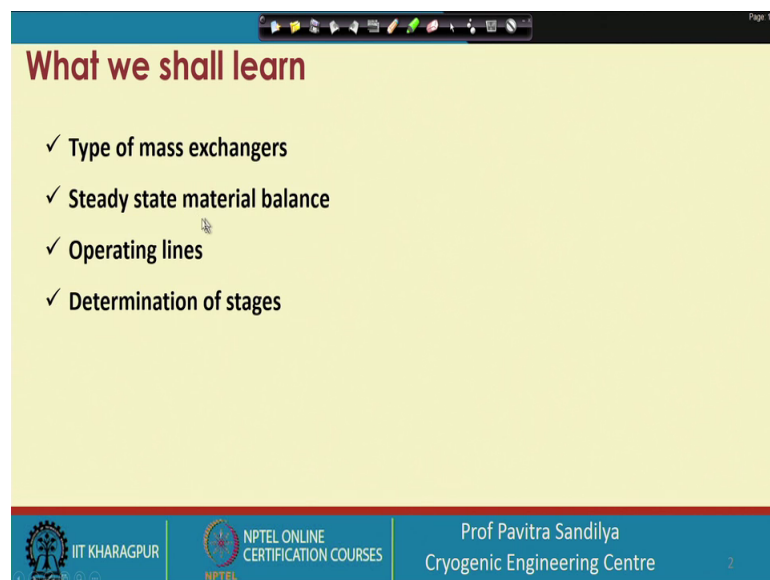


**Upstream LNG Technology**  
**Prof. Pavitra Sandilya**  
**Department of Cryogenic Engineering Centre**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 36**  
**Multistage single component equilibrium separation**

Welcome. Today, we shall be looking into the separation of a component using the multistage analysis, earlier, we have learnt extensively about distillation separation. Now today's lecture will be a generalized one which can be applied to various types of separation process. So, this lecture is multistage single component equilibrium separation.

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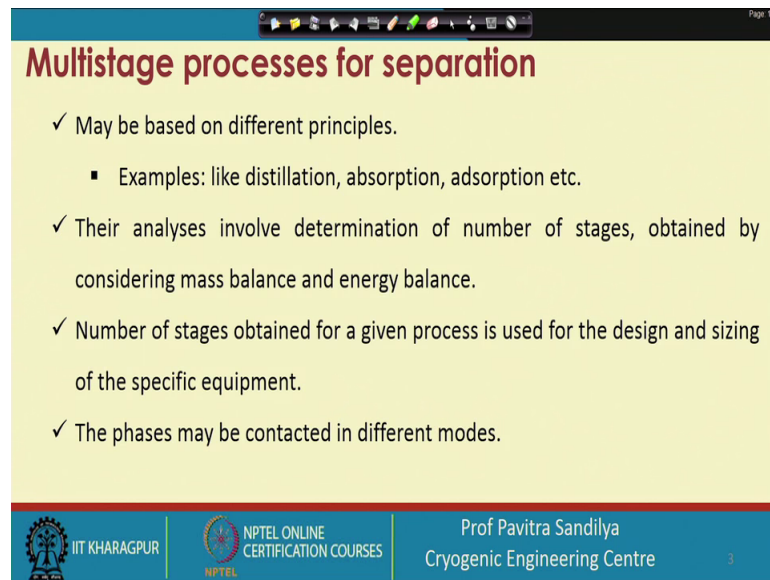
**What we shall learn**

- ✓ Type of mass exchangers
- ✓ Steady state material balance
- ✓ Operating lines
- ✓ Determination of stages

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Now, in this lecture, we shall be learning about various types of mass exchanger, then steady state material balances operating lines and determination of stages for the separation.

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**Multistage processes for separation**

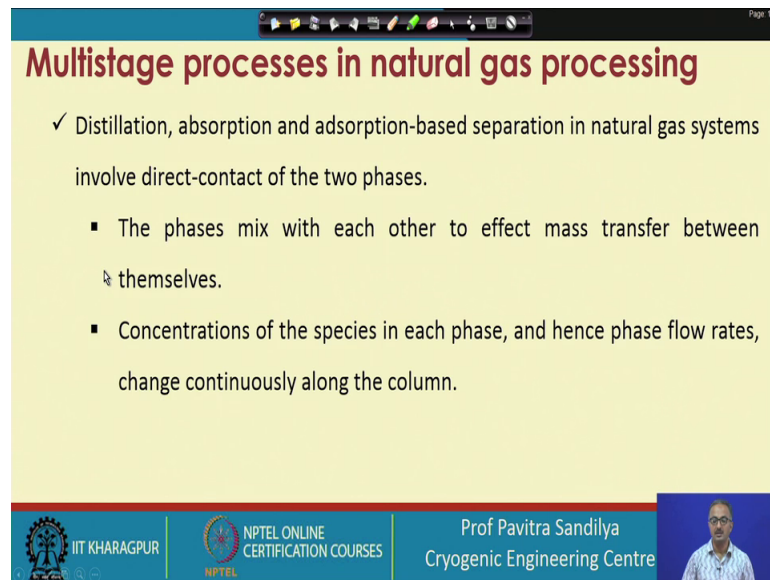
- ✓ May be based on different principles.
  - Examples: like distillation, absorption, adsorption etc.
- ✓ Their analyses involve determination of number of stages, obtained by considering mass balance and energy balance.
- ✓ Number of stages obtained for a given process is used for the design and sizing of the specific equipment.
- ✓ The phases may be contacted in different modes.

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Now, when we talk on the multistage process, we know that there could be various types of principles of separation and some of them, we have studied already and we have examples like distillation absorption adsorption, then when we are trying to understand this multi state process with the analysis involve the determination of the number of stages that is obtained by considering the mass balance and energy balance equations, this also we have learnt under distillation.

So, by obtaining the number of stages, what we shall be obtaining, we shall be having the design and the sizing of the specific equipment and the phases may be contacted in different manners.

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The slide is titled "Multistage processes in natural gas processing" in a bold, dark red font. Below the title, there is a checkmark followed by the text: "Distillation, absorption and adsorption-based separation in natural gas systems involve direct-contact of the two phases." This is followed by two bullet points: "The phases mix with each other to effect mass transfer between themselves." and "Concentrations of the species in each phase, and hence phase flow rates, change continuously along the column." The slide has a yellow background. At the bottom, there is a blue footer bar containing the IIT Kharagpur logo, the NPTEL Online Certification Courses logo, the name "Prof Pavitra Sandilya" and "Cryogenic Engineering Centre", and a small video inset of the professor.

**Multistage processes in natural gas processing**

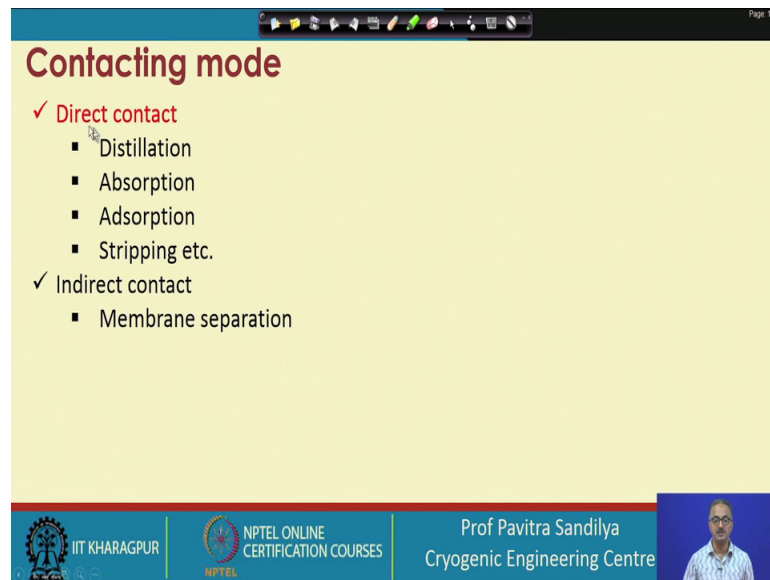
- ✓ Distillation, absorption and adsorption-based separation in natural gas systems involve direct-contact of the two phases.
  - The phases mix with each other to effect mass transfer between themselves.
  - Concentrations of the species in each phase, and hence phase flow rates, change continuously along the column.

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So, let us see that if you talk about distillation absorption adsorption based separation, in this natural gas systems, they involve direct contact of the two phases and what happens that the when the two phases are contacted directly they mix with each other and there by the mass is exchanged that is some species get transferred from one phase to the other preferentially and that is how we are able to achieve separation of the various species.

And in this way, what we find that the concentrations of the species in each phase and hence the phase flow rates keep continuously changing along the column because as the species at the concentration changes; what happens? We find that the effective properties or physical properties also like density, they also change. So, because of this changes which what we find that the flow rates also keep changing along with the composition all along the particular separation equipment.

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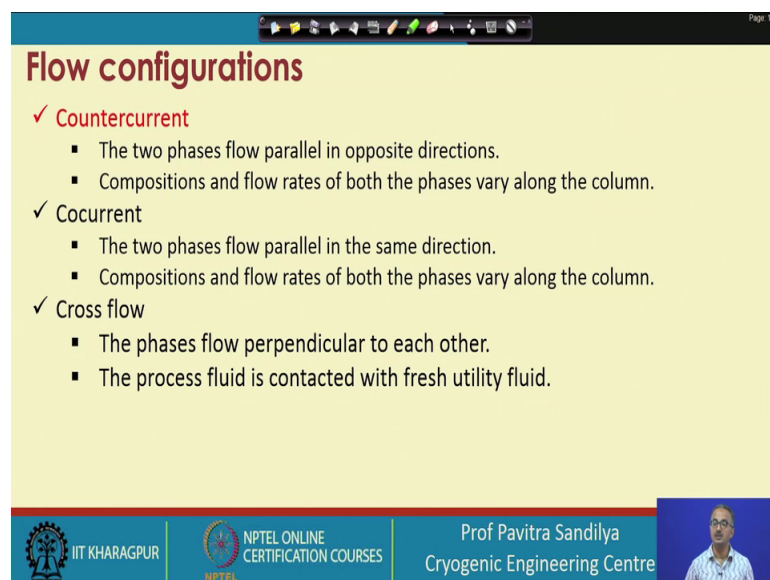
**Contacting mode**

- ✓ **Direct contact**
  - Distillation
  - Absorption
  - Adsorption
  - Stripping etc.
- ✓ **Indirect contact**
  - Membrane separation

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Now, there are various modes of contacting the things, either we cannot direct contact and direct contact is used in distillation, absorption, adsorption, stripping, etcetera, whereas, we can also have separation that is indirect contact that is through membrane separation and about which we have learnt separately. So, in this particular lecture, we shall be focusing on only the direct contact mass exchangers.

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**Flow configurations**

- ✓ **Countercurrent**
  - The two phases flow parallel in opposite directions.
  - Compositions and flow rates of both the phases vary along the column.
- ✓ **Cocurrent**
  - The two phases flow parallel in the same direction.
  - Compositions and flow rates of both the phases vary along the column.
- ✓ **Cross flow**
  - The phases flow perpendicular to each other.
  - The process fluid is contacted with fresh utility fluid.

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Now, there could be various flow configurations; that means, how this two phases are flowing with respect to each other. So, we can have counter current flow rate, you have

co current or you have cross flow, this is analogous to what we learnt in the heat exchangers, there also we have similar kind of flow configurations. So, in the counter current, what we find the two phases flow parallel to each other, but in opposite directions and compositions and flow rates of both the phases vary along the column.

In case of cocurrent the two phases again flow parallel, it each other, but in the same direction unlike the counter current process and in this case, like the counter current process the compositions and a flow rates of both the phases vary along the column in case of cross flow the phases flow perpendicular to each other and generally, what happens the process fluid flows one after the other process continuously in to the inside the particular system.

Whereas, the utility fluid that is the fluid which is used and as an agent to separate some components flows in a perpendicular direction and it is always taken has fresh fluid; ; that means, the process fluid continuously changes its composition along the column whereas, fresh fluid is inducted into the column at different points as the agent of separation.

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✓ Let  $L$  and  $V$  denote the heavier and lighter phases respectively.

Heavier phase ( $L$ )	Lighter phase ( $V$ )	Separation
Liquid	Vapor	Distillation, absorption, stripping
Solid	Liquid	Adsorption, leaching
Solid	Gas	Adsorption
Denser liquid	Lighter liquid	Extraction

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Now depending on the kinds of fluids, we have we have different types of processes. So, here we have denoting by  $L$  and  $V$ , the heavier and lighter phases this should not be confused with liquid and vapor. So, this  $L$  represents heavier phase and  $V$  represents the lighter phase.

So, let us see , but if the heavier phase is liquid and the lighter phase is vapor, we can have different set of separation and the examples are distillation absorption stripping. So, if you have liquid and gas or liquid of vapor we can have this kind of systems.

Similarly, if you have solid and liquid system then we can have adsorption or we can have leaching is also used in a metallurgical processes then we may have solid as heavier phase and gas has a lighter phase and this finds application in adsorption and then we may have two types of liquids with different densities.

So, one is denser liquid and as it may be lighter liquid and which are invisible for example, oil and water they are two different liquids with two different densities and they are immiscible. So, they will form two phases. So, that is how we say that we can have a denser liquid and a lighter liquid which are immiscible. So, this can also effect separation and that kind of separation is obtained in extraction.

(Refer Slide Time: 06:32)

### Countercurrent separator

- ✓ Typical process is shown.
- ✓ Only one solute ( $A$ ) is being exchanged between the two phases  $L$  and  $V$ .
- ✓ Conventionally, 1 and 2 denote the bottom end (or bottom terminal) and top end (or top terminal) respectively.

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So, here we have a counter current separator a typical conffiction, we see here that here we assume that there is only one solute a which is being exchanged between the two phases L and V and here we have shown 2 n as 1 and 2 this 1 and 2 denote the bottom end and top end or bottom terminal or top terminal of the particular column and here, we have the various types of flow involve; we find that the means heavier phase is coming from the top to the bottom while the lighter phase is going from the bottom to the top..

And as the flow along this by column, they are getting contacted with each other and due to the mixing of this two phases what is happening the particular solute a is getting exchanged between the two phases and at some any intermediate position, we are giving some arbitrary a compositions xy.

Here you can see there are various a notations involved like a V 2 Vs y 2 y this capital Y 2 along these things.

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**Notations used in countercurrent separator**

- ✓  $L$  and  $V$  are the molar phase flow rates.
  - These vary all along the column.
- ✓  $L_s$  and  $V_s$  are the molar phase flow rates on solute-free basis.
  - These remain constant all along the column.
- ✓  $x$  and  $y$  are mole fractions of  $A$  in  $L$  and  $V$  respectively.
- ✓  $X$  and  $Y$  are mole ratios of  $A$  in  $L$  and  $V$  respectively.

$$Y = \frac{y}{1-y}, V_s = \frac{V}{1+Y}, X = \frac{x}{1-x}, L_s = \frac{L}{1+LX}$$

Thus whereas  $0 \leq x$  or  $y \leq 1$ ,  $X$  and  $Y$  vary between 0 and  $\infty$

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So, in this, now your fusion see what kind of notations, you are having this  $L$  and  $V$  are the molar phase flow rates and these vary along the column this has to be very clear that molar flow rates are changing along the column because of some one of the phases is losing the particular solute, whereas, the other phase is gaining the solute and that is how the phase flow rates keep changing.

On the other hand, we have another notation  $L_s$  and  $V_s$ , these are the molar phase flow rates on solute free basis solute free basis means that when we are calculating  $L_s$  and  $V_s$ , we are excluding the solute which is getting transferred; that means, if we exclude the solute which is in transferred assuming that other species are not getting transferred, it means that the  $L_s$  and  $V_s$  will remain constant all along the column..

And why we are doing this we shall see later that by assuming the flow rates on a solute free basis, we are able to simplify our analysis and then by small  $x$  and small  $y$ , we

represent the mole fractions of the species  $a$  in  $L$  and  $V$  respectively, whereas, by capital  $X$  and capital  $Y$ , we are representing the mole ratios of species  $a$  in  $L$  and  $V$  respectively and what is mole fraction, we know that mole fraction of a component in a mixture is the number of moles of the particular component per unit number of moles in the mixture.

On the other hand, the mole ratio means that it is a ratio of the moles; that means, mole ratio is always with respect to at least 2 components so; that means, the if I say that mole ratio of component 1 with respect to component 2 is equal to the mole of component 1 divided by the mole of component 2. So, this is the mole ratio.

So, in this case, we if we assume that only species  $a$  is getting separated; that means, we can take rest of the species as one single pseudo component. So, when we talk of the mole ratio, it would mean that the mole ratio of component  $a$  with respect to the rest of the components means the number of moles of component  $a$  divided by the number of moles of the rest of the species in the mixture.

Now, this mole ratio and the mole fractions can be easily correlated and here I have shown this relationship between the mole ratio and the mole fraction and this can be dealt very easily that this mole ratio is the number of moles of the species  $a$  and  $1 - y$  represents the number of moles of the rest of the species.

Similarly, we have the solute free basis flow rate and that is represented by this total flow rate divide by the  $1 + y$  ok. So, naturally we see that the solute free basis flow rate will be always less than the actual flow rate because we are taking out one of the components from the whole flow rate

So, similarly we have the mole ratio in the  $L$  phase related with the mole fraction in the  $L$  phase and this is the solute free flow of the  $L$  phase related to the solute free thing of the this is will be  $x$ , this is the mole ratio. So, this is how we are able to correlate the mole ratio with the mole fraction and the solute free flow rate with the total flow rate.

Now, here we have to remember that  $x$  or  $y$  that is this mole fractions this mole fractions are between always 0 and 1 that we know, but we see that the mole ratios can vary between 0 and infinity even easily see that if you put  $y$  as 0, then this capital  $Y$  is 0 and if you keep  $y$  as 1, then the bottom one becomes  $1 - 1$  that is 0. So, this  $y$  tends towards infinity.



So, that is how we find that mole ratio can go from 0 to infinity where is mole fraction will stay only between 0 and 1.

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**Analysis of countercurrent separator**

- ✓ Phases  $L$  and  $V$  are assumed immiscible, and are carrier of solute  $A$ .
- ✓ Steady state mass balance around Control-volume 1:
 
$$V_2 Y + L_2 X_2 = L_1 X_1 + V_1 Y_1$$
- ✓ Rearranging we get
 
$$Y = \frac{L_1}{V_1} (X - X_1) + Y_1$$
- ✓ This is the operating line equation which is a straight line with a slope of  $L_1/V_1$  and  $y$ -intercept of  $Y_1$ , and passing through  $(X_1, Y_1)$ .

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Here in this slide, now we go for the analysis, now what we do here is this that first we locate one particular control volume which encompasses that terminal 2 and at some arbitrary height of the particular column, here we see that in going streams are this vapor from the this thing and the liquid from the top.

So, now if we do a steady state material balance assuming that the two phases  $L$  and  $V$  are immiscible, then we find that this is the incoming stream that is  $V_1$  into the column  $Y_1$  is the amount of species  $A$  going in and this  $L_1$  is into  $X_1$  is the amount which is going in and the coming out is the  $V_2$  into  $Y_2$  and the  $L_2$  into  $X_2$ . So, this is how we are writing the, whatever is going in is equal to whatever is coming out at steady state there is no accumulation.

Now, of course, we please understand that we can also put this thing in terms of the mole fractions if we put into terms of mole fractions then we have to use the total flow rate now because  $L_1$  and  $V_1$  are constant and then we can rearrange this equation, we find that this is the mole ratio in the  $V$  phase and these are the mole ratio in the  $L$  phase and this is the terminal mole ratio in the  $V$  phase and if because  $L_1$  and  $V_1$  are constant.

So, this particular  $L_s$  by  $V_s$  is also a constant. So, the slope is a constant. So, we reduce that this is an operating line equation because this correlates at any given height, it correlates the composition of the L phase and V phase.

So, this at operating line and the straight line with the slope of  $L_s$  by  $V_s$  and the Y intercept is  $Y_2$  and it passes through  $X_2$  and  $Y_2$ ; that means, if you put  $X$  equals to  $X_2$ , then we will find  $Y$  equal to  $Y_2$ . So, this is the straight line operating operate line equation for this particular column.

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**Analysis of countercurrent separator**

✓ Steady state mass balance around Control-volume 2:

$$V_s Y_1 + L_s X_2 = L_s X_1 + V_s Y_2$$

✓ Rearranging we get

$$Y_1 - Y_2 = \frac{L_s}{V_s} (X_1 - X_2)$$

✓ Thus

- $(X_1, Y_1)$  also satisfies the operating line equation, and
- Operating line joins  $(X_1, Y_1)$  and  $(X_2, Y_2)$ .

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Now, what we do we look at the column as a whole. Now here we choose another control volume 2 which is encompassing the whole height of the column and now when we right again the study state mass balance over here we find we have is  $V_s$  into  $Y_1$  plus  $L_s$  into  $X_2$ ; that means,  $V_s$  into  $Y_1$  plus  $L_s$  into  $X_2$  is equal to  $V_s$  into  $Y_2$  plus  $L_s$  into  $X_1$ . So, that is how we writing the study state mass balance for this control volume and again rearranging this we are getting this particular equation.

Now, what we find here that  $X_1$  and  $Y_1$  ok; that means, this  $X_1$   $Y_1$ , they also satisfy the operating line equation that is this equation is of satisfying by  $X_1$  and  $Y_1$  and these; that means, operating line passes through both  $X_1$   $Y_1$  and  $X_2$   $Y_2$ ; that means, the operating line will connect both the terminal concentrations of the two species in the two phases.

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**Graphical operating line for countercurrent separator**

✓ Equilibrium curve and operating lines are drawn.

- Nonlinear equilibrium relationship in terms of mole fractions may be linear in terms of mole ratios, and vice versa.
- For linear equilibrium curve

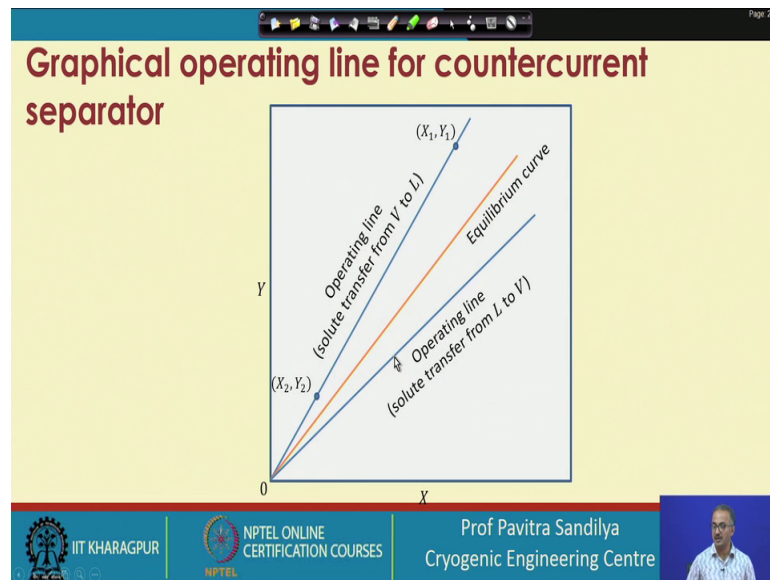
$$Y_n = \alpha X_n$$

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Now let us see that how to graphically represents this kind of a system. Now we need both the equilibrium curve and the operating lines and the, we understand that the equilibrium relationship may be non-linear in terms of mole fraction, but when we represent in terms of the mole ratios that may come has linear. Similarly, if we have the equilibrium relationship in terms of mole fraction has linear that in terms of mole ratios it may turn out to be a non-linear.

Now, let us assume for the time being that we have a linear equilibrium curve without loss of generality. So, we find that this is the wave we are at any given stage this  $n$  represents the stage that any given stage we are equilibrium, equilibrium relationship between  $Y_n$  and the  $X_n$  this is given by this equation and this  $\alpha$  is the equilibrium constant or the distribution coefficient.

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Now, once we have done this now what we find that if we would now try to plot it. Now here we find that on the X axis, we are plotting the mole ratio in the L phase on the Y axis, we are plotting the mole ratio in the y phase and it both of them start from 0 and you can see that we have not put any kind of number or the other end of the particular curve because unlike the mole fractions this mole ratios can go to any number ok.

Now, in the first; what we do we put this particular line is the equilibrium curve and then we can have different types of the operating line depending on, what is the direction of the species transfer now here we have one operating line which is also passing through  $X_1 Y_1$  and  $X_2 Y_2$  and this is the operating line we shall be obtaining if the species transfer is taking place from phase V to phase, this you can see very easily.

On the other hand, if the species is getting transferred from phase L to phase V, then we find the operating line will be below the equilibrium curve and you on this operating line also, we can locate the point  $X_1$  and  $Y_1$   $X_2$   $Y_2$ . Now please understand that this  $X_1$  when the species is coming from V  $Y_1$   $X_2$   $Y_2$  will always be more than  $X_2$   $Y_2$  that you can see easily from this particular a graph that if the thing is comma V  $Y_1$   $X_2$   $Y_2$  means this particular V phase, it getting enriched with the is did getting depleted by the particular solute and is L phase is getting the solute.

So, what is happening this  $X_1$  is going to be more than  $X_2$ , but on the other hand, you may go to the other way round, then what will find that the V phase will get enriched whether L phase L phase get depleted.

So, that is why we find the locations of  $X_1$  and  $Y_1$  and  $X_2$   $Y_2$  also varies and in the particular curve. So, here we have find  $X_1$   $Y_1$  is more than  $X_2$   $Y_2$ , whereas, in case of L to, we find that  $X_2$   $Y_2$  will be here and  $X_1$   $Y_1$  will be on this side.

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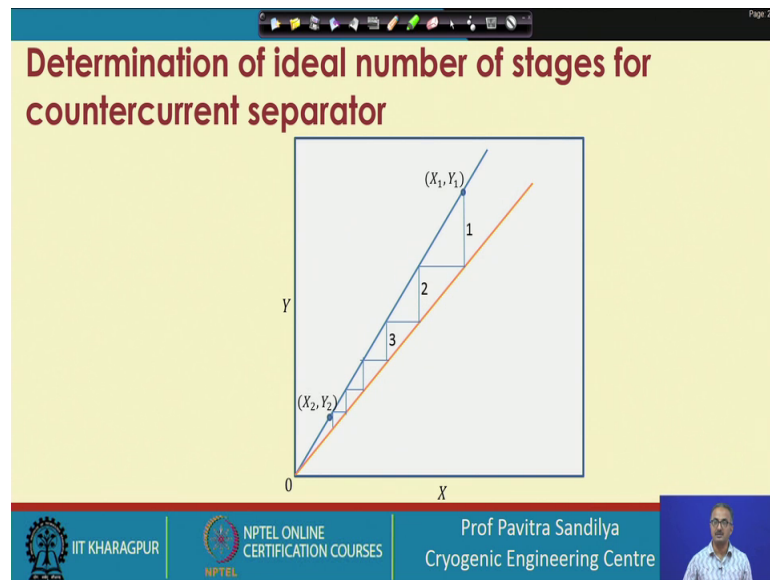
**Determination of ideal number of stages for countercurrent separator**

- ✓ Stair case construction is done between  $(X_1, Y_1)$  and  $(X_2, Y_2)$ .

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Once, we have known this, now what we do that has we have done in case of binary distillation column analysis.

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We can also make some kind of stair case construction this I shall not explain, you can refer back to the lecture on the binary restoration and here we simply do that he assuming that the their transfer is from V to L what we do that here, first, we draw again the equilibrium curve and then we have draw the this a operating line then what we do.

Let us suppose we start from the  $X_1 Y_1$  and we first we draw down to this equilibrium curve and then we keep on doing stare case to go to the next level and similarly, we keep on moving this way and then what we do wherever we are crossing this particular given composition, we shall be taking this small fraction ; that means, out of this whole equilibrium stage we shall be considering only the small fraction over here.

So, this is also the same thing we did for the binary phase distillation column separation using the McCabe Thiele method. So, in the similar fashion what we do that we count the number of equilibrium ideal stages has whichever has connecting the operating line with this particular thing. So, this is counted has 1, 2, 3 and so on and so forth.

So, this is exactly the same whatever we have learnt under the McCabe Thiele method. So, this is now, we are finding the number of equilibrium stages now there is another way of finding the algebraically the number of ideal stages.

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**Algebraic determination of ideal number of stages**

- ✓ Can be determined if both operating line and equilibrium line are straight.
- ✓ Kremser (or Kremser-Brown-Souders) equation.
- ✓ For transfer from V to L

$$N = \frac{\ln \left[ \left( \frac{Y_{n+1} - \alpha X_0}{Y_1 - \alpha X_0} \right) \left( 1 - \frac{1}{\bar{A}} \right) + \frac{1}{\bar{A}} \right]}{\ln \bar{A}}$$

Where  $\bar{A}$ : Absorption factor =  $L_s/V_s\alpha$ , and  $(1/\bar{A})$ : Stripping factor (or  $\bar{S}$ )

- ✓ For transfer from L to V

$$N = \frac{\ln \left[ \left( \frac{X_0 - (Y_{n+1}/\alpha)}{X_N - (Y_{n+1}/\alpha)} \right) (1 - \bar{A}) + \bar{A} \right]}{\ln(1/\bar{A})}$$

- ✓ In case  $\bar{A}$  varies between top and bottom of the column,  $\bar{A} = \sqrt{\bar{A}_1 \bar{A}_N}$ ,  $\bar{A}_j = L_j/V_j\alpha$ ,  $j = 1$  or  $N$

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And without any derivation, I shall give you the final form of this equations and this have been derived by Kremser and this is known as Kremser's equation or sometimes Kremser's Brown Souder's equation.

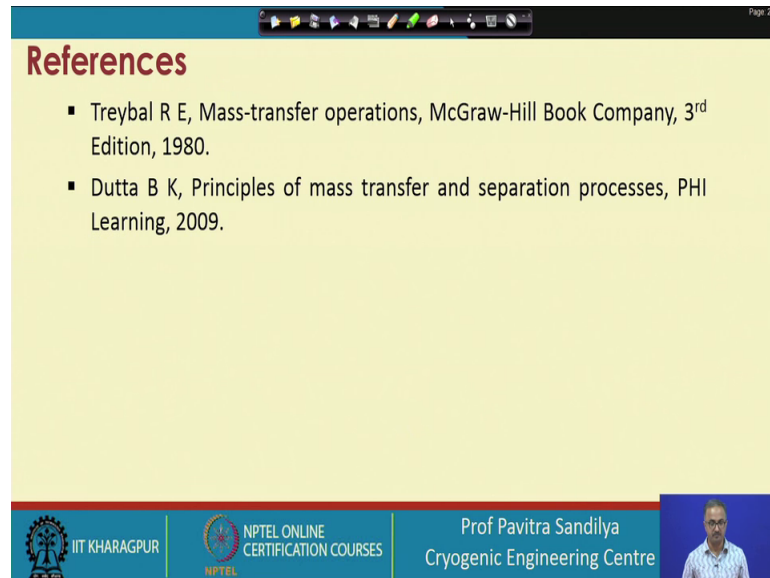
So, these equations are giving us the ideal number of stages and here we find if the transferred is from V phase to L phase. So, this is the particular expression this is used for finding the number of ideal stages. On the other hand, if the transfer is from phase L to phase, then this is the equation which is used for finding the number of ideal stages in this two equations, we have another parameter that is a over bar and this a over bar is the absorption factor and defined as  $L_s$  by  $\alpha V_s$  and this  $1$  by a over bar that is the inverse of the reciprocal of the absorption factor is called the stripping factor sometimes denoted by  $S$  over bar.

So, in terms of is absorption factor stripping factor, we are able to express the number of ideal stages algebraically, now if we find that the absorption factor varies along the column from top to bottom drastically, then what we need to do that we shall be taking some kind of average and it is generally a geometric average is taken between the absorption factor at the top and at the bottom and here we have finding that one you representing stage one and n representing stage n.

So, this  $A_j$  at; that means, if j is 1 or n  $A_j$  will be  $L_j$  by  $\alpha V_j$ . So, in this case, we are assuming that alpha may be constant or it may not be constant, it may be also varying

from stage to stage. So, this is how we are finding the algebraically the number of theoretical stages.

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Now, to more know more about this, you can refer to this to books on the mass transfer.

Thank you.