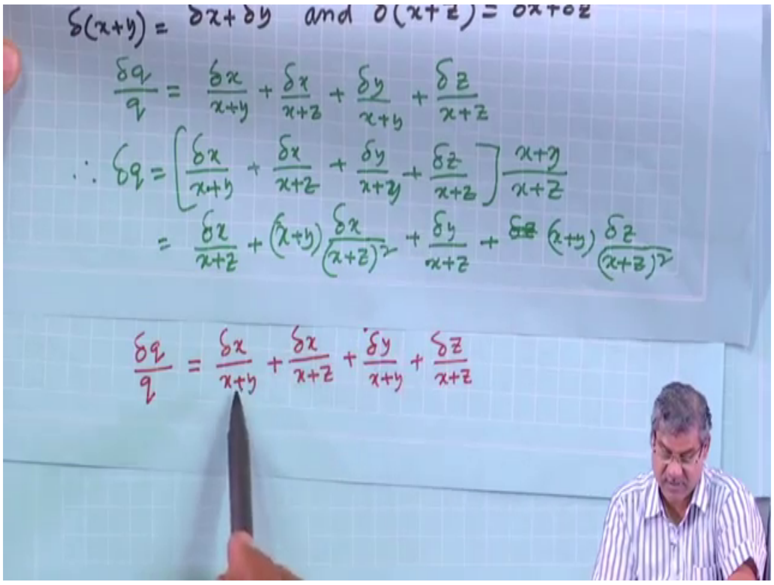

So, if I multiply with the q, so what I will get? So, this delta q by q equal to this, whatever we got it and just multiplied with q is x plus y divided by x plus z ok. So, if you multiplied, so you will get in this form. So, this will be the absolute error over the q, ok.

So, this alternate way one has to one can use. So, if you think this is the simpler one, so it is in your hand which way you will calculate. So, all rules are known to us now. So, now, let me take slightly complicated physical parameter say which is related like this, q equal to x, then multiplied with y minus z sin u, ok.

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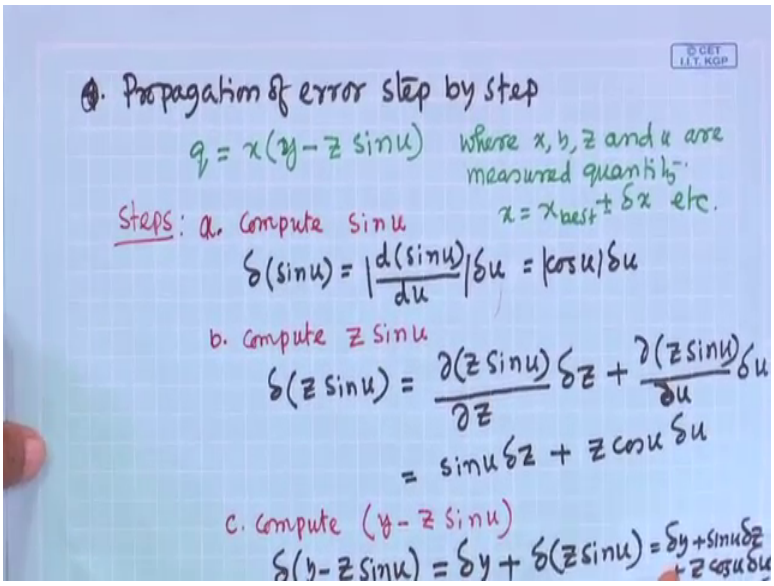

So, this one continuous one function is there sin function. So, in this case, measured parameter is basically x, y, z and u. So, so this measured parameters, so x equal to x best plus minus delta q y, similarly y etcetera ok.

So, now how to calculate the error over this q? So, here you should proceed step by step, then it will be simpler for you to find out the final error over delta q over q that is basically delta q. So, here just it is a you see here this I think this x, y, z, sin u, right. So, four measured parameters are there. So, sin u is a function of u, right. So, let us calculate this error over the sin u. So, so for sin u what will be the error, del sin u. So, del sin u, so this you can take how to proceed, say q equal to sin u ok. So, q equal to sin u how go calculate that I have shown you earlier. So, q equal to sin u. So, this is the function of one variable. So, we can u, so this, so this function of one parameter, one variable. So, we can differentiate it. So, this will be del q equal to q is sin u equal to d q by d u delta q ok. So, d q th q is sin u. So, of you calculate this one. So, this differentiation of sin u is basically cos u and then delta u ok, delta u is known from the, this is the uncertainty over measured parameter this u. So, this will be known to you and u also known. It is called a particular value of u, what is the delta u that is known. So, you can find out cos value of that u, ok.

So then, next step is, this part z sin u, z sin u. So, again this is the function of two parameters. So, say q equal to z sin u, this is the function of two parameter. So, del q equal to del q by del z delta z plus del q by del u delta u, right, delta u. So,, so just q you replace with this z sin u and then, just do the calculation differentiate it. So, as a function of z, so it is a sin u delta z plus this partial differentiation and this is z cos u delta u, ok. So, so this part is done, then let us go for this part. So, here yes, so let us go for this part compute y minus this. So, this following it will follow summation rule right, it will follow the summation rule. So, delta y minus z sin u equal to delta u plus delta of this part z sin u, ok.

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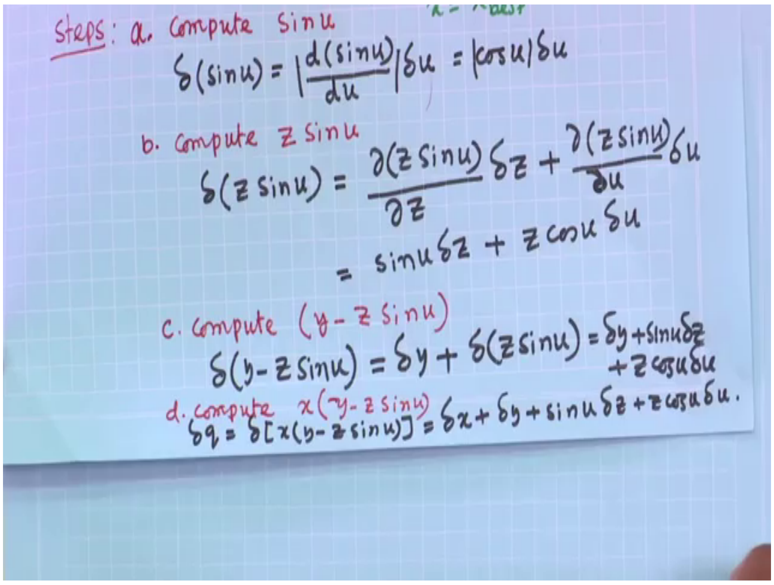

So, already we know from here, this what is delta sin z sin u. So, delta z sin u. So, that is already known. So, del u delta u plus this part ok, this part. So, that is what I have written. So, this part is done, this part is done ok. So now, final part x into this one, now, it is multiplication right multiplication. So, again it will be summation of this two part. So, delta q equal to delta of q is this. So, that is what I have written. So, this would be summation over this two part; one is x another is in bracket ok. So, delta x plus delta of this part, y minus z sin u, this part, so this part already we have we have found. So, this part we can directly write here del x plus this part for this one this is del y plus sin u delta z plus z cos u delta u. So, this is the final form of the error over the q. So, this is the function. So, if you start just without doing step by step, if you dire tly of you do it. So, it may it may looks complicated. So, one can do also step by step as then, looks it a simpler. So, to find out the final error delta q, so,delta q equal to this, ok. So, this is the another tricks I told you. So, how we can calculate the error step by step?

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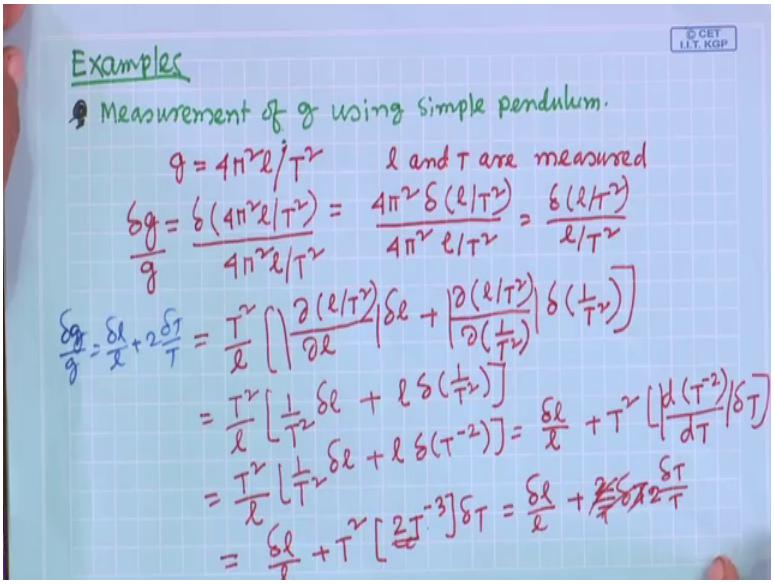

So, yeah so, one very, one very popular experiment in laboratory that is the to find out the measurement of g, gravitational acceleration using simple pendulum ok. So, if you have one to find out the g using simple pendulum. So, this formula for g gravitational acceleration that g equal to 4 phi square l by T square ok. So, how it comes you know ok? So, where l and T are measured parameters, l and T are measured parameters. So, T is time period and l is the length of the pendulum. So, in pendulum this mass is hanged with a thread from a support. So, l is length between this between this between this support and the center of the of the mass ok. So, that is the l. So, so that is what we will measure. So, from the instrumental precision or least count we will get delta l and from watch stop clock, we will when we will measure time period T. T is the just one complete oscillation. So, what is the time for a complete oscillation that is the T. So, from clock or stop watch, we will get the least count.

So, that is the delta t for this. So, it is basically if you find out the error of our g, so then it is you can take this it is in multiplication form if you consider multiplication form. So, this you can write this fractional, so, fractional error will be added, right. So, fractional error over the g, so that will be the summation of the fractional error of other parameters right measured parameters. So, del g by g you can write, del delta g by g equal to delta g is this part by g ok. So, if you proceed, so 4 phi square it will come out, it is the constant right because if constant is multiplied. So, it is error is just multiplied with this constant as we have shown the examples earlier. So, this 4 fh 4 phi square 4 phi square will go basically. So, here basically delta l by T square by l by T square.

So, this part it is if you proceed for this part, so you can take basically yeah. So, in this case delta l by T square, if you use product rule, then basically it product rule comes a producter or division rules comes it will give you it will give you in fractional form. So, I think this what we are doing. So, this we have taken out. So, now, for this part we can write. So, delta q, so this you can take delta q now q is l by T square. So, then del q by del l, del q by del l delta l plus del q by del. So, it is we I have taken 1 by T square, here I have taken two parameter l and another one by T square. So, del 1 by T square, delta 1 by T square.

So, here T square by l and then from this part differentiation partial differentia differentiation over l. So, it will be 1 by T square delta l plus here again, it will be l into delta 1 by T square, delta 1 by T square because this is we are differentiating with respect to 1 by T square. So, it is it will give 1, l into 1 so, this part into this part ok, delta 1 by T square. So, find we have to find this delta 1 by T square. So, l is there, so this we can write T minus 2. So, this just it is you can take; it is q is a function of one parameter as q is a function of x. So, here it is a function of T. So, just it will be del q equal to del q by dq by dt delta T ok. So, if you differentiate it, so it will be minus 2 minus 2 T to the power minus 3. So, , so it is in mod. So, we have to keep in mod because we have to take all the time we have all there, we have seen that we have putting in mod because we have to take positive value. So, that is why minus 2 we have taken, this plus 2 ok.

So, from where, here you are getting T square 2 by T q, so basically 2 by T we are getting 2 by T 2 by T, yes. So, del l delta l by l plus 2 delta T by T. So, this is your basically delta q by delta g by g equal to del l by l plus 2 delta T by T. So, this is the final form of the your this is the final form of your error over the g. So, this is the relative this is the fractional error relative error. So, it will find out want to find out this absolute one fractional, absolute one. So then, delta g equal to, so this would be multiplied with g. So, this g it is in this form. So, generally we do not write multiply with the g with that one. So, what we do, just we if you want to find out first we find out this delta g by g here, thus put this find out the numerical value and find out the numerical value of g, what is the parameter is given. So, if you want to find out this absolute error, so then just numerical value here and numerical value here is known. So, if you multiplied with this one with this numerical value, then you will get delta g, ok. So, instead of doing analytically here itself so that you can do after calculating each part, calculating each part delta g by g as well as g and from there, you can find out delta g which is absolute error, ok.

So, this is the, this is all about the error analysis error calculation. So, this is the rules basically I have thought you. So, now, we will we will go for different experiment laboratory experiment and for each experiment, we have to calculate this percentage error or most probable errors and. So, basically, we apply all this rules in our laboratory and yes we will do many more calculations over this error. So, I think, I will stop here.

Thank you for your attention.