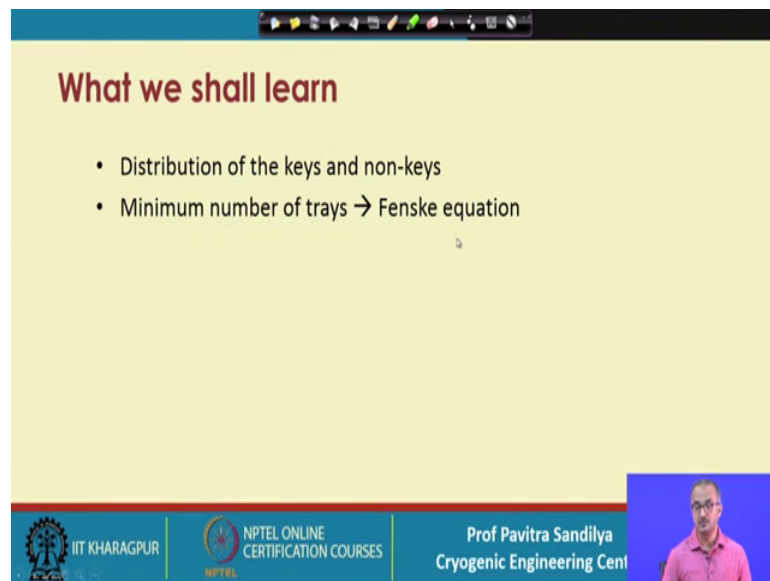


**Upstream LNG Technology**  
**Prof. Pavitra Sandilya**  
**Department of Cryogenic Engineering Centre**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 62**  
**Tutorial on multicomponent distillation – I**

Welcome, we have learnt about some basic analysis procedure for the multicomponent distillation and we have learnt about how to use the various type of shortcut methods and the correlations to find out the minimum reflux, the minimum number of ideal stages and the actual number of stages depending on the actual reflex. And, now in this particular lecture we shall be looking into some of the problems for the multi component distillation this is part I and we shall be following it up with another set of problems.

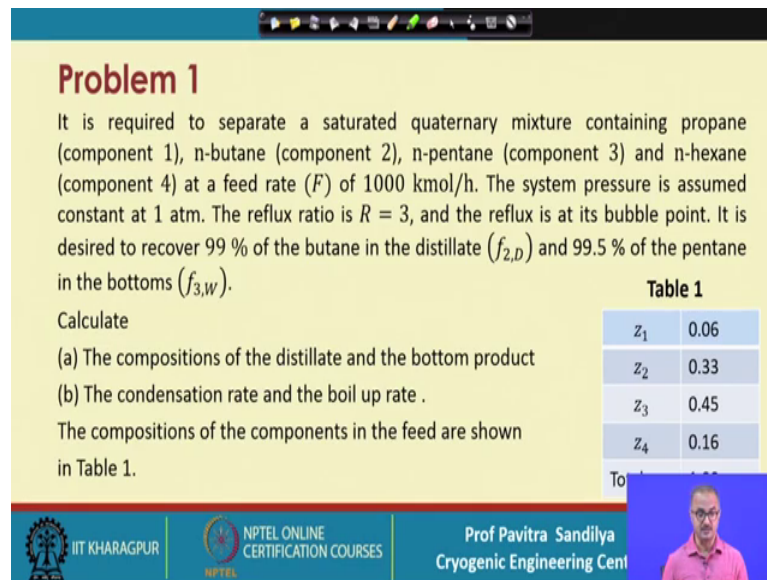
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The slide is titled "What we shall learn" and lists two bullet points: "Distribution of the keys and non-keys" and "Minimum number of trays → Fenske equation". The slide is part of a presentation by Prof. Pavitra Sandilya, as indicated by the footer. The footer also includes the logos of IIT Kharagpur and NPTEL Online Certification Courses.

So, in this particular lecture we shall be learning about the distribution of the keys and non keys and how they are going to be used for analysis of the multicomponent. distillation problems. And we shall be also looking into the determination of the minimum number of trays using the Fenske's equation.

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**Problem 1**

It is required to separate a saturated quaternary mixture containing propane (component 1), n-butane (component 2), n-pentane (component 3) and n-hexane (component 4) at a feed rate ( $F$ ) of 1000 kmol/h. The system pressure is assumed constant at 1 atm. The reflux ratio is  $R = 3$ , and the reflux is at its bubble point. It is desired to recover 99 % of the butane in the distillate ( $f_{2,D}$ ) and 99.5 % of the pentane in the bottoms ( $f_{3,W}$ ).

Calculate

(a) The compositions of the distillate and the bottom product

(b) The condensation rate and the boil up rate .

The compositions of the components in the feed are shown in Table 1.

$z_1$	0.06
$z_2$	0.33
$z_3$	0.45
$z_4$	0.16

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So, first we take up this particular problem. It is required to separate a saturated quaternary mixture. Quaternary means the a mixture containing four components. So, this particular mixture contains four components and these components are propane that is labelled as component 1 then n butane it is component 2, n pentane component 3 and n hexane that is component 4. And this mixture this feed mixture has a flow rate of 1000 kilo mole per hour. The system pressure is taken to be constant at 1 atmosphere and the reflux ratio is taken to be 3.

And it is said that the reflex is at its bubble point. Bubble point means it is a saturated liquid. And it is desired to recover 99 percent of the butane in the distillate that means, whatever butane is going in the feed of that quantity 99 percent should be recovered in the distillate. And this we call the recovery, recovery as we know that this is defined as the amount in the distillate divided by the total amount which is present in the feed for the distillate recovery and similarly we have 99.5 percent recovery of the pentane is desired in the bottom. And, here this the recovery means the amount of the pentane which is in the bottoms divided by the total amount of pentane which is entering the system.

And as you can see in this particular problem that this propane, butane, pentane and hexane; they are coming in the increasing order of their molecular weight as well as boiling points. So, we can figure out that the n hexane is the heaviest or the densest of

the component and it is has the maximum boiling point where as propane has the minimum boiling point and the least or the least of the molecular weights. So, here we have to calculate the composition of the distillate and the bottom products, the condensation rate and the boil up rate. And we have for do to do this we have been given the feed composition in this particular table.

Here we find the  $z$  represents the mole fraction of the various components 1 2 3 4 are given and you can see they are given as 0.06 of propane, 0.33 of n butane then 0.45 of n pentane and 0.16 of the n hexane. And this is totalling to 1, this totalling is very important for us because whenever you are given any kind of composition and before you go on to solve any problem it is always good practice to check that whether the summation of the mole fraction is coming out to be unity or not. So, as we see that there is a summation is coming to unity that means, this data it can be used for further analysis.

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**Solution :**

(a) Since the desired separations are given in terms of n-butane ( $-0.5^{\circ}\text{C}$ ) and n-pentane ( $36^{\circ}\text{C}$ ), and since n-butane has lower boiling point than n-pentane, the LK is n-butane and HK is n-pentane.

(b) Since n-propane ( $-42^{\circ}\text{C}$ ) has the lowest boiling point, and n-hexane ( $69^{\circ}\text{C}$ ) has the highest boiling point, n-propane is the LNK, while n-hexane is the HNK.

$F = 1000 \text{ kmol/h}$

$L_0 = 3D$

$D$

$D x_{1,D} = F z_1$   
 $D x_{2,D} = (0.99) F z_1$   
 $x_{4,D} = 0 \text{ (assumed)}$

$W$

$W x_{1,W} = (0.99) F z_3$   
 $W x_{4,W} = F z_4$   
 $x_{4,D} = 0 \text{ (assumed)}$

$z_1 = 0.06, z_2 = 0.33$   
 $z_3 = 0.45, z_4 = 0.16$

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Now, here we first make a schematic of the particular system. So, here we see this is the distillation column with is a having a condenser and the reboiler. And, what we find that the vapour from the column is going to the condenser getting condensed and a part is taken out as a distillate and there the rest of the things is sent back to the column as the reflux.

And similarly on the other side on the on the bottom side we find the liquid comes out of the column, it is taken to a reboiler here and it is vaporized and the vapour portion is

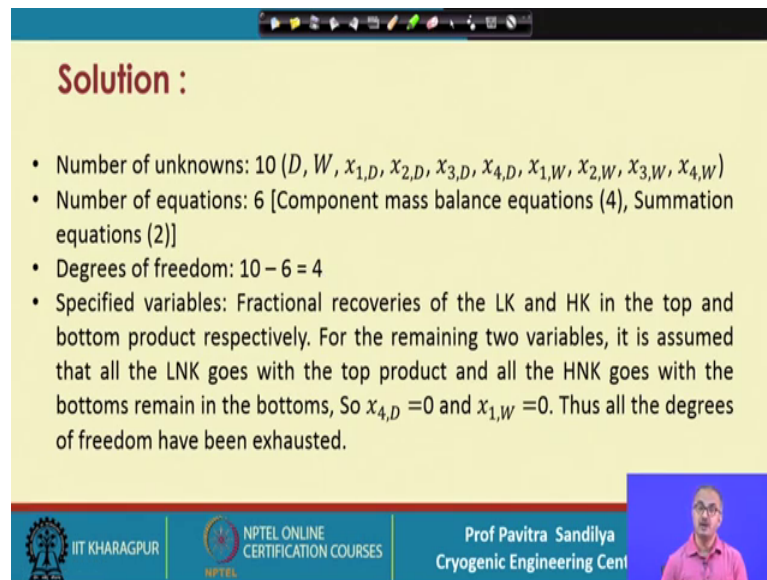
taken back to the column and if it is partial reboiler that means, the rest of the things which is not getting vaporized will be coming out as the bottom product in the liquid phase. So, that is how we are having the operation of the column and on this side we have the feed and this is we have written the feed flow rate and here we have written the feed composition. On this side these all this things we shall be just seeing that how we are getting this equation, this is coming from the mass balance for the component 1 and these are the other mass balances given here.

And we are assuming that there is no  $x_4^D$  equals to 0 means we are assuming that there is no n hexane coming in the distillate and here also we are assuming that there is no n hexane is not going. So, where are assuming that also the pentane is not coming in the bottom product. So, since the desired considerations are given in terms of n butane which is having a boiling point of minus 5 degree centigrade and n pentane with a boiling point of 36 degree centigrade. And since n butane has lower boiling point than n pentane the, we choose what to do we choose n butane as our low key component and n pentane as our high key component that means, we have reduced the quaternary mixture in terms of this pseudo binary mixture.

And whatever whichever component has lower boiling point than the butane will be taken as the low key component and or and whichever has higher than the high key component will be going with the high key component; unless they distribute themselves between the two. So, here we find the n propane has a boiling point which is much lower than the boiling point of n butane. So, there is no chance that n pentane can go into the bottom products.

And similarly we find n hexane has a boiling point of 69 degree centigrade which is much more than the boiling point of pentane. So, there is no chance of going this n hexane into the distillate. So, we can assume there this amount of n propane in the in the bottom as 0 and n hexane in the distillate as 0. So, this n propane is the LNK that is the light non key while n hexane is the HNK that is heavy non key.

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**Solution :**

- Number of unknowns: 10 ( $D, W, x_{1,D}, x_{2,D}, x_{3,D}, x_{4,D}, x_{1,W}, x_{2,W}, x_{3,W}, x_{4,W}$ )
- Number of equations: 6 [Component mass balance equations (4), Summation equations (2)]
- Degrees of freedom:  $10 - 6 = 4$
- Specified variables: Fractional recoveries of the LK and HK in the top and bottom product respectively. For the remaining two variables, it is assumed that all the LNK goes with the top product and all the HNK goes with the bottoms remain in the bottoms, So  $x_{4,D} = 0$  and  $x_{1,W} = 0$ . Thus all the degrees of freedom have been exhausted.

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Now, we see that before we go to the problem let us see that how many equations and how many unknowns you have to solve for. Here we find the number of unknowns are 10. And what are these 10 unknowns? This is the distillate product rate, the bottoms rate the various mole fractions in the distillate and the various mole fractions in the bottoms. And, now we find that we have only 6 equations and what are these 6 equations, we have 4 equations for the component mass balance. And then we have 2 summation equations that is summation means so, summation of this  $x_i D$  is equal to 1 and summation of  $x_i W$  equal to 1 that is the summation of the mole fractions is always unity.

So, that is the meaning of this summation equations in effect we have 6 number of equations. So, degrees of freedom is coming out to be 10 minus 6. So, we need four more variables to be specified before we can have the degrees of freedom to be 0 so, that we can have a unique set of solutions. So, here we see that how to specify those variables the fractional recoveries of the LK HK in the top and bottom product has been specified. So, these become two more specifications. So, out of these four we reduce it by two and now for the rest two specifications as we said that we are assuming that there is no n hexane in the distillate and no pentane propane in the bottoms.

So, that is how we are making this two more assumptions which based on our understanding. So, that is how we are exhausting all 4 degrees of freedom. And now we have we are able we can solve the set of equations uniquely.

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**Solution :**

✓ 99 % of component 2(LK) is recovered at the top, that is,

$$f_{2,D} = \frac{Dx_{2,D}}{Fz_2} = 0.99 \Rightarrow Dx_{2,D}$$

$$= (0.99)(1000)(0.33) = 326.7 \text{ kmol/h}$$

Mass balance for component 2 over the distillation column,

$$Wx_{2,W} = Fz_2 - Dx_{2,D} = (1000)(0.33) - 326.7$$

$$= 3.3 \text{ kmol/h}$$

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Now let us come into those numerical solutions we have been given that 99 percent of the component 2, that is the low key is recovered at the top. And so, we put the definition of this recovery and this the amount which is present in the distillate and this is the amount which is going in with the feed for component 2 and we find this is 0.99. So,  $Dx_{2,D}$  can be found out from this particular formula.

And we get this has to be this particular value this kilo mole per hour and then we do a mass balance here we have say see a control volume. So, we can do a overall mass balance over this whole control volume for component 2. And, if you do that mass balance we find that this is how we can find out how much this  $Wx_{2,W}$  shows this is the amount of this component 2 coming out as in the bottom product. So, we find this value is coming out to be this from the overall material balance for component 2.

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**Solution :**

- ✓ 99.5 % of component 3 (HK) at the bottom. Hence,
 
$$f_{3,W} = \frac{Wx_{3,W}}{Fz_3} = 0.995$$

$$Wx_{3,W} = (0.995)(1000)(0.45) = 447.75 \text{ kmol/h}$$
 Mass balance for component 3 over the distillation column,
 
$$Dx_{3,D} = Fz_3 - Wx_{3,W} = 2.25 \text{ kmol/h}$$
- ✓ Now  $x_{1,W} = 0$
- ✓ Mass balance for component 1 over the distillation column,
 
$$Dx_{1,D} = Fz_1 - Wx_{1,W} = (1000)(0.06) - 0 = 60 \text{ kmol/h}$$

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Now, we come to the next one that is we take component 3 for which we have been given 99.5 percent recovery, again we go back to this equation for the recovery. And then we find that this is the amount of the component 3 in the bottoms and this is coming out after plugging in the values this value. And we then do an overall material mass balance for this component 3 for the whole column and we find this is the flow rate of or the amount of the component 3 coming out with the distillate. And this we have taken already as 0 this x 1 W ok.

Here that is the there is no propane in the bottoms and when we do this mass balance for this propane mass balance. We find that we are getting this is the amount of the propane which is going out with the distillate.

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**Solution :**

- ✓ Also  $x_{4,D} = 0$ , so  $Dx_{4,D} = 0$
- ✓ Distillate flow rate is obtained from

$$D = \sum_{i=1}^4 Dx_{i,D} = 60 + 326.7 + 2.25 + 0 = 388.95 \text{ kmol/h}$$

- ✓ Overall mass balance for the column gives:

$$W = F - D = 1000 - 388.95 = 611.05 \text{ kmol/h}$$

- ✓ Now,  $x_{1,W} + x_{2,W} + x_{3,W} + x_{4,W} = 1$

$F = 1000 \text{ kmol/h}$   
 $z_1 = 0.06, z_2 = 0.33$   
 $z_3 = 0.45, z_4 = 0.16$

$L_0 = 3D$   
 $Dx_{1,D} = Fz_1$   
 $Dx_{2,D} = (0.99)Fz_1$   
 $x_{4,D} = 0 \text{ (assumed)}$

$x_{1,W} = \text{(assumed)}$   
 $Wx_{3,W} = (0.99)Fz_3$   
 $Wx_{4,W} = Fz_4$

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And this is we find that there is we assume that there is no hexane going with the distillate. So, this value is 0. So, there is nothing come out of the distillate. And now, to find out the distillate flow rate what we do? We simply add up the amount of each of the components in the distillate. And we find this is the amount, this is the flow rate of the distillate. So, it is quite simple we are just using the mass balance equation and then the summation equation to find out the total distillate flow rate.

And then we if we do a overall mass balance over the whole column we find that the feed is equal to D plus W from which we get the W equal to F minus D and the if we plug in the various values we find this is the flow rate of the bottom product from the column. Now we know that the summation of the mole fractions in any stream is equal to 1. So, we apply this to the bottom stream.




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**Solution :**

✓ Solving the mass balance equations and summation equation together, we get the distillate and bottoms compositions as

Component	$z_i$	Distillate ( $x_{i,D}$ )	Bottom product ( $x_{i,W}$ )
1	0.06	0.1543	0.0000
2	0.33	0.8400	0.0055
3	0.45	0.0057	0.7327
4	0.16	0.0000	0.2618

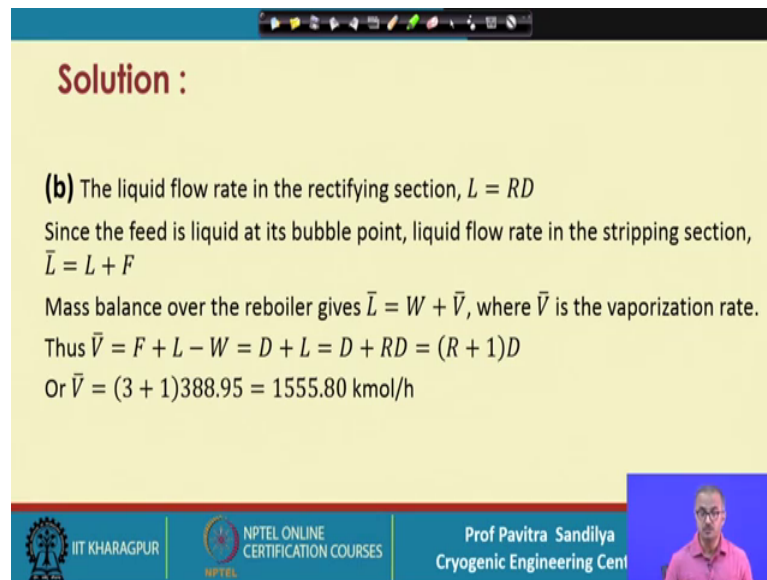
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And then we solve all those equations and we find that is that simple algebraic equation you are having. Now, if you solve them one by one we shall be having the various solutions for all the products and distribution of the products in the distillate and the bottoms.

So, this is showing that if we make this table here we find that this is the composition of the feed and this is the composition of the distillate and this is the composition of the bottom product. And, in this we find that there is no n hexane and in this we find there is no propane. So, that is how we are able to use the mass balance and the summation equation to find out the composition of the distillate and the bottoms.

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**Solution :**

**(b)** The liquid flow rate in the rectifying section,  $L = RD$

Since the feed is liquid at its bubble point, liquid flow rate in the stripping section,  $\bar{L} = L + F$

Mass balance over the reboiler gives  $\bar{L} = W + \bar{V}$ , where  $\bar{V}$  is the vaporization rate.

Thus  $\bar{V} = F + L - W = D + L = D + RD = (R + 1)D$

Or  $\bar{V} = (3 + 1)388.95 = 1555.80 \text{ kmol/h}$

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Next we have been asked to find out the boiling operate. So, here we have it that we first we find that the liquid flow rate in the rectifying section is coming out taken from this that we know the reflux ratio is equal to  $L$  by  $D$ . And we are also assuming that the liquid flow rate is remaining constant inside each section. So, this is a liquid flow rate in the rectifying section that is  $L$  equal to  $R$  into  $D$  and because the feed is liquid at its bubble point to that means, the feed will be going only downwards it cannot go upward.

So, what we find in the stripping section of the column the liquid flow rate will be the flow rate which will be obtained by adding the feed flow rate with the liquid flow rate from the rectifying section. So, that is how we will find that the liquid in the stripping section is liquid flow rate using stripping section is more than the liquid flow rate in the rectifying section.

And now we do a mass balance over the reboiler which is obtained like this. That whatever liquid is going in the reboiler is getting distributed for the bottom product and the boiler. So, this is the  $V$  is the vaporization rate. So, this we are again using another mass balance and now what we do we simply make this  $V$  over bar is equal to this  $V$ .

Instead of this  $L$  over bar we writing  $F$  plus  $L$  and minus  $W$  and we know that  $F$  minus  $W$  is nothing, but  $D$  from the overall material balance and then we put for  $L$  we put  $R D$  here. And we get the vaporization rate is coming out to be  $R$  plus  $1$  into  $D$  we plug in the

value of R and the value of the D and we get the vaporization rate in the column. So, this is also a very simple application of the mass balance to find out the vaporization rate.


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**Problem 2**


A feed mixture containing six components (see Table 1) is to be separated by distillation at a feed rate ( $F$ ) of 1000 kmol/h so that 98.5 % of component 3 goes to the distillate ( $f_{3,D}$ ) and 98 % of component 5 goes to the bottom product ( $f_{5,W}$ ). Determine the minimum number of trays required

The average relative volatilities with respect to component 5 are given in the Table 2.

Table 1		Table 2	
$z_1$	0.032	$\alpha_{15}$	3.15
$z_2$	0.068	$\alpha_{25}$	2.75
$z_3$	0.17	$\alpha_{35}$	2.35
$z_4$	0.30	$\alpha_{45}$	1.40
$z_5$	0.32	$\alpha_{55}$	1.00
$z_6$	0.11	$\alpha_{65}$	0.75
Total	1.00		




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Now we go for a second problem. In this problem we have a feed mixture with six components and here we have been given the composition of this feed with the six components and again to see to it that the composition is right we again make it a total of it and we find it is coming out to be 1.

So, this is alright. So, we and this is separated by distillation and the feed rate is again 1000 kilo mole per hour and here we say that 98.5 percent of component 3 goes to the distillate where as 98 percent of component 5 goes to the bottom product. So that means, we are given the recoveries of component 3 and component 5.

And we are required to find out the minimum number of trays. And in this particular table we have been given the relative volatilities with respect to component 5. So, this data will be used to find out this minimum number of trays.

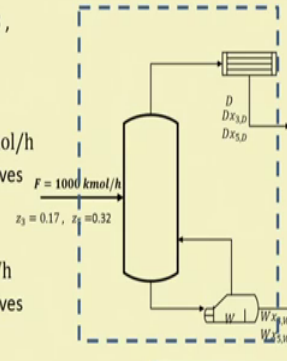
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**Solution :**

- ✓ From the given information, Light key (LK): Component 3 ,  
Heavy key (HK): Component 5

(a)

- ✓ 98.5 % of component 3 (LK) goes to the distillate.  
 $Dx_{3,D} = (0.985)Fz_3 = (0.985)(1000)(0.17) = 167.45 \text{ kmol/h}$   
 Mass balance of component 3 over the distillation column gives  
 $Wx_{3,W} = Fz_3 - Dx_{3,D} = 170 - 167.45 = 2.55 \text{ kmol/h}$
- ✓ 98 % of component 5 (HK) goes to the bottom.  
 $Wx_{5,W} = (0.98)Fz_5 = (0.98)(1000)(0.32) = 313.6 \text{ kmol/h}$   
 Mass balance of component 5 over the distillation column gives  
 $Dx_{5,D} = Fz_5 - Wx_{5,W} = 320 - 313.6 = 6.4 \text{ kmol/h}$



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So, again we put this column and again we are putting the whatever is given to us this  $z_3$   $z_5$  and this feed flow rate and first we identify the low key component which is taken as component 3 and heavy key component we take it as the component 5. And, we make this particular control volume and on the first side we find that we have been given the recovery. So, without detailing now as we have done it in the previous problem, we find out the flow rate of component 3 in the distillate and this is coming out to this value.

And then we make a mass balance for component 3 for the whole column and we find out the mass of the component 3 coming out with the bottoms. And then we go to component 5, we know its recovery 98 percent and we find out the amount of the component 5 coming out with the bottom. And then again we make a mass balance for component 5 for the whole column and this value is coming out to be this is the amount of the component 5 which is coming out with the distillate.

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**Minimum no of stages :**

(i) Using Fenske equation :

$$N_{\min} = \frac{\ln\left[\frac{(x_{3,D} \cdot x_{5,W})}{(x_{5,D} \cdot x_{3,W})}\right]}{\ln \alpha_{35}}$$
$$= \frac{\ln\left[\frac{(Dx_{3,D} \cdot Wx_{5,W})}{(Dx_{5,D} \cdot Wx_{3,W})}\right]}{\ln \alpha_{35}}$$
$$= \frac{\ln\left[\frac{(167.45 \times 313.6)}{(6.4 \times 2.55)}\right]}{\ln 2.35}$$
$$= 9.45$$

(II) In terms of fractional recoveries of key components (Using Fenske equation),

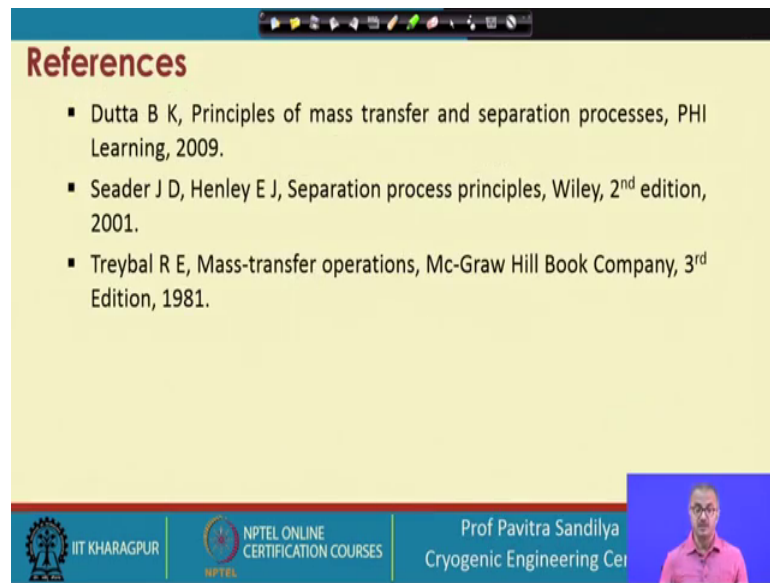
$$N_{\min} = \frac{\ln\left[\frac{(f_{3,D} \cdot f_{5,W})}{\{(1 - f_{3,D})(1 - f_{5,W})\}}\right]}{\ln \alpha_{35}}$$
$$= \frac{\ln\left[\frac{(0.985 \times 0.98)}{\{(1 - 0.985)(1 - 0.98)\}}\right]}{\ln \alpha_{35}} = 9.45$$

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And, now we go for the Fenske equation which is given like this to find out the minimum number of the ideal stages. In this equation now it become very simple we just plug in the values of this various mole fractions and then we put it in terms of this D and W and what we find that we put this alpha 3 5 value. And we put it and we get the value that minimum number of ideal stages is coming out to be 9.45. And, if we do in terms of the recovery because we have learnt in our theory: two ways of finding out this minimum number of stages either in terms of the flow rates or in terms of recovery.

So, again we put this thing in terms of recovery and we find this is the thing we are getting. So, we are finding they are coming out is same, please remember that do not never try to round the top or make any kind of things like 9.45 do not put it that 9 or 9.4 or 10; you just put as 9.45 the number of ideal stages can always be fraction. And to know the real number of phrase we have to use the efficiency to find out the real number of phrase. So, here we are finding the number of ideal stages required for the given separation.

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Now, these are the various books you can consult for further detailing and explanations.

Thank you.