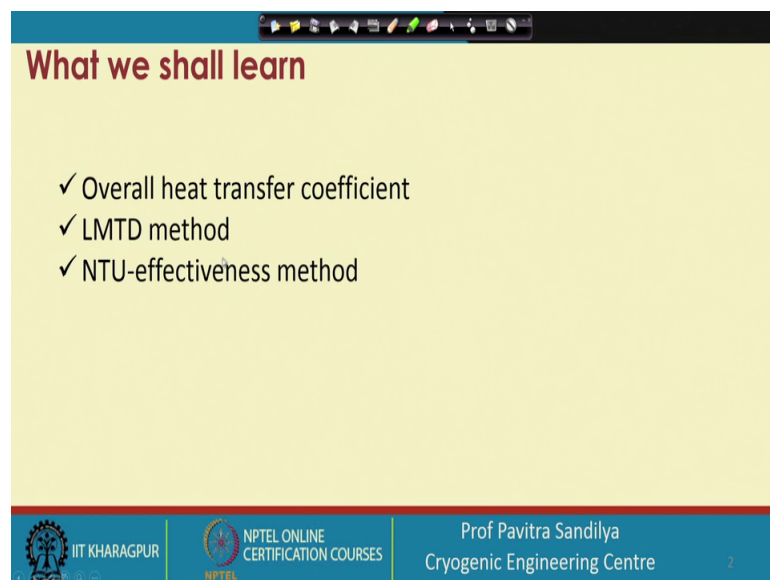


**Upstream LNG Technology**  
**Prof. Pavitra Sandilya**  
**Department of Cryogenic Engineering Centre**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 27**  
**Analysis of heat exchangers in natural gas systems**

Welcome after learning about some fundamentals about heat transfer and heat exchangers; in this lecture we shall be looking into the Analysis of the heat exchangers in the natural gas systems.

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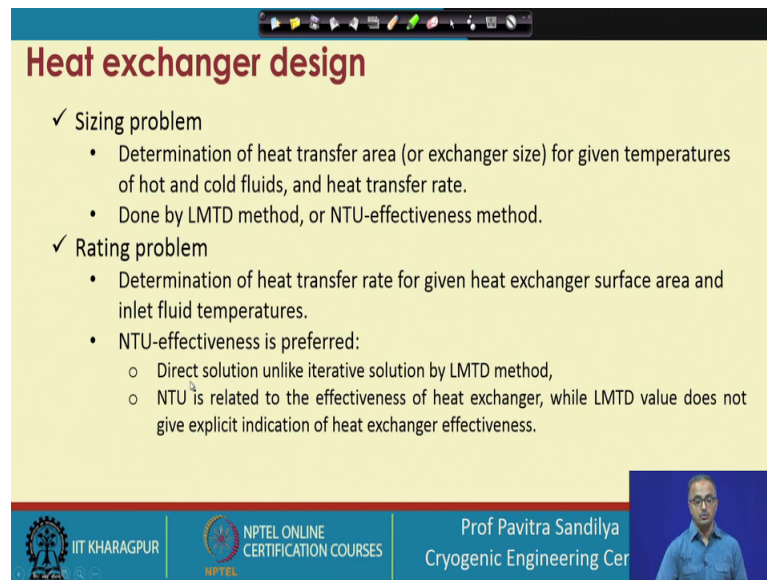
**What we shall learn**

- ✓ Overall heat transfer coefficient
- ✓ LMTD method
- ✓ NTU-effectiveness method

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In this lecture, what we shall be learning about the overall heat transfer coefficient, the LMTD method and the NTU-effectiveness method for the analysis of the heat exchangers.

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**Heat exchanger design**

- ✓ Sizing problem
  - Determination of heat transfer area (or exchanger size) for given temperatures of hot and cold fluids, and heat transfer rate.
  - Done by LMTD method, or NTU-effectiveness method.
- ✓ Rating problem
  - Determination of heat transfer rate for given heat exchanger surface area and inlet fluid temperatures.
  - NTU-effectiveness is preferred:
    - Direct solution unlike iterative solution by LMTD method,
    - NTU is related to the effectiveness of heat exchanger, while LMTD value does not give explicit indication of heat exchanger effectiveness.

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So, first let us see that what kind of design problems we shall be having. One is a sizing problem another is the rating problem; in the sizing problem we have to determine the area for heat transfer and; that means, that we have to find out the size of the heat exchanger for some given temperatures of the hot and cold fluids and some heat transfer rates. And in the rating problem we have to determine the heat transfer rate for a given heat exchanger surface area and some inlet fluid temperatures. The difference is this in one this rating problem; we are evaluating the performance whereas, in the sizing problem for a given performance we have to find out the exchanger size.

Now this sizing problem may be done either with LMTD method or the NTU effective this method, but the rating problem is generally done with the NTU effectiveness method; Because this is a direct solution whereas, if we use LMTD method for this rating problem it becomes an iterative solution. And this NTU is directly related to the effectiveness of the heat exchanger whereas; there is no direct correlation between the effectiveness and the LMTD. So, that is why this NTU method is preferred to the LMTD method for the rating problem.

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## Heat exchanger design

- ✓ Sizing problem
  - Determination of heat transfer area (or exchanger size) for given temperatures of hot and cold fluids, and heat transfer rate.
  - Done by LMTD method, or NTU-effectiveness method.
- ✓ Rating problem
  - Determination of heat transfer rate for given heat exchanger surface area and inlet fluid temperatures.

Heat exchanger effectiveness is direct measure of how closely a heat exchanger approaches the best possible performance.

- NTU is related to the effectiveness of heat exchanger, while LMTD value does not give explicit indication of heat exchanger effectiveness.

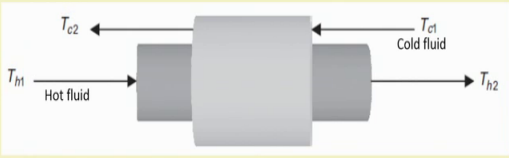
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And what is effectiveness? The heat exchanger effectiveness is a direct measure of how closely a heat exchanger approaches the ideal behavior; that means, how a closely it approaches the best possible performance under some given operating conditions.

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## Heat transfer in a double pipe heat exchanger

- ✓ Consider a section of the double pipe heat exchanger.
- ✓ Heat transfer occurs by
  - Convection hot fluid to the wall of the inner pipe
  - Conduction through the pipe wall
  - Convection from the inner wall to the cold fluid



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Now, to develop these equations first let us considered this double pipe heat exchanger in this for example, we have chosen a counter current flow and we had be fine that the hot fluid is flowing from the left to right with user temperature of  $T_{h1}$  and outlet of  $T_{h2}$  whereas, the cold fluid is going from the right to left within the temperature of  $T_{c1}$  to  $T_{c2}$

c 2; that means, this  $T_{h2}$  is less than  $T_{h1}$  whereas,  $T_{c2}$  is more than  $T_{c1}$ . And during this process of the flowing the hot flow is heat giving up its heat to the cold fluid.

Now, the heat transfer occurs between the 2 fluids by these 3 methods one is convection or in the hot fluid to the wall of the inner pipe this is the inner. So, this is the inner pipe; so, what happens? The hot fluid is giving out the heat to the inner pipe. And then within this pipe wall there is conduction and after this from the wall of the inner pipe it is going to the cold fluid by convection; that means, we are having a combined convection conduction effect to determine the overall heat transfer rate.

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**Rate equation: LMTD Approach**

✓ Heat transfer rate between two fluids:

$$Q = UA\Delta T_m$$

where  $U$ : Overall heat transfer coefficient based on the heat transfer area  $A$ ,  
 $A$ : Surface area of the inner pipe; may be based on either the inner diameter ( $D_i$ ) or the outer diameter ( $D_o$ )  
 Generally  $D_o$  is used so that  $A = A_o = \pi D_o L$ .  
 On the other hand,  $A_i = \pi D_i L$   
 Considering  $U$  to be independent of the position in the heat exchanger, we have  
 $\Delta T_m$ : Log Mean Temperature Difference (LMTD) defined as

$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)}$$

where  $\Delta T_1$  and  $\Delta T_2$ : temperature differences at the two ends of the heat exchanger.

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Now in the LMTD approach what we do? We first write the heat transfer rate equation in terms of the overall heat transfer coefficient. The surface area of the inner pipe and the some delta  $T_m$  and this is the lmtd temperature difference. So, let us see one by one that the surface area is of the inner pipe through which the 2 fluids are exchanging their energies.

Now, this surface area of the inner tube may be expressed in terms of inner the inner diameter or the outer diameter. However, generally outer diameter is taken; so, outer diameter is  $\pi D$  naught into  $L$  where  $L$  is the length of the pipe and we can also have the inner area surface area as  $\pi d_i$  into  $L$ .

Now, we see that if we consider the overall heat transfer coefficient to be independent of the position of the heat exchanger; that means, it remains constant throughout; then it can be derived that this is the log mean temperature difference this can be derived and you can find the derivation in any basic heat transfer books.

So, ultimately what we will find? That this is how we define the LMTD temperature; so, this particular  $\Delta T_1$  and  $\Delta T_2$  are the temperature differences at 2 ends of the heat exchangers. So, this is called the log mean temperature difference because of the presence of the log logarithmic here.

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**Rate equation: LMTD Approach**

- ✓ Thermal resistance ( $R_{th}$ ):
$$R_{th} = \frac{1}{UA}$$
- ✓  $R_{th}$  has three components
  - Convective resistance between hot fluid and the inner pipe wall
  - Conductive resistance in the pipe wall
  - Convective resistance between pipe wall and the cold fluid
- ✓ Thus
$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{k_w A_i} + \frac{1}{h_o A_o}$$

where  $h_o$  and  $h_i$  : Heat transfer coefficients for the outer fluid and inner fluid respectively

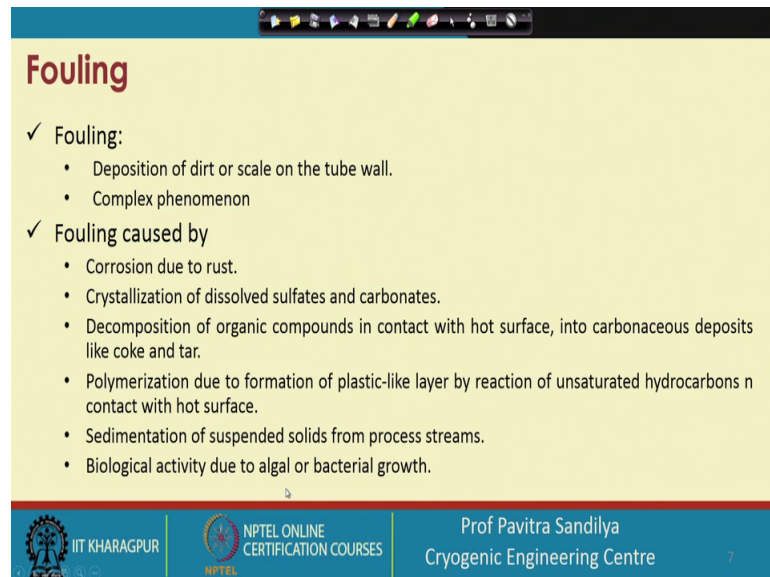
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So, now, in this particular LMTD approach what we find? That we first define the thermal resistance and we find this is the reciprocal of the product the overall of the, what are the transfer coefficient and the surface area. And this, the resistance has again 3 components as we have found that if the heat transfer is going by convection and conduction.

And so, first we have the conductive resistance between the hot fluid which is in the inside tube to the inner pipe wall and then the conductor resistance in the pipe wall. And again we find convective resistance between the inner pipe wall and the cold fluid. So, this way we find that 3 resistances and then what we do that we represent the overall resistance to the heat transfer in terms of these 3 individual resistances. Now, this first represents the convective heat transfer resistance in the inner fluid then this represents

the conductive resistance in the pipe wall and this represents the convective heat transfer resistance in the or with respect to the outer fluid. And these may heat transfer coefficients may be found suitably we can we will look into this at the later of the lecture.

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**Fouling**

- ✓ Fouling:
  - Deposition of dirt or scale on the tube wall.
  - Complex phenomenon
- ✓ Fouling caused by
  - Corrosion due to rust.
  - Crystallization of dissolved sulfates and carbonates.
  - Decomposition of organic compounds in contact with hot surface, into carbonaceous deposits like coke and tar.
  - Polymerization due to formation of plastic-like layer by reaction of unsaturated hydrocarbons in contact with hot surface.
  - Sedimentation of suspended solids from process streams.
  - Biological activity due to algal or bacterial growth.

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Now, there is an important consideration whenever we are talking about the heat transfer coefficient's that is fouling. So, this fouling happen because of the deposition of various dirt or scale on the tube wall over a period of time and this fouling is a very complex phenomenon. So, what we do that we because whenever there is a fouling; if there will be a an increase in the resistance to the heat transfer and due to this what we will find that the size of the heat exchanger will be affected.

So, more the fouling more will be the size of the heat exchangers for a given heat duty. So, that is why it is important for us to know the resistance offered due to the fouling. So, first let us see what are the causes of fouling? It may be due to corrosion of the under rusting and this rust will be settling on the tube wall and it could be due to the crystallization of some dissolved sulfates and carbonates and understand this in the petroleum industries in the natural industries there is a high possibility of having such kind of particulate matters or such kind of chemical matters in the fluid.

So, then we may be having decomposition of the organic compounds then they come in contact with the hot surface and what happens? They convert they convert into some

carbonaceous deposits like coke or tar. Or there could be some polymerization reactions which will be giving rise to some kind of plastic kind of layer and which will be some unsaturated hydrocarbons will be there on the hot surface.

So, this plastic like where is very difficult to remove, but they will increase resistance to the heat transfer. And then we could have sedimentation of the suspended solids from the process streams and later we can have some biological activity due to some algae or bacteria that may be there in the fluid streams. So, all these things are may cause them some kind of fouling resistance.

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**Rate equation: LMTD Approach - Correction for fouling**

- ✓  $R_{D_i}$  and  $R_{D_o}$ : Thermal resistances due to fouling on inside and outside of the inner pipe.
- ✓ For inner dirt film
 
$$R_{D_{th}} = \frac{R_{D_i}}{A_i}$$
- ✓ If  $U_D$  is overall coefficient after fouling, then
 
$$\frac{1}{U_D} = \frac{D_o}{D_i h_i} + \frac{D_o \ln(D_o/D_i)}{D_i h_i} + \frac{1}{h_o} + \frac{R_{D_i} D_o}{D_i} + R_{D_o}$$
- ✓ Fouling factors are obtained from published data.

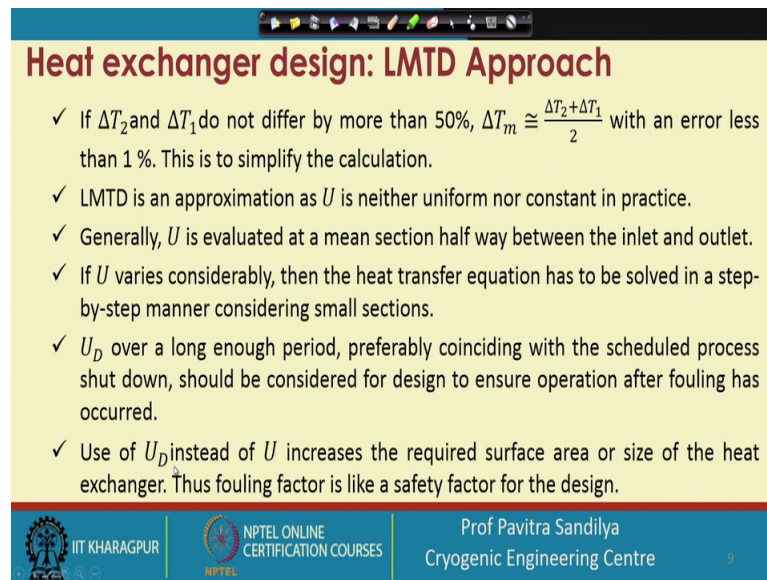
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To account for the fouling resistance what we do again we define 2 more resistances  $R_{D_i}$  and  $R_{D_o}$  these are resistances due to fouling on the inside and outside of the inner pipe. Please note that we are not concerned here about the outer pipe because that is not coming in picture as far as the heat exchange between the 2 fluids are concerned.

So, we are looking at the fouling resistance on the inner the wall and the outer wall of the inner pipe alone. So, for the dirt film we write that this is the thermal resistance. So, this is with respect to the inner pipe wall and then we modify our overall heat transfer coefficient by including the 2 resistances at the inner and outer pipe walls. And these fouling factor that is this  $R_{D_i}$  values are obtained from the published data.



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### Heat exchanger design: LMTD Approach

- ✓ If  $\Delta T_2$  and  $\Delta T_1$  do not differ by more than 50%,  $\Delta T_m \cong \frac{\Delta T_2 + \Delta T_1}{2}$  with an error less than 1%. This is to simplify the calculation.
- ✓ LMTD is an approximation as  $U$  is neither uniform nor constant in practice.
- ✓ Generally,  $U$  is evaluated at a mean section half way between the inlet and outlet.
- ✓ If  $U$  varies considerably, then the heat transfer equation has to be solved in a step-by-step manner considering small sections.
- ✓  $U_D$  over a long enough period, preferably coinciding with the scheduled process shut down, should be considered for design to ensure operation after fouling has occurred.
- ✓ Use of  $U_D$  instead of  $U$  increases the required surface area or size of the heat exchanger. Thus fouling factor is like a safety factor for the design.

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Now, some comments on these heat LMTD and the overall heat transfer coefficient first thing one is thing is this that when the delta T 2 and delta T 1 do not differ by more than 50 percent, then this logarithmic mean temperature difference may be approximated by taking the arithmetic average of this to delta T 2 and delta T 1. And by taking this kind of average we find that the error we incur is about less than 1 percent. And by taking the arithmetic mean what we essentially do we are simplifying our calculations and then this LMTD is an approximation as we said that it is obtained only by considering this overall heat transfer coefficient to be constant, but in practice this U is neither uniform nor constant.

And generally this may heat transfer coefficient is evaluated at halfway at some section at the halfway between the inlet and the outlet and if the U varies considerably then we cannot use this LMTD we have to go step by step taking the actual temperature difference between the 2 fluids. Now this U D that is the overall heat transfer coefficient in the presence of fouling is preferred we generally taken to coincide before design. Because if we take the only U then what happens it will whatever a heat transfer area we estimate that will be kind of an underestimation in presence of the fouling.

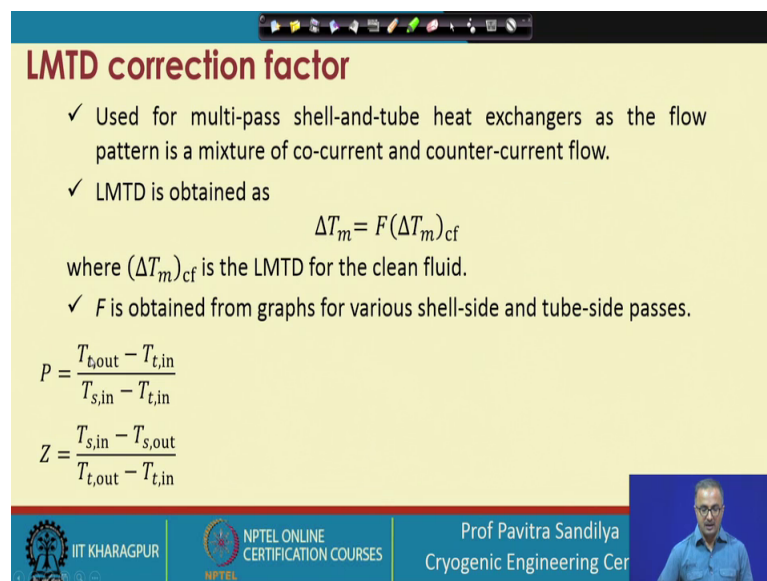
So, whatever what we do is this we always design our heat exchangers based on the U D and not on based on U so, that the heat exchanger initially may be running with clean fluids. So, at that time it will be smaller which will be need it will not be needing all the



heat transfer area, but over a period of time; as there will be fouling then we find that the whole heat transfer area will be utilized for the to encounter the resistance due to the fouling.

So, the design is based on the U D. So, what we do? Generally this U D is taken in the may that it preferably coincides with the scheduled process shutdown. Why because anyway whenever there is fouling and if there is too much of fouling then what we have to do? We have to shut down the whole plant and take out all the tubes clean them and again fix them back. So, until we are going for shutdown we should see to it the heat exchanger is performing to the desired extent. That is why we always make the design with respect to U D and this even though we are using U T what happens is essentially gives a higher heat transfer area. So, this fouling factor may be taken to be some kind of a safety factor for the heat exchanger design.

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**LMTD correction factor**

- ✓ Used for multi-pass shell-and-tube heat exchangers as the flow pattern is a mixture of co-current and counter-current flow.
- ✓ LMTD is obtained as

$$\Delta T_m = F(\Delta T_m)_{cf}$$

where  $(\Delta T_m)_{cf}$  is the LMTD for the clean fluid.

- ✓  $F$  is obtained from graphs for various shell-side and tube-side passes.

$$P = \frac{T_{t,out} - T_{t,in}}{T_{s,in} - T_{t,in}}$$
$$Z = \frac{T_{s,in} - T_{s,out}}{T_{t,out} - T_{t,in}}$$

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Now, there with this some kind of correction factor for the LMTD because the basic derivation was made based on only a single set of inner and outer tubes. But in practice we have seen earlier that there could be multiple passes in the heat exchangers. So, the definition of the LMTD will not be valid, we have to put some correction factor to the LMTD to account for the change in the configuration of the heat exchanger.

So, and we find that whenever we talk of the multiple passes; there is a combination of co current as well as counter current flow. So, what we do that LMTD is modified by

taking this factor F and then here this cf is the one which is with the clean fluid. So, this F is obtained from some graph for the various shell side and tube side passes.

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**LMTD correction factor**

- ✓ Used for multi-pass flow
- ✓ LMTD is obtained from the graph

where  $(\Delta T_m)_{cf}$  is the LMTD for the counter flow

- ✓  $F$  is obtained from the graph

$$P = \frac{T_{t,out} - T_{t,in}}{T_{s,in} - T_{t,in}}$$

$$Z = \frac{T_{s,in} - T_{s,out}}{T_{t,out} - T_{t,in}}$$

Source: TEMA

1 shell pass, 1 tube pass

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Now if we define these 2 parameters P and Z like this and then we find we have radius graph these are given in TEMA. In TEMA we find that these kind of graphs are given we may now here we find this value P is there and P we can see there is T t out and team these are the tube side the T represents tube side and s s represent shell side.

So, this temperature difference this is that this temperature difference and this temperature difference are coming into picture to define this P. So, first we find the value of P, locate the P here and then we find the value of Z and locate the Z from one of the curves or and then we find that from P and Z value wherever suppose we will get some P value.

And we go to some Z value and then we read the value of the F from the y axis. And this way we can make the correction for the LMTD it is small correction here that the cf here is not the clean fluid it is for the counter flow counter current flow. So, this cf will be taken to be the counter LMTD for the counter current flow.

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**LMTD correction factor**

- ✓ Used for multi-pass heat exchangers
- ✓ LMTD is obtained from the graph

where  $(\Delta T_m)_{cf}$  is the LMTD for cross flow

✓  $F$  is obtained from the graph

$$P = \frac{T_{t,out} - T_{t,in}}{T_{s,in} - T_{t,in}}$$

$$Z = \frac{T_{s,in} - T_{s,out}}{T_{t,out} - T_{t,in}}$$

the flow passes.

2 shell passes,  
4 tube passes

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Similarly, we have some other values of this L; this is for 2 shell pass and for tube passes.

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**LMTD correction factor**

- ✓ Used for cross flow heat exchangers
- ✓ LMTD is obtained from the graph

where  $(\Delta T_m)_{cf}$  is the LMTD for cross flow

✓  $F$  is obtained from the graph

$$P = \frac{T_{t,out} - T_{t,in}}{T_{s,in} - T_{t,in}}$$

$$Z = \frac{T_{s,in} - T_{s,out}}{T_{t,out} - T_{t,in}}$$

Cross flow

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And then we have for some cross flow. So, we find that these are some representative diagrams to find out the value of  $F$  for various configurations. There are many more configurations possible for which we have to locate the particular graph to find out the value of the  $F$ .

Sometimes these values of the F are also given in a tabular form. So, we find that we will find some tables also for this values F.

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**Rate equation: NTU-effectiveness method**

- ✓ Heat exchanger effectiveness ( $\epsilon$ )
 
$$\epsilon = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}}$$

$$0 \leq \epsilon \leq 1$$
- ✓  $\Delta T_{\max}$  for either of the fluids:  $T_{h,in} - T_{c,in}$
- ✓ Maximum possible heat transfer
 
$$Q_{\max} = C_{\min}(T_{h,in} - T_{c,in})$$

where  $C = \dot{m}c_p$ , capacity rate.  
 For hot fluid,  $C_h = \dot{m}_h c_{ph}$ .  
 For cold fluid,  $C_c = \dot{m}_c c_{pc}$

- ✓  $C_{\min} = \min(C_h, C_c)$  is used because the fluid with  $C_{\max}$  can accept  $Q_{\max}$ , but the fluid with  $C_{\min}$  cannot reject the same quantity of energy ( $Q_{\max}$ ) unless it cools down to a temperature that is not permissible by the Second Law of Thermodynamics.

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Now next we come to the, another method that is the NTU-effectiveness method. In this first, we define the heat exchanger effectiveness like this that is the actual heat transfer to the maximum possible heat transfer and we find that this epsilon value will always be lying between 0 and 1. Here we first define the maximum temperature difference between the 2 fluids and this is the  $T_{h,in} - T_{c,in}$  that is the inlet temperature the hot fluid and the is the temperature of the cold fluid. So, these are maximum driving force.

So, the maximum possible heat transfer is this  $C_{\min}$  into  $T_{h,in} - T_{c,in}$ ; now what is  $C$ ?  $C$  is the capacity rate that is the product of the mass flow rate and the specific heat. And what is  $T_{c,in}$  mean we shall see now; that we find first for hot fluid and cold fluid because they will be having they may be having different mass flow rates and also different specific heats. So, we find that for the hot fluid and a cold fluid; we will define this capacitated separately and whichever is the lower will be taken as the  $C_{\min}$ .

Now, why we are taking  $C_{\min}$ ? It means that where in this particular heat transfer we say that whatever energy is given out by the by the hot fluid should be taken by the cold fluid. Now, if it is not possible; that means, if the hot fluid is generating more energy then the cold fluid can take; that means, the external it will be dissipated.

Similarly, if the cold fluid is taking more energy than the hot fluid can give up then what is find that the it will be needing more amount of energy from also other surroundings or somewhere to go to the required temperature. And if we take C max; C max if you take in this particular equation what we find? It means that the fluid with the C max can accept Q max, but the fluid with C min cannot reject the same amount of energy that is Q max one is able to take, but another is not able to rejected.

So, there will be some imbalance between the heat exchanged between the 2 fluids. So, what will happen that if this is only possible that if the particular fluid cools down below a temperature that will not be permissible by the by the second law of thermodynamics. Because the maximum temperature the cold fluid can reach can be the outlet temperature of the hotter fluid. Similarly the lowest temperature the hot liquid can reach will be the high the outlet temperature of the cold fluid.

So, in that way we find that if this is being not being followed then we will not be able to have the proper heat transfer between the 2 fluids and that will be violating the second law of thermodynamics. So, from that angle we take this C min to find out the value of the maximum heat transfer possible.

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**Rate equation: NTU-effectiveness method**

- ✓ Thus
 
$$\epsilon = \frac{C_h(T_{h,in} - T_{h,out})}{C_{min}(T_{h,in} - T_{c,in})} = \frac{C_c(T_{c,out} - T_{c,in})}{C_{min}(T_{h,in} - T_{c,in})}$$
- ✓ Actual heat transfer
 
$$Q = \epsilon C_{min}(T_{h,in} - T_{c,in})$$
- ✓ Number of transfer units (NTU):
 
$$NTU = \frac{UA}{C_{min}}$$
  - Indicates the heat exchanger size
  - The larger the value of NTU, the closer the approach to thermodynamic limit by the heat exchanger.

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Now, once we know this now we find this epsilon in these two terms if the hot fluid is determining the C min; then this is the way we write it and if the cold fluid is doing it then we write it in this fashion. So, here is a small correction that it will be for the cold

fluid. So, the actual heat transfer is calculated by taking this equation that we with the maximum possible heat transfer, we multiply with the effectiveness factor to get the actual heat transfer and now with these definitions we define the NTU like this.

So, this is the ratio of the UA to the C min and what it means and this particular these ratios can be also derived from theory. And this NTU determines the heat exchanger size and the larger the value of the NTU, the closer is the approach to the thermodynamic limit of the heat exchanger.

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**Rate equation: NTU-effectiveness method**

$r \equiv C_{\min}/C_{\max}$      $NTU \equiv UA/C_{\min}$

Exchanger Type	Effectiveness Equation
Counter flow	$\epsilon = \frac{1 - \exp[-NTU(1-r)]}{1 - r \exp[-NTU(1-r)]}$ ( $r < 1$ )
	$\epsilon = \frac{NTU}{1 + NTU}$ ( $r = 1$ )
Parallel flow	$\epsilon = \frac{1 - \exp[-NTU(1+r)]}{1+r}$
1-2	$\epsilon = 2 \left[ 1 + r + \beta \left( \frac{1 + \exp[-\beta \times NTU]}{1 - \exp[-\beta \times NTU]} \right) \right]^{-1}$ where $\beta = \sqrt{1+r^2}$
N-2N	$\epsilon = \left[ \left( \frac{1 - \epsilon^* r}{1 - \epsilon^*} \right)^N - 1 \right] \times \left[ \left( \frac{1 - \epsilon^* r}{1 - \epsilon^*} \right)^N - r \right]^{-1}$ ( $r < 1$ )
(For a 2-4 exchanger, $N = 2$ ; etc.)	$\epsilon = \frac{N\epsilon^*}{1 + (N-1)\epsilon^*}$ ( $r = 1$ )

where  $\epsilon^*$  is the effectiveness for a 1-2 exchanger with the same value of  $r$  but with  $(1/N)$  times the NTU value

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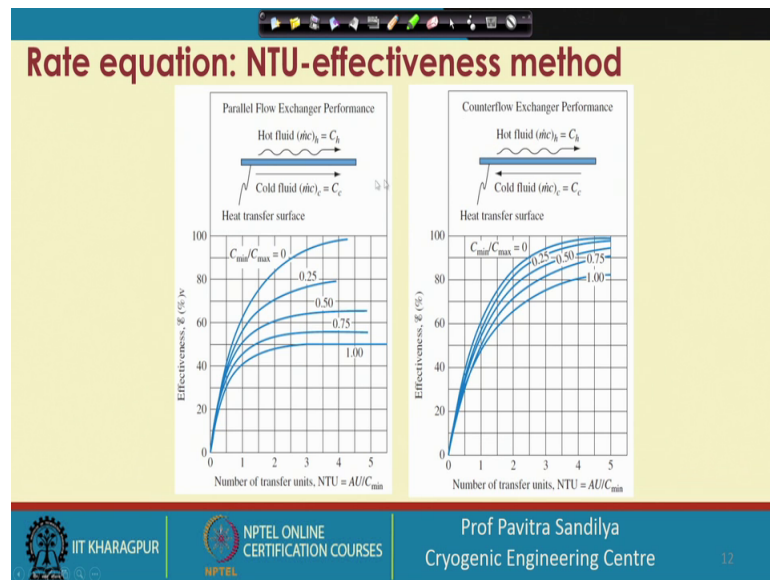
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Now, you find that this table gives some relationship between the epsilon and the NTU ok. So, and here we are defining this value of the r as C min by C max and NTU as UA by C min.

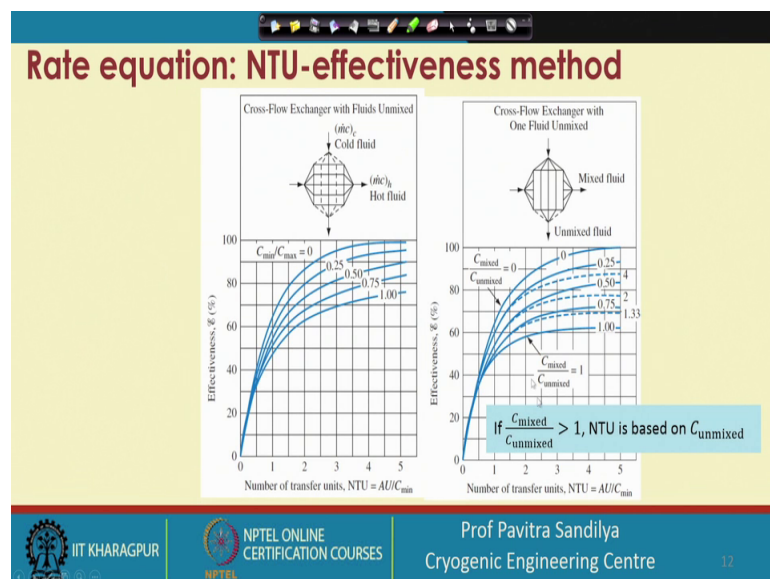


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And here in this particular table we have the graphical representation for say parallel flow, then for counter current flow for the effectiveness and the NTU for the various values of  $C_{min}$  and  $C_{max}$ .

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So, these particular figures may also be used to find out the value of the effectiveness factor. So, this is for the cross flow, this is for the unmixed and this is for the mixed cross flow. And in this in this particular unmixed and mixed combination we find that if  $C_{mixed}$  by  $C_{unmixed}$  is more than 1, then the NTU is based on the unmixed  $C$ .



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### NTU as a function of effectiveness

Configuration	Relationship
Counterflow For $C_R = 1$	$NTU = \frac{1}{1-C_R} \ln \left( \frac{1-C_R \epsilon}{1-\epsilon} \right)$ $NTU = \frac{\epsilon}{1-\epsilon}$
Parallel flow	$NTU = \frac{1}{1+C_R} \ln \left[ \frac{1}{1-(1+C_R)\epsilon} \right]$
Cross flow $C_{min}$ mixed; $C_{max}$ unmixed	$NTU = \frac{1}{C_R} \ln \left[ \frac{1}{1-C_R \ln \left( \frac{1}{1-\epsilon} \right)} \right]$
$C_{min}$ mixed; $C_{max}$ unmixed	$NTU = \ln \left[ \frac{1}{1 - \left( \frac{1}{C_R} \right) \ln \left( \frac{1}{1-C_R \epsilon} \right)} \right]$
Shell and tube (1 shell pass, 2 tube passes)	$NTU = \frac{1}{\sqrt{1+C_R^2}} \ln \left[ \frac{2 - \epsilon(1+C_R - \sqrt{1+C_R^2})}{2 - \epsilon(1+C_R + \sqrt{1+C_R^2})} \right]$
All exchangers with $C_R = 0$	$NTU = \ln \left( \frac{1}{1-\epsilon} \right)$

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Now, in this particular table we give the NTU in terms of the epsilon. So, the order table was giving the epsilon in terms of the NTU and this table gives you the value of the NTU in terms of the epsilon; that means, we can find out the value of NTU and epsilon from either of the values; either from these analytical expressions given in this particular table or from the graphs. So, any of these things can be used to find out the value of the NTU is NTU and the epsilon.

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### Heat transfer correlations for incompressible flow of liquids and gases through pipes and tubes

✓ **Dittus-Boelter correlation**

$$\frac{hD}{k} = 0.023 Re^{0.8} Pr^n$$

$n = \begin{cases} 0.4 & \text{for heating } (T_s > T_b) \\ 0.3 & \text{for cooling } (T_s < T_b) \end{cases}$

$6000 < Re < 10^7$ ,  $0.5 < Pr < 120$ ,  $(L/D) > 60$   
 All properties are evaluated at the bulk fluid temperature ( $T_b$ ) except where noted.

✓ **Sieder-Tate correlation**

Applicable for significant property-variations due to ( $T_s - T_b$ )

$$\frac{hD}{k} = 0.027 Re^{0.8} Pr^{1/3} \left( \frac{\mu_b}{\mu_s} \right)$$

All properties are evaluated at the bulk fluid temperature ( $T_b$ ) except  $\mu_s$  which is evaluated at surface temperature ( $T_s$ ).  
 $6000 < Re < 10^7$ ,  $0.7 < Pr < 10000$ ,  $(L/D) > 60$

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Now we have found that in determining the overall heat transfer coefficient, we need the heat transfer coefficient individually for both the hot fluid and a cold fluid. And because these fluids are passing through the tube tubes; so, we need some heat transfer correlations to find out this heat transfer coefficients. So, here I have shown some representative heat transfer coefficients and please note there are many more correlations proposed in the literature for the evaluation of the heat transfer coefficient.

Here we have found given some common ones that is first is details about your equation. And this is given in terms of results number that is the ratio of the product of the heat transfer coefficient and some representative length. And in this case the length is the diameter of the tube and  $k$  is the thermal conductivity of the fluid and this is in given in terms of the Reynolds number and the Prandtl number.

And here we find this value of the  $n$  which is there for the power of the Prandtl number is 0.4 for heating that is when the surface temperature is more than the bulk temperature of the fluid and it is 0.3 for cooling that is when the surface is colder than the bulk fluid. And this particular correlation is valid for this range of Reynolds number this range of Prandtl number and this range of the  $L$  by  $D$  ratio that is kind of aspect ratio for the particular pipe. And the all the properties are evaluated at the bulk fluid temperatures that is  $T_b$ .

Now, in the Sieder Tate correlation where we this is another correlation which is also quite applicable, but this correlation is different from Dittus-Boelter because it takes care of some property variations with respect to this temperature difference between the surface and the bulk fluid.

So, this is the correlation that Nusselt number is given in terms of this Reynolds number Prandtl number this almost the same except that here this another correction factor which has been used to account for the difference between the bulk temperature and the surface temperature in terms of the viscosity; that is  $\mu_b$  is the viscosity at the bulk fluid temperature and  $\mu_s$  is the viscosity at the surface temperature. And this the correlation is valid for this range of Reynolds number, for this range of Prandtl number and this the aspect ratio.

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**Heat transfer correlations for incompressible flow of liquids and gases through pipes and tubes**

✓ **Gnielinski correlation**  
Accounts for both variable properties and entrance effect in both transition and fully developed flow regions

$$\frac{hD}{k} = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)} \left[ 1 + (D/L)^{2/3} \right] K$$
$$K = \begin{cases} (Pr_b/Pr_s)^{0.11} & \text{for liquids} \\ (T_b/T_s)^{0.45} & \text{for gases} \end{cases}$$
$$\text{Friction factor, } f = \frac{1}{(1.82 \log Re - 1.64)^2}$$

$2300 \leq Re \leq 5 \times 10^6$ ,  $0.5 < Pr < 200$ ,

*In all correlations, for non-circular ducts, hydraulic diameter should be used.*

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Next we come to another correlation that is Gnielinski correlation and this is this correlation is taking care of the variable properties. And the entrance effect in both transition and fully developed flow regions. As we know that whenever a fluid entering pipeline or tube, it takes some time for the fluid to have a fully developed flow. So, initially there is an entrance region where the flow is developing and then it goes to a developed flow region. So, in these 2 regions the heat transfer characteristics are different. So, this particular correlation accounts for both these regions and this is the kind of correlation proposed.

In this correlation we again find bit of Reynolds number, Prandtl number this D by L ratio and extra is that we have the friction factor and another parameter K. So, this K is given in terms of the ratio of the Prandtl number to the power 0.11 for liquids and ratio of the temperatures of the bulk fluid and the surface 0.45 for gases. And the friction factor is obtained from this particular equation for this correlation. So, this particular correlation is valid within this range of Reynolds number and within this range of the Prandtl number and so, far we have seen all the pipes are taken to be circular. But in case we have non circular pipes we have to use the hydraulic diameter.

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**References**

- Barron R F , Cryogenic Heat Transfer, Taylor & Francis, 1999.
- Serth R W, Lestina T G, Process Heat Transfer – Principles, Applications and Rules of Thumb, Elsevier, 2<sup>nd</sup> Ed., 2014.
- Kreith F, Manglik R M, Bohn M S, Principles of Heat Transfer, Cengage Learning, 7<sup>th</sup> Ed., 2011.

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And these are the references which can give you that for more details about these topics. Also you can refer to any book on heat transfer and heat exchanger.

Thank you.