Experimental Physics I
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Lecture – 18
Basic analysis (Contd.)

So, in last class, I was discussing the propagation of error means that this physical quantity generally we do not we cannot measure directly in our laboratory. So, we calculate the physical quantity from the relation with the measurable quantity measured quantity in the laboratory. So, if this relation is in form of summation and differences then how to find out the error on the physical quantity that I have discussed and that is the first rule for propagation of error. Now, I will discuss if this relation is in product or ratio form product or and quotients form then what would be the, what will the error of the physical quantity.

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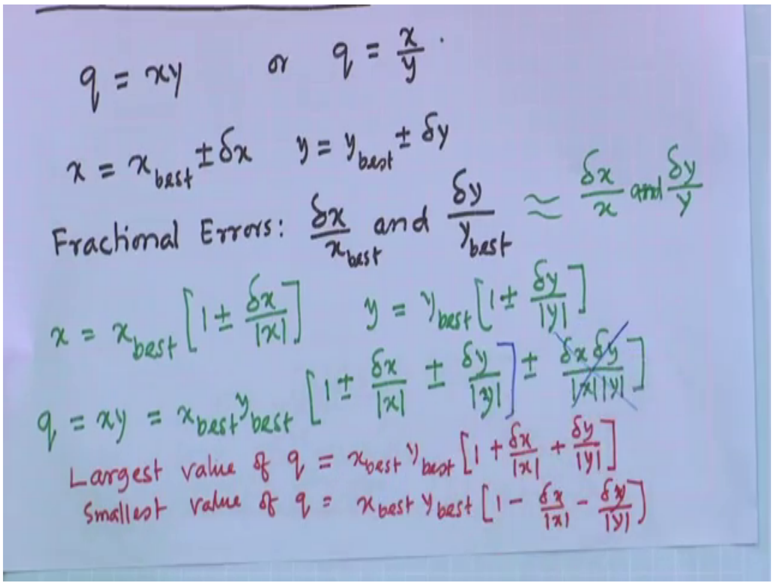

So, same way if we proceed so, this for products and questions say; that means, relation this q equal to xy or q equal to x by y, ok. So, then let us see this what will be the error delta q. So, x as usual x best plus minus del delta x, y equal to y best plus minus delta y, ok. So then in this case in case of summation we just discuss or consider the error that is called absolute error, but in this case we are considering fractional error, or yes what it is called relative error, ok. So, that is defined by delta x by x best and for y this delta y by y best. So, this we generally write just delta x by x and delta y by y.

So, this is there if error in form of fractional or relative. So, why we have taken consider the fractional error that you will understand just after this calculation. So now, so, this by the measured parameter and I know the delta x, I know the delta y, ok. So, I can define find out the fractional error delta x by x and delta y by y. So now, x equal to basically x best plus minus delta x; so, if I take common this x base I can write 1 plus minus delta x by x.

So this I have put in mode. So, whether x is positive or negative, but we will take positive value. So, that is why mod x. Similarly, y equal to y best if we take common then 1 plus minus del y by mod y. So, then q equal to xy x is x best, so, I put this two value here then what I will get? x best y best then 1 plus minus del x by mod x plus minus del y by mod y, then plus minus del x del y by mod x mod y. So, this just I cancelled it, I have not considered this one because is the second order we tell the second order error means first order, second order, third order we say higher order error is a it is value will be very very small compared to this other two first order error, ok.

So, forget first order second order, but from here you can see because this is the del x del y very small now it is a in decimal point generally. And, now multiplication of this two, ok, obviously, del x del y is again it is smaller you say it will be very very small compare to this one and this denominator also this multiplication of this two, ok. So, it will be high value. So, then basically this term last term is very very small compare to this other this two terms. So, that we do not consider this one we neglect it.

So, then ultimately so, q equal to your you are getting x best y best and 1 plus minus del x by x plus minus del y by y. So, if we compare with this q best, so, from here you can tell this limit of this q value largest value of q and smallest value of q; obviously, largest value of q if you take plus of both cases and smallest value if you take minus in both cases.

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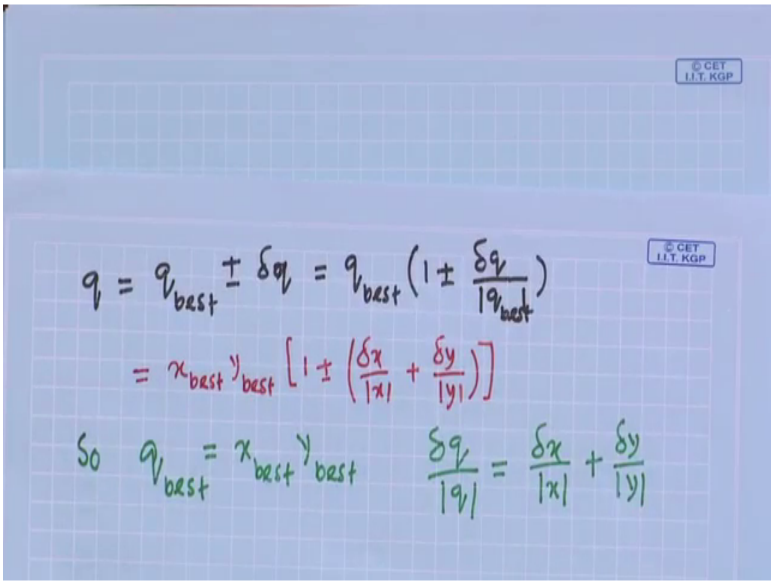

So, your q equal to q best plus minus delta q equal to q best plus minus delta q equal to q best. If you take common 1 plus minus delta q by q best mod q best, so, generally we take just like q. So, if you compare with this one whatever I have we calculated from here, ok. So, if you compare with this one, so, that is x best y best 1 plus minus del x by x plus minus or I have taken common. So, plus del y by y. So, if you compare then q best equal to x best y best; q best equal to x best y best and delta q delta q by q best that is just we generally write q, mod q then that will be equal to del x by x plus del y by y.

So, in this case what we are seeing? In this case we are seeing that this in case of product or quotients, or ratio the error again it is the summation of the individual error, but it is not absolute error, it is in form of relative error. So, in case of product and quotients the error on the physical quantity will be the summation error on the physical quantities are. So, that is relative error or fractional error on the on the physical quantity will be the summation of the fractional errors of the individual parameters or quantities or measured quantities.

So, that is why in this case we have taken this consider the fractional error and other case we have consider the absolute error. So, this is basically second rule in case of product or quotients the relative error of the physical quantity will be equal to the summation of the relative error or fractional errors of the measured quantities, ok. So, whereas this first rule what we have seen the for summation and difference absolute error or summation of the absolute error of the of the measured parameters or measured quantities that will be the error of the physical quantities. So, this so that I showed you just for calculation I have showed you for product.

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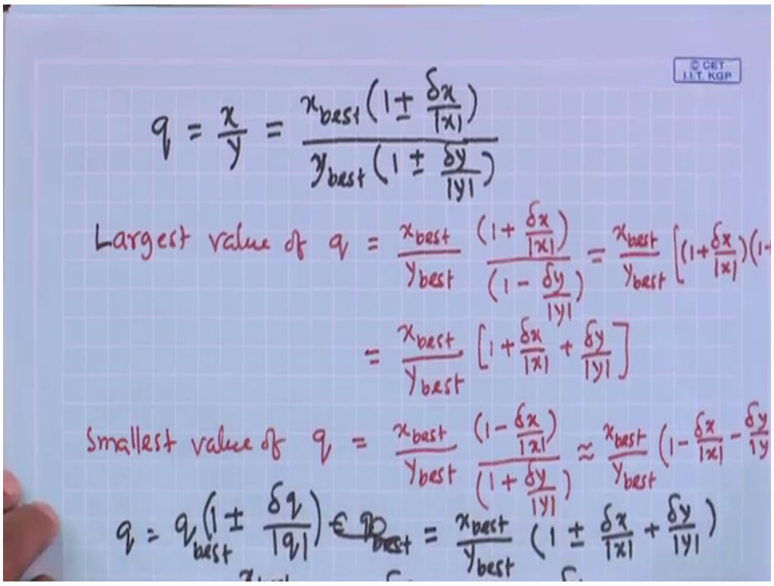

So, here I have calculation for the for the ratio also quotients and same way we have showed we have shown that in this case also this result is same just quickly I will show you this calculation. So, q equal to x by y; see, that means, x best 1 plus minus del q by x y best divided by y best 1 plus minus del y by y.

So, then from here largest value of q what will be? So, largest value means this numerator has to be larger denominator has to be small. So, I will consider here plus and I will consider here minus, ok. So, then this value will be large value and from here I have written x best by y best then you see this 1 plus or minus something, if it is very very small than one then binomial theory you can use. So, this we can write 1. So, this we can take up see this is basically 1 minus this minus 1 power minus 1. So, that that is equal to the 1 plus del y by y. So, from binomial theory you can write. So, that is what I have written 1 plus del x by x and this 1 1 by this 1 is basically 1 plus del y by y.

So, again this here you can see this for largest value these both are plus and similarly and this if you multiply then neglecting this second order term you will get this one. Similarly, for smallest value of q so, this numerator has to be smaller and denominator has to be higher. So, this here we are consider minus here we have consider plus. So, if you take up so, this will be one minus this into 1 minus del y by y. So, from there you will get 1 minus del by del x by x minus del y by y. So now, this del x by x plus del y by this is the this 1 minus of this one, ok.

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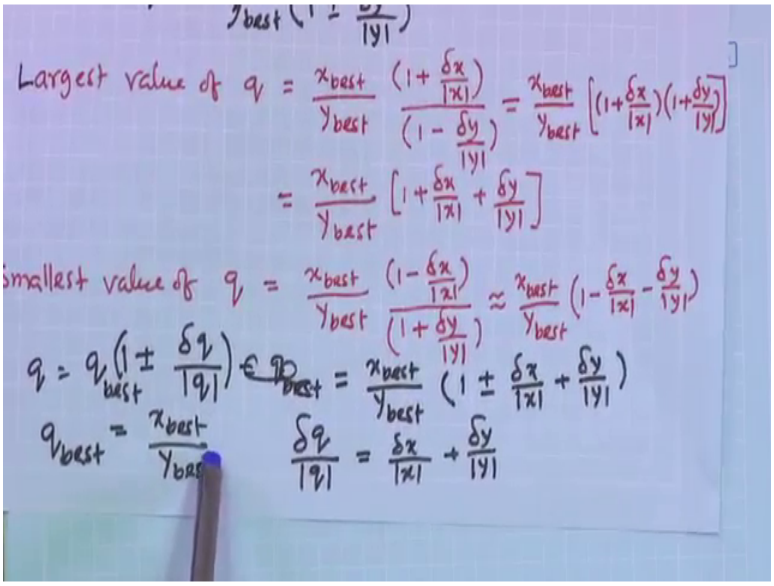

So, in this case also you are getting del q by q if you just proceed here calculation steps are given. So, del q by q equal to del x by x plus del y by y. So, in case of ratio also we are getting this summation of the summation of the individual error relative error of the measured parameters, that will be the error relative error on the physical quantity.

So, in both cases I as I told this in for product as well as for quotient rule is that the relative error of the physical parameter is equal to the summation of the relative errors of the measured parameters, ok.

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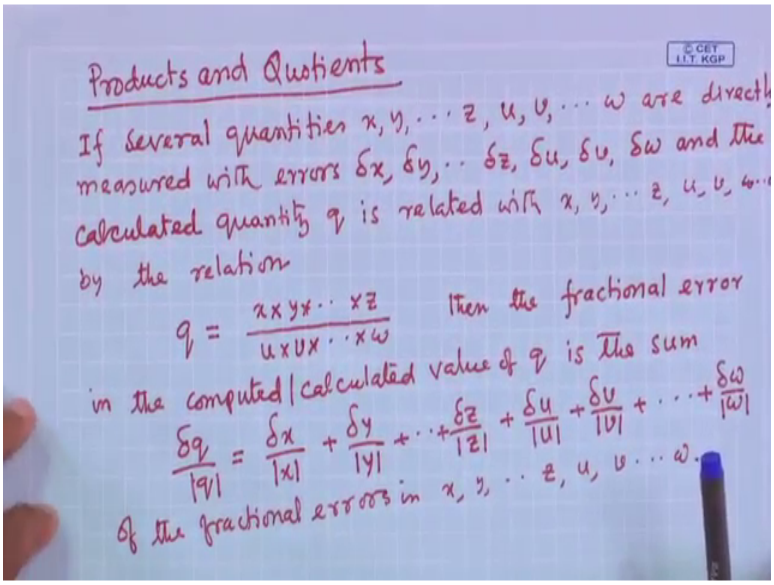

So, rule in general then rule is for product and quotients these are second rule. If several quantities x, y, z, u, v, w are directly measured with errors del x, del y, del z etcetera del u, del v, del w and the calculate quantity q is related with x, y, z, u, v, w by relation q equal to this way, ok. So, here both multiplication as well as divisions are there.

Then the fractional error in the computed or calculated value of q is the sum delta q by q equal to this sum of all individual errors relative errors that will be the fractional errors. These are the basically, yeah summation of the fractional errors in x, y, z, u, v, w, ok. So, this is the second rule, or for propagation of error. So, we already we discussed about the summation and difference or subtraction or product and quotients or divisions or ratios.

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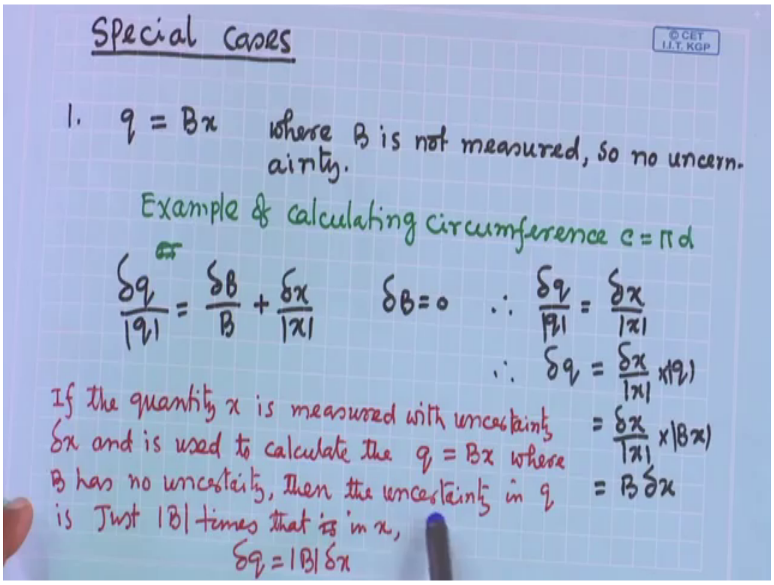

So, let us consider some special cases. We will we need to consider because in real example in laboratory different kind of relation will see and they are the special whatever we will discuss this type of relation also may be there not only summation and different products and quotients. So, other special relations may be also there. So, here let me discuss few of them.

So, if relation is like this q equal to Bx; not xy, Bx where b is constant basically ok; means, B is not measured, it is the given value or some constant ok, then we can tell the no uncertainty in B uncertainty with B is 0 means del B equal to 0 delta B is equal to 0. So, in this case what will be the error on q, ok. So, as for example, this for this in case of if you if you if you are asked to calculate the circumference of a of a of a circle hm. So, then relation is c equal to circumference of the circle c equal to pi d; d is a diameter, or 2 pi r r is a r will be the radius, ok.

So, basically for that what you have to do if you want to find out the circumference of the circle you have to measure the. So, what you will measure? You are not measuring circumference directly. Basically you are measuring d, diameter of the circle. Now, from this relation you will find out the circumference ok. So, what is the error on this circumference? So, here basically you can take it as a just product form B and x product form of B and x. So, for product what happens, so, relative error for on both cases will be added. So, I can write del q by q equal to del B by B plus del x by x del B equal to 0. So, then del q by q equal to del x by x.

So, del q, one can write del x by x into del q into q mod q. So, if you if you put this value of q that is Bx, ok. So, you will get x, x will go and then you are getting basically equal to B delta x. So, from here what you have seen? You have seen that that if the quantity x is measured with the uncertainty delta x and is used to calculate the q equal to Bx, where B has no uncertainty. So, then uncertainty absolute uncertainty in q is just the B times that in x, ok; that means, delta q equal to Bx.

So, from here the from this calculation based on the our known rules rule 3 rule 2, for product case just considering that one. So, here we found that if one measured parameter is multiplied with a constant which is not measured. So, uncertainty is 0 on that constant, then the absolute error will be just that that constant into the error in the measured quantity that is delta x.

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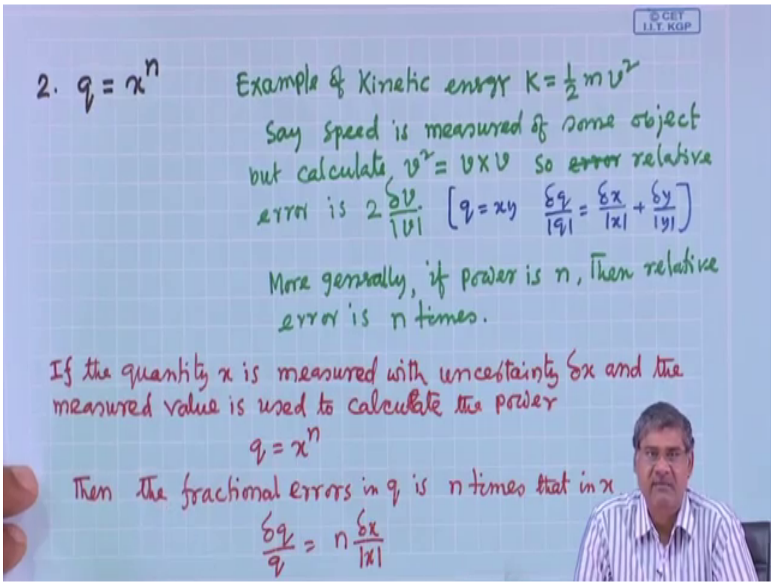

So, this is one case if we face another case like this form q equal to x to the power n, q equal to x to the power n. So, just consider this one example that is what is the kinetic, what is the kinetic energy of a body, if it is velocity is v? So, kinetic energy I can write half mv square, ok. So, for this case for this case now we want to find out what is the error on K. if so, then here basically we are measuring our measure measured parameter of quantity is v, and then we have to we have to we have to calculate v square. So, v square is nothing, but v into v, forget half m, right term because they are constant and you know this if constant is multiplied means, so, already we know. But, now square of it so, but we are measuring v, but here is relation v square is there, ok.

So, here we are how to what will happen for this v square so, that is what we are discussing. So, v square one can write v into v. See it is simple this multiplication of the two (Refer Time: 20:29) although if they are same, but it is like xy form x into y form, but both are same they are not different so, but rule will be applied on this. So, obviously, relative error will be del for this del v by v. So, del v by v plus del v by v, summation of the summation of the relative error ok. So, that will be 2 del v by v.

So, then we can generalize is that if power is n then relative error will be n times of this relative error will be n times, right; will be n times when a n. So, n times of the relative error of the measured parameter. So, obviously, for this case del q by q will be equal to n del x by x. So, that is what here. So, this what we what we got, if this type of power is there. So, this error will be this power times this error relative error of the measured quantity measured parameter.

So, I think this we are. So, we do not need to consider this, multiplication etcetera. So, this from knowing these rules of multiplication from there itself, this two cases I showed you this one is if constant is multiplied with the parameter if this parameter have power. So, then what will be the error. So, directly we can use this just multiplied with constant in case of absolute error multiplied with this power in case of relative error, ok; so, for this two cases.

So, this way there are there are more special cases, but whatever I discussed or whatever rules I discussed using that one you can calculate for most of the I think for all relations of the measured parameters with the with the physical quantity. So, there is some I think.

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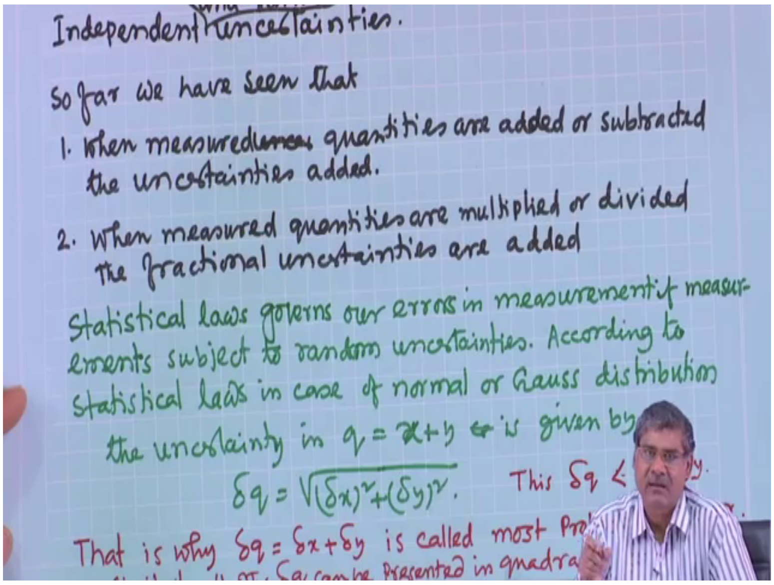

So, far what we got? So, far what we got? What we have seen? When measured quantity are added or subtracted the uncertainties are added absolute uncertainties are added, right. When measured quantities are multiplied or divided the fractional uncertainties are added, ok. These two rules we have we have seen we have verified we have proved. So, now, if the measured parameters are independent and random, if measured parameters are independent and random, ok; in that case, so, statistical laws governed in that case basically we can modify this we can modify this that error whatever so far we have seen. So, that is what we said discussion is we are doing we will discuss about it just I will mention basically I will not go for that much depth.

So, basically statistical laws because random error mostly it is a statistical in nature and statistical laws governs our errors in measurement if measurements subject to random uncertainties. So, in case of random uncertainties, so, this statists statistical laws governs the errors in measurements. So, there I think this statistical part I will not discuss, but you know this if you measure, if you repeat the experiments or yes so, basically. And, many datas if you collect many datas and if you plot it so, all data is suppose to be if it will be very accurate if all data if get of same value, but this never happens. This datas are distributed. So, and these distribution generally in different form it is in it may be Gaussian distribution or normal distribution or I think Lorentzian distribution etcetera, ok.

So, in that case generally this standard deviation we will consider the error as a standard deviation and that standard deviation you know this in Gaussian form, in Gaussian form this there is some calculation the height we consider or this yes, the width we consider at a particular height where, the 70 percent data will be within the 68 percent will be within the within the error range, ok. Plus minus the standard deviation, if it is sigma and then your best value generally a peak value you take that is a x best plus minus that is sigma, and this standard value sigma it is generally comes you know this in square root form, in square root form, ok.

So, if your measurement is statistical in case of random uncertainties. So, then the uncertainty in q equal to x plus y can be written or is given by del q equal to square root of del x square plus del y square. So, earlier we have considered del q equal to del x plus del y. But, instead of it in when this your experiment is in statistical nature then the error will be square root of this square root of the square of this errors on x and square of the plus square of the errors on this, ok.

So, whatever we have calculated just if you put square root of it taking the square of this individual errors then that is that will be the error. Now, obviously, this is less than del x plus del y. So, here basically it is telling that if you consider this del x plus del y then your error is your over estimating your error. But, actual error should be like this and now only restriction this it is only valid for independent and random uncertainties, independent and random uncertainties. So, error is in random type as well as the parameters are related with the q or physical parameter related to the physical parameter. So, they are independent, ok. So, in that case only it so.

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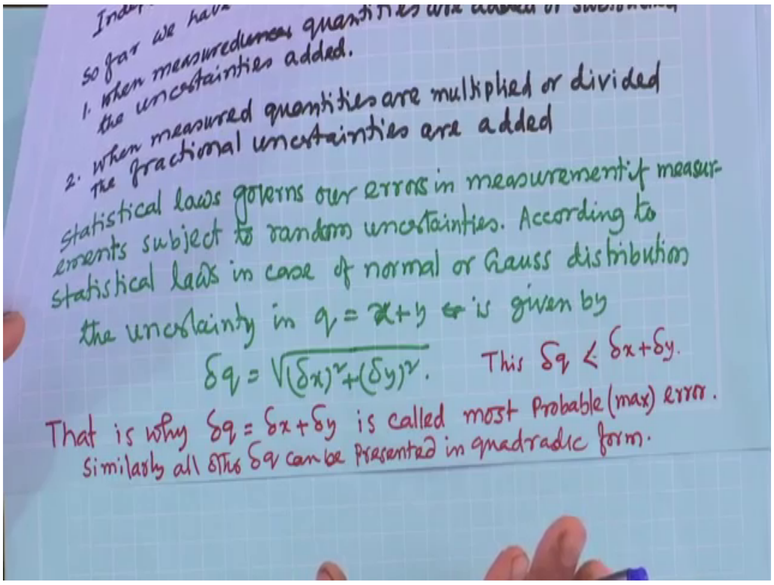

So, but generally you know laboratory whether it is statistical in nature or it is a normal experiment in all cases we consider we do not consider this one square root of this we consider del x plus del y. So, that is why this and this is the maximum possible pro possible error. So, that is why the error this del q equal to del x plus del y in this form in our laboratory we will take that is why this error is also called most probable error or yeah it is called most probable errors, ok. We tell this, percentage error most probable error. So, in our laboratory basically we will find out the most probable error because this as here just I discuss to tell you that the error we are whatever we are the way we are finding out, and taking the error on the physical quantity.

So, that is the del q equal to del x plus del y so, that is basically over estimated that is why it is the maximum error, and error can be smaller than this if it is in statistical nature, but in general that will not considered. So, that is why in laboratory we will use whatever we have seen in the form of the rules whatever we have established so, that we will follow in our laboratory also. So, that will be the overestimated, but this is the maximum value of the error we can tell this that will be the maximum value of error, but error can be actual error can be smaller than that, ok.

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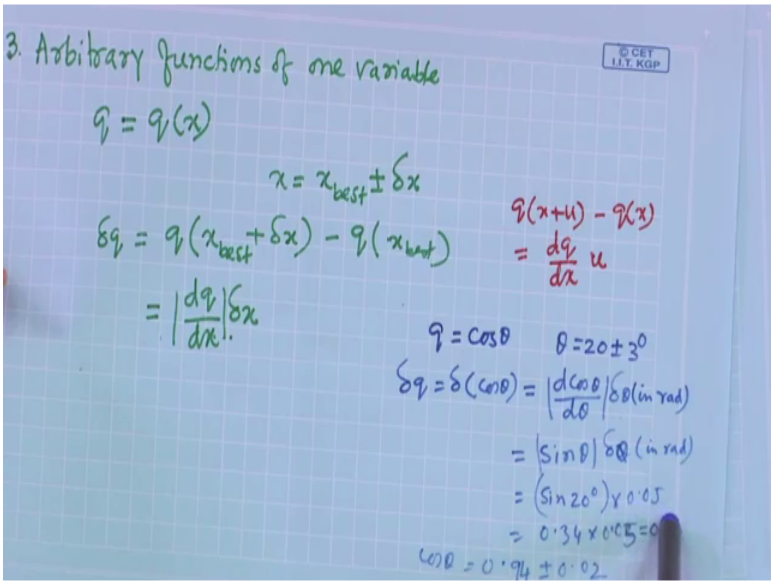

So now, just we considered this yes; so, some relation summation or difference or product or quotients. So, here if this if you deal this parameter q physical parameter q is not a is the relation is not with one or with more individual parameters, measured parameters, but it is a function of one variable, ok. If so, then how to find out the error? So, q equal to this is a function of x, ok. So, q equal to q function of x and x is measured parameter. So, x equal to x best plus minus del x, right.

So, del q; now we are interested to find out this error on this q. So, del q, we can write this q is a function error. So, q x is x best plus, I think it should be plus minus, ok. So, I think we are taking the maximum value here, yeah. So, q is we are we are taking this maximum value here q that is a q best plus del q, minus q of q best q is a function of x. So, for x value x best plus minus del x. So, that x here so, this del q we can take this, for this value of the q for x best plus del x minus q x best. So, this is the function you know and then this is this is nothing, but the one can write dq by dx change of q with respect to x into this delta x, ok.

So, this is the way basically we find out the differential form. So, differential form of this so, you know you are familiar with this. So, this del q is basically one can write dq by dx into delta q fine. So, it is q x plus u minus q x it is nothing, but equal to dq by dx u, ok. So, that is that is the things here we have written. So, as for example, this if q equal to cos theta physical parameter q that is the is a cos function, it is a cos theta it is a function and it is a cos function, it is theta, we are measuring theta and theta equal to say 20 plus minus 3 degree.

So, what will be the error on q? So, I can write del q equal to I know del q equal to this. So, del q means del cos theta equal to the d q by d theta means d cos theta by d theta del theta and they said here only we have to see that is del theta when we are writing that is a basically in radian we have to take in radian. So, d cos theta by d theta is mod we are taking. So, it is basically yes minus sin theta, but we are taking mod. So, sin theta of del q del theta in radian. So, sin 20 degree and it is a 3 degree in radian it is a 0.05 and sin 20 degree 0.34 into 0.05 equal to basically 0.02.

So, this del q is this. So now, cos theta this q is cos theta. So, cos theta for this value we can write the result equal to 0.94 if you add this two 0.94 plus minus 0.02. So, this way one can calculate the error in case of function form of the physical parameters. So, this way one can basically calculate of different calculate of a event for function form of the variables. So, there are many many things, but what about main things I am discussing and it will be. So, this rules will use for when we will do experiment in laboratory and if we if I find some more examples where this rules are not enough so, I think I will discuss some other special case cases also because this is huge subject.

So, it is difficult to cover, but then it will be complicated. But, our purpose is to tell you my purpose is to tell you whatever you need in your laboratory at least for teaching laboratory at least this much concept I should give you. So, I think yeah I will stop here and then there are some more steps for propagation of errors I will discuss in next class.

Thank you.