Introduction to Non-Linear Optics and its Applications
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Lecture – 08
Basic Linear Optics (Contd.)

So, welcome student to the next class of Introduction to Non-Linear Optics and its Application. So, in the previous class we have learned about the polarization and something related to susceptibility. So, we will continue with that because that is important part in non-linear optics and the knowledge of susceptibility is important. So, let us check what we have in today's lecture.

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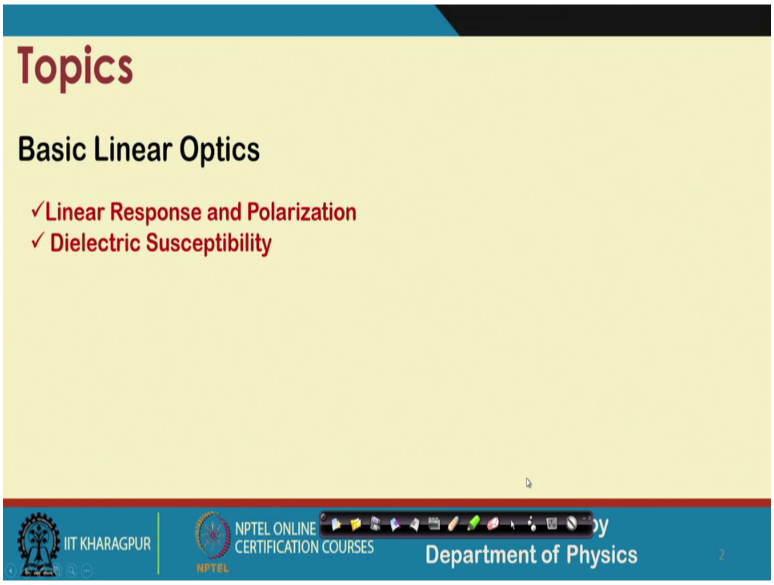

So, linear response and polarization that is the main part we will going to cover today and also dielectric susceptibility. What is dialysis susceptibility, we will learn basics of that and how it effects the non-linearity we will going to learn in the future classes. So, that is why the initial concept of the susceptibility is important.

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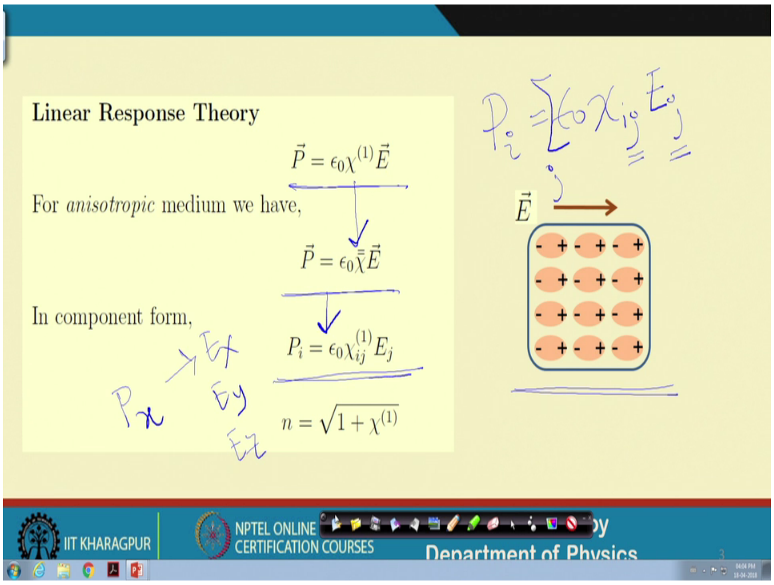

So, first this is the old slide I am using. This is related to linear response theory. So, let me write this equation. So, this equation let me mark this equation rather that P is equal to epsilon 0 chi 1 E, that means, P here is the polarization and this polarization is now related to electric field and from this equation we can see that is the vector equation and if E is a vector P has to be a vector and this is a very simple case where the direction of the electric field and the direction of the polarization both are in same direction. So, they are same; that means, these vectors component P x is directly related to component of E x, P y is directly related to component to E y and P z is also directly related to the component of E z.

But this is not the case we know that in anisotropic system in the next equation we can see that the relationship is slightly different and in state of writing chi we write chi bar. So, we also mention these things in our earlier classes that this is related to some kind of tensor thing. So, tensor property are there, so, that means, the P x is now not depend on the component of the E x rather it will also depends on E y and E z component. In the right hand side we have a schematic figure if I applied the electric field E, then we find that the dipoles will be arranged, ok.

Before going to that let me write this component form also let me explain that as I mentioned from this to this P is now related to different component of E that means, P x is now related to E x, E y and E z.

If this is the case then one can write this in a component form. So, that is why this component form is important. So, P i-th component is now related to this quantity and then E j and it is over a summation this summation is over j because we know that j is the repetitive indices and we know that if there is repetitive indices then the sum is over that, this is the Einstein's notation. So, we have already discussed these things in our earlier classes ok. So, if I launch an electric field so, what happen inside the material is important.

So, right now we are discussing the system in simple isotropic medium; that means, there is no dependency of P x with other component of E y and E z rather both are both P and E are in same direction. So, if I launch an electric field what happened, the dipoles will be arrange in this fashion. So, what are the dipoles we will going to see in the next slide. So, let me go back to the next slide I think it will be easier then yeah we have the figure here in the slide also.

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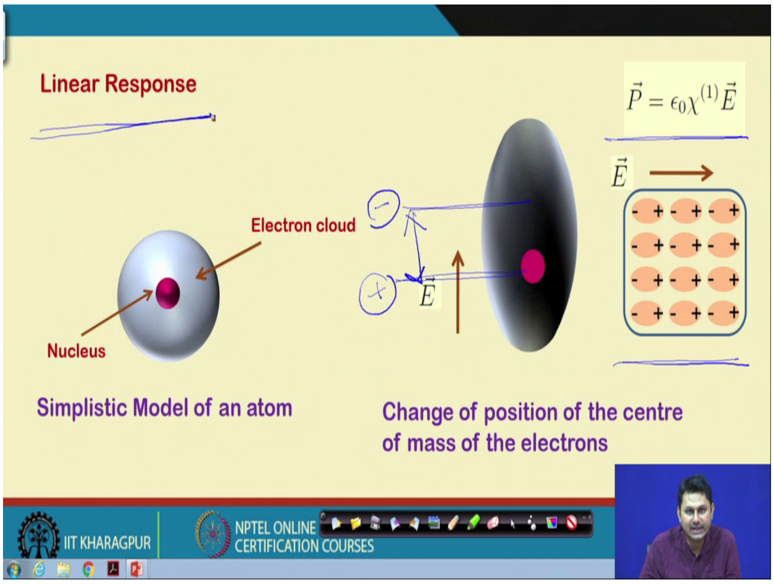

So, in linear response what happened that if I launch an electric field without any electric field this is the structure without any electric field we have electron cloud we have electron cloud here and a nucleus here. So, in electron clouds are surrounded the nucleus is surrounded by the electron cloud. This is the simplistic model of atom we know.

So, the question is what happened to this system when we will launch an external electric field in the next figure we can see that when I launch an electric field this electron cloud is some sort of distorted. Now, it is not a sphere shape some kind of elliptical shape is there this is just schematic figure to so, how these things can happen and then what happened that if I launch an electric field or if electric field are there then there will be then there will be some kind of shift of this thing. So, this animation if this animation works I will like to show the animation once again that when the electric field are there, there is a shift of this electrons cloud.

So, once this electron cloud is shifting once again if I see there is a shifting of electron cloud once the electron cloud is shifting from this nucleus which is defined by the red dot hear what happened thus there is a separation between the charge. So, the charge will be separated because the centre of mass of the electrons are now moved in a different position somewhere here if I want to find out say nucleus is sitting here nucleus is sitting here, somewhere here and the electrons are shifting in upward sides. So, there is a difference between the charges. So, this is minus charge and this is plus charge. So, there is a charge separation and because of this charge separation what happened that we will get some kind of dielectrics.

So, these are the schematic figure in the right hand side what happened to entire system that means, in time material that tiny dipoles will be aligned when the electric field is launched and we know that this polarization is nothing, but dipole moment per unit volume. So, number of dipole per unit volume will give you the value of the polarization and this polarization is now depend on the electric field with this law or this equation and we from this equation we can see that the polarization P is now proportional to electric field E.

So, now, we try to find out what is this linear response because here we find in the beginning we write something called linear response what is response and why it is called linear let us try to find out and after understanding the situation that if I launch an electric field the dipole will be align and will have some kind of polarization. So, now try to understand in a more detailed way and more in mathematical way that what is going on.

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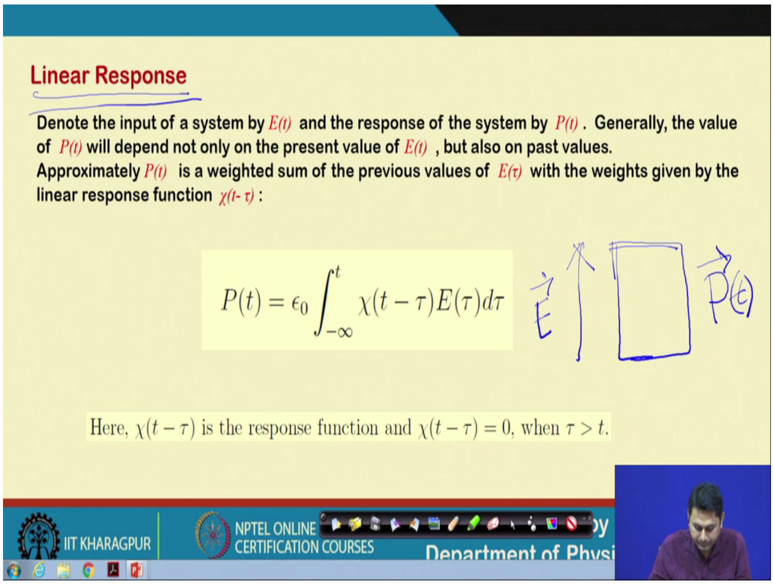

So, here what happened that if I write this polarization P. So, this polarization let me understand or let us understand what is this linear response first. So, when we launch an electric field the material will going to response that is the first thing that I applied an electric field, I applied an electric field and as a result if this is a material the material will response. So, here the response is in terms of polarization. So, I launch an electric field this is my cause is a vector and as a result I am getting something which is the polarization this is the effect of that.

So, launching electric field is a cause and I am getting some kind of polarization inside the system of the material that is the effect if I understand the matter in this way then we can see whatever is written here that input of the system here is electric field. So, that means, I am launching something into the system that is a input and the response of the system is P as I mentioned this is a polarization. Generally, the value of this polarization will depend not only the present value of E t, but also the past value this is a very important statement and what is the meaning of that we will going to understand mathematically actually.

So, if I want to find out what is the polarization at time t that does not means that this polarization is appearing because of the launching of the electric field at time t rather I launch the electric field before and there is a effect after that I am getting the polarization.

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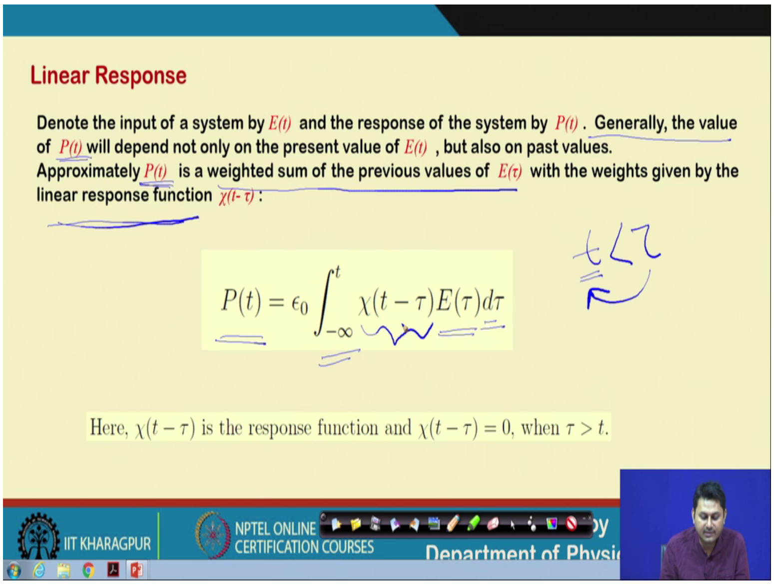

So, what happened here if I look back to our slides, then P t which is the effect of the system. Here P t means the polarization is the weighted sum of the previous value of E tau. So, all the previous value t less than tau, t is a present time and tau is a time that is before some t and throughout the time if I write tau to t, then this is the time span and throughout that time span whatever the electric field is there, there will be a sum of all the electric field to get my final polarization at time t.

So, that means, E t is something here E tau is something here, then I will add up all the E tau over dT the d T is a time span and this is from minus infinity to t, t is present time and minus infinity is the minus infinity time long back is the general formalism and then there is a term here this is chi t minus tau; this is the weightage factor or the response the response factor these factor basically gives you what should be the response what could what should be the response of the system that means, I integrate these things over that and as a result I am getting something in terms of P here this is the polarization.

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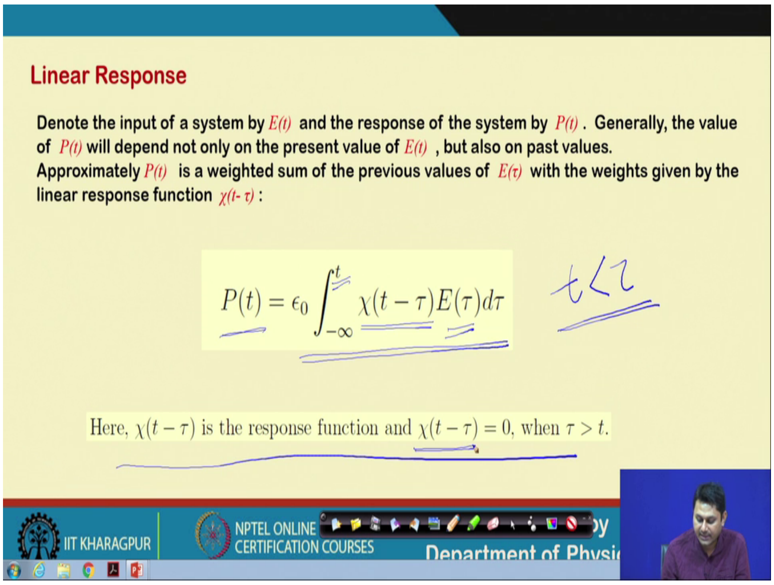

So, I am getting the polarization after integrating the electric field over the entire time and this time span is t less than tau this is the condition the tau is some before time and I integrate it over and when I integrate it over I will have I must have some weight factor or linear response function here.

So, why it is linear because P and E both order of one there is a possibility that P can be depending on E and also it can depends on the E square then the situation is slightly different and complicated and in that case this kind of response we called the non-linear response. So, we will come back to this point when we start our exact non-linear optics problem, but here in the linear optics we need to learn about this response function. So, what do we understand here? So, P t is a response, E t is a cause, E tau is a cause and I want to find out because of the launching of electric field at time tau what is a polarization at time present time t.

If I want to find out it will be the total effect that is why I need to integrate it over minus infinity to t; mind it t is the upper limit because I cannot go over t that is why we write a statement hear that my response function xi t minus tau has to be 0 when tau is greater than t; that means, I should not have any kind of response before applying my electric field. So, that is natural is a causality effect.

So, because of this causality effect what happened that my chi is 0, when the argument is negative. So, negative argument of the chi is not valid here it should be 0, if that is the case then in particular this system what happened I will have the response before apply any kind of electric field. So, polarization will have into the system before applying the electric field which is not possible.

Well, after having a knowledge of this linear response this is as very special kind of form we have P t is integrate over chi t minus tau then E t tau.

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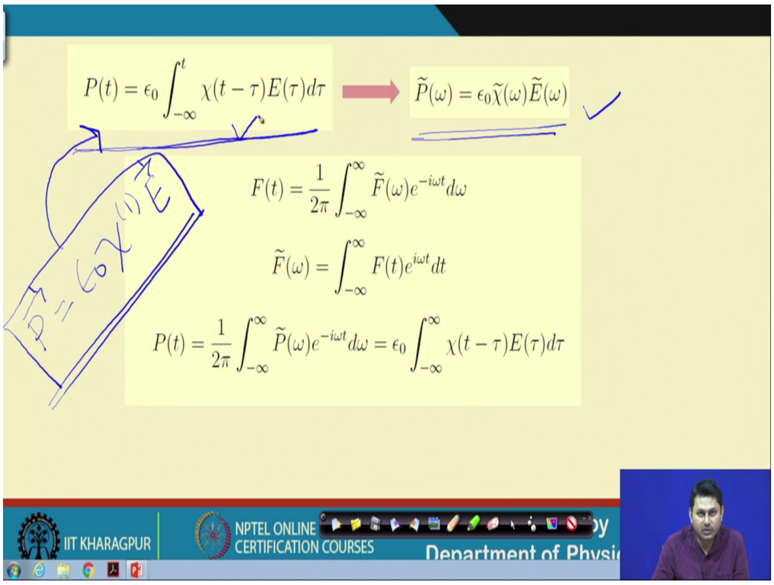

We know that this is some sort of convolution thing and I believe those who are taking this course they are aware of this convolution, but any way I will do this things will in detail way, so that we can understand that in time domain this is my equation. This is my equation in time domain. So far we are writing the expression like P is equal to epsilon 0 chi 1 E that was my expression all the time if I put the vectors sign fine let me put the vector sign, but I never mention about what is the is the function. P is a function of t; E is a function of t. If that is the case we can we should write this equation in this form because this is the correct form to represent the relationship between the polarization and applied electric field when they are in time domain.

So, there are two kind of domain I should mention here one is time domain and another is frequency domain. So, that means, when we try to find out what is happening in time then this should be the equation one can write, but also there is a frequency domain expression of P and E which is relatively easier and this expression is nothing, but whatever is written here in this place. So, whatever I am writing hear that P is equal to epsilon 0 chi one E is nothing, but this expression, but only thing you should mention that P tilde and E tilde and these chi tilde; that means, all these components are the Fourier components of P, E and chi.

So, that means, whatever the equation we have here in time domain I can rewrite the same equation in the frequency domain and in the frequency domain the expression is relatively simpler because they are just really relate with the multiplication, but the multiplication quantity are the Fourier components or the Fourier transform of P, E and tau whatever we have in time domain. So, we can prove that we can prove that very easily. So, let me try to find out how to prove.

So, first thing is what is Fourier because they are related to Fourier components of first thing here in this two equation I write if F is a function in time domain how it is related to it is Fourier components. So, this F omega is a Fourier transform or the Fourier omega in frequency domain this is the function. So, F t is 1 divided by 2 pi integration of minus infinity to infinity these E to the power I this is the representation of F in terms of it is Fourier component.

I can return back to F, F tilde omega and if I do then I will just reverses this is the simple Fourier transform I am I am doing and when we do the Fourier transform these are the relation now I will going to apply these to our function P. So, my function P is here and I can write this P t in terms of it is Fourier component because at the end of the day I need I need to go from here to here as I see that.

So, P t is a function and time domain which is the polarization now I write in the Fourier component. It is exactly like same the example is already given in the upper two lines and then what happened I write P t in terms of electric field and chi they are the this is the relation this is the relation; that means, this relation I am using here. So, in the left hand side I have something with Fourier component, in the right hand side I have writing something in time component.

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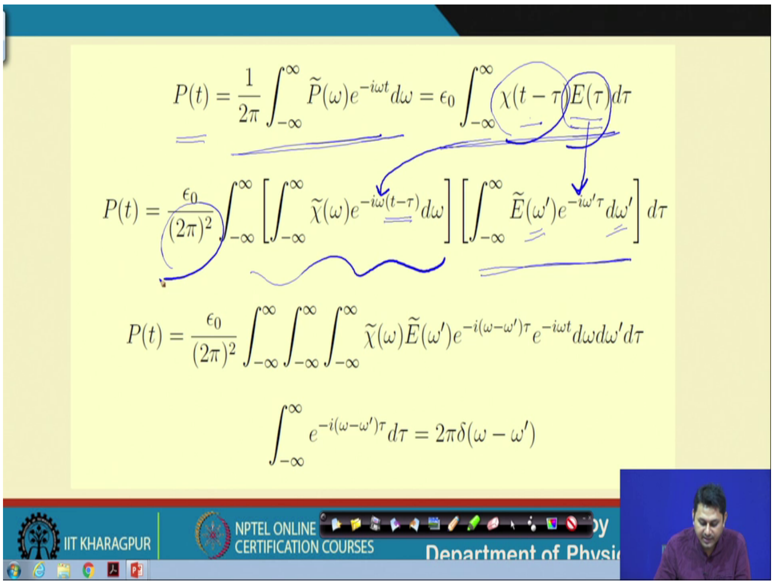

So, next I will go further I will go further. So, P t is this and which is equivalent to this quantity this is our last equation. So, next what we will do we will make a Fourier transform of these and this quantity. So, if I do then what happened for these case I have a Fourier transform and for these case E tau I have another furrier transform. So, this is the Fourier transform of the chi t minus tau. So, chi t minus tau if I make the Fourier transform then I should have chi tilde omega this should be the Fourier component then E to the power I omega and whatever the argument here t minus tau is the argument I should write t minus tau here it should be very careful one should be very careful to write because I am not making tau chi t the Fourier transform of that rather I am making chi t minus tau. So, this t minus tau value should be here. So, I am making one Fourier transform.

Second Fourier transform, that means, for E t, I will do the same thing only one thing you should known that in state of using omega I am writing omega prime and I can do that I can do that because omega is a dummy indices. So, you know integrate it your integrating over omega. So, it does not matter whether you are writing omega or omega prime, but it is essential that when you have two multiplication kind of thing inside summation, if you use one indices omega then the other indices should be use in a different one; that means, omega prime.

So, when we do that so, Fourier transform of these and Fourier transform of these I just put the Fourier transform and Fourier transform component here and here the entire thing is same and because of this two Fourier transform I have 1 by 2 pi term from both the cases. That is why 1 by 2 pi square term is coming square because of the two cases I have 1 by 2 pi multiplied by 1 by 2 pi.

The next case what we have done? In the next case in the next step so, I will I have taken all this integration in one side. So, the all this integration is one side and write this term this term exponential term all this term inside this all integration and then d omega, d omega prime and d tau in the last. So, after doing that I have one important term that is coming now and that is E to the power of minus i omega minus omega prime tau if you do that you will find it is come naturally here you have E to the power i omega t minus tau here we have E to the power minus i omega prime tau. So, if I write these two things then I will have something like this and once I have these two things then I can write this quantity in terms of delta function.

So, this is this is the definition of the delta function that if I write delta function in terms of one can write delta function in a different way, but this is one way to write the delta function in integration minus infinity to infinity to the power minus i omega minus omega prime tau d tau is nothing, but 2 pi of delta omega minus omega 0. So, we will going to use this delta function; that means, when omega is equal to omega 0 then we will have some kind of weightage here over the integration and in other case it will not be there, ok. So, let us see what happened in the next slide, fine.

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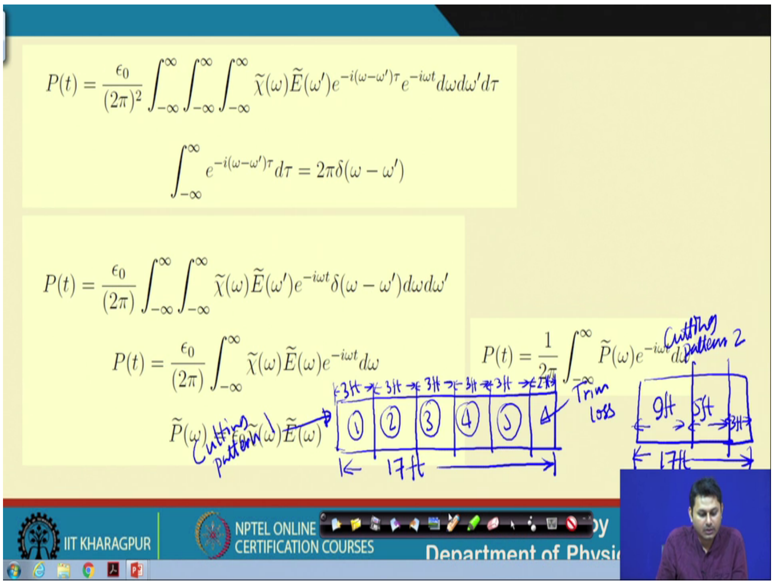

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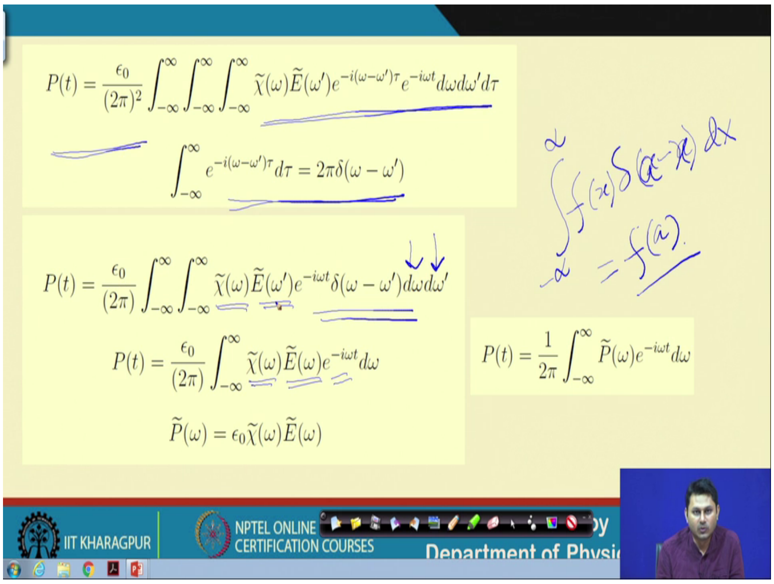

So, so, I have this quantity P t is equal to epsilon 0 divided by 2 pi integration of these. And, then I replace I will going to replace the c to the power minus I omega minus omega prime tau in the form of delta function. When I do that when I write this delta function and then integrate it over then what happened that one integration will gone and this d tau integration is gone because in place of d tau integration I put 2 pi omega minus omega, omega minus omega prime.

So, only two integration we have one is over d tau d omega and another is over d omega prime d tau omega d d tau integration is gone because I am using this delta function. Now, if I integrate these over say d tau prime. So, this delta function make the system so, only when omega is equal to omega prime we know that if I have some kind of function here as a function of x.

And, there is delta function say x minus a and if I integrate it over say m say a minus x and if I integrate it over d x minus infinity to infinity then I will eventually have function of a here we are using the same principal that when we these things all the omega will be omega prime will be replaced by omega and this integration will gone if I do that then will find this term is not depend on omega prime. So, this term will remain same E tilde was depending on omega prime. So, it will be this one and because of this thing I will have intact whatever we have. So, E omega prime will remain E omega, after integration and because of the presence of delta function.

So, finally, what we have P t. So, this is interesting. So, finally, we have P t is equal to this quantity if you remember here it is written that P t was already defined in terms of it is Fourier component and that is 1 by 2 pi integration of minus infinity to infinity P tilde omega and this quantity. So, if I compare these two things then we can find that these quantity should be to this quantity multiplied by epsilon 0 other terms will be cancel out. So, this is the form final form I am getting and this form suggest that in frequency domain the relationship between E that means, electric field and the polarization P will be linear. So, they are related to a linear relation.

Well, so, after having a knowledge of these all these Fourier transform I mean all response function and what should be the value of these thing in time domain and frequency domain we have a idea that the susceptibility chi is now function of omega that is important; that means, if I change the frequency of the electric field the response of the system may change and what should be the response that will going to see in the next slide.

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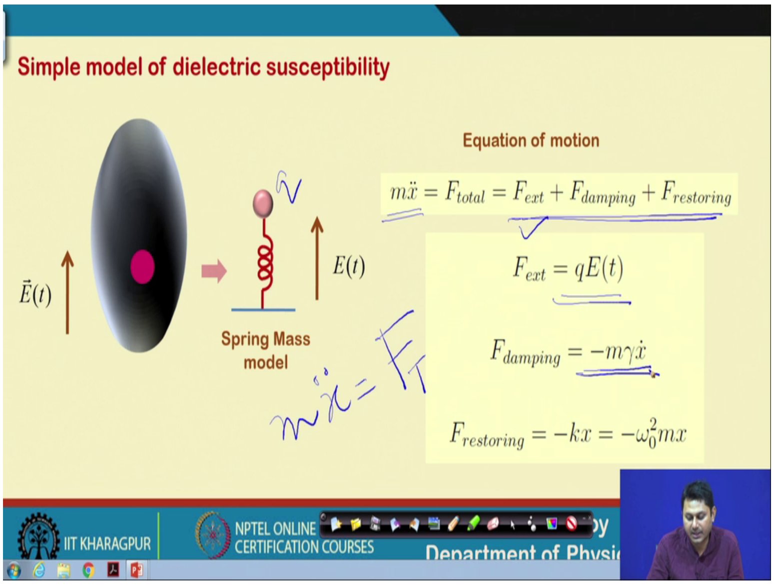

So, we will have a simple model here to find out the susceptibility of the system. We will have a very simple model and the simple model suggest that if I apply the electric field here so, what happened that there is a shift of the entire electron cloud in one atom. So, what happened? So, go back, so, I will launch an electric field. So, what happened things shifting; that means, the electron clouds are shifting. These things can be assumed as a spring mass model simple spring mass model as if I am launching some kind of electric field means I am launching or I am putting some kind of force in a spring mass system and what happened the spring will now vibrate.

Why it is vibrate? Because they launched electric field now may have some kind of frequency components. So, the launch electric field this launch electric field may have some kind of frequency component. If it has some frequency component, so, what happened that, it will make these spring mass vibrate along some kind of equilibrium. So, this is equilibrium position and what happen it will start vibrating.

So, this is simple spring mass model. So, what we are trying to do we try to find out a spring mass model system we try to impose a simple spring mass system over a system over this electron models. So, electron models are there. So, I launch an electric field over atom and what happened electron cloud it shift will be shifting and we make these things equivalent to a spring mass model and now try to find out what are the governing equation of the motion. So, equation of the motion is something where I launch an electric field.

So, the total so, total force should be equal to m F. So, m F is here x dot is equal to F total. If I write F t F total so, this is the simple equation this is second I means this is the Newtonian if I use the Newtonian mechanics this is the second law of motion. So, this law of motion suggest if you applied some kind of force over that so, P multiplied by the acceleration will be equal to the force. So, if this is the case then my x is a position of the system mass m is the mass of the electron and total force can be divided into three different part; one is external force, that means, what I am applying, what kind of force I am applying from the outside here I am applying the electric field if E is a electric field and q is a charge here then q multiplied electric field should be the force that is excited the system it should be the mass should experience this external force.

Then, apart from that there should be two independent forces one is damping force the negative sign suggest that this is damping. This damping force basically because of the damping coefficient gamma. So, if the system has some sort of damping then it will try to die out these constant oscillation and that is why it will give some kind of force and these kind of force is called damping force. So, damping force will be simply minus m multiplied by gamma into x dot and we know that damping force is always velocity dependent. So, that is why x dot term will be there because it is velocity dependent term and finally, I have some kind of restoring force.

If I launch the electric field what happened it will it will change the location of the electron, but because of these restoring force this electron try to come back to it is original position and because of these things some kind of force should be there and this force is coming through the spring constant, if I consider this is a spring mass system if I launch if I give some kind of force over this mass. So, mass will go up and then this spring constant try to make this mass to it is original position and as a result we will have some kind of restoring force. So, this restoring force is nothing, but minus k into x simply it will be a distance dependent force if I if x is high then the restoring force is high this is some sort of Hooke's law.

And, then if I try to write this k in terms of the resonance frequency omega 0 it should be simply omega 0 square m multiplied by x because we know that omega 0 is a resonance frequency.

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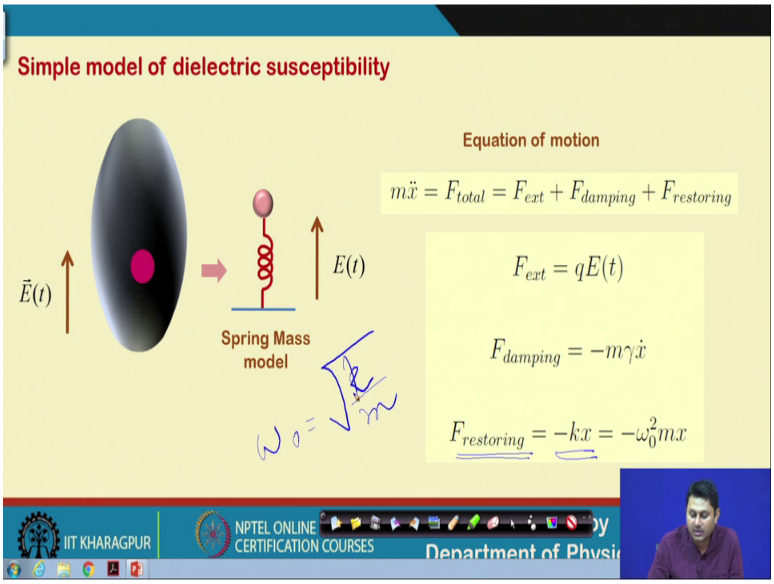

It is nothing, but root over of k by m in the spring mass system, where m is a mass of the system and the value k is a spring constant. So, spring constant divided by mass and route over of that thing is nothing, but the resonance frequency we have.

So, this is the entire equation so, this equation one can solve very easily.

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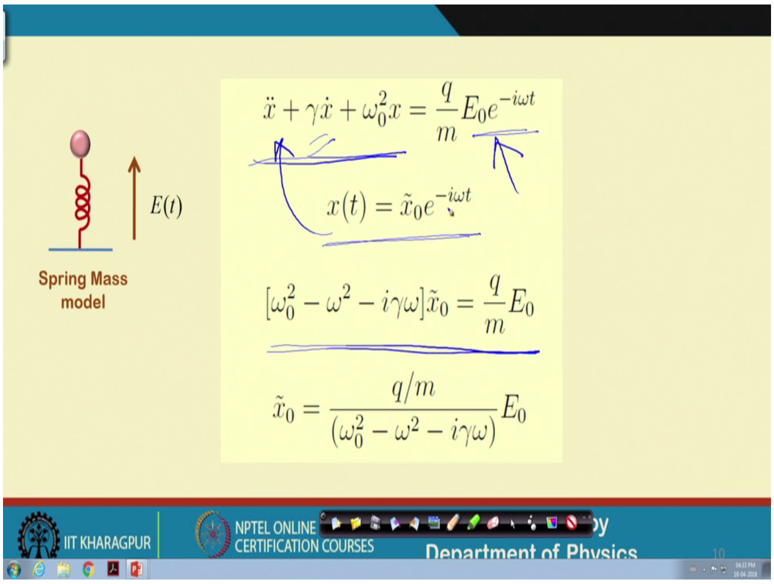

So, if I go back to the next slide we will quickly use this Newtonian equation. So, this is the equation we have. So, now, my launching electric field has a form E 0 e to the power minus omega t and because of these frequency dependent term in the force we have my position also frequency dependent and I can write this position or the x the distance in the same form that of the applied electric fields. So, this is a force damped system simply a forced damped system we have some kind of force in the right hand side and this is a damp coefficient and I divided everything with respect to m, I will have this. If I put the solution here so, this is the solution I am looking for.

So, this solution if I put here I will have the equation like this E to the power I minus omega t E to the power i minus omega t both the case from both the side it will cancel out and eventually I will have on the pick distance x 0 it is q divided by m omega 0 square omega square minus i omega. When you put that is make a derivative we will have one omega 0 and if you make two derivative you will have omega square. So, this is a very simple equation and you will get these things.

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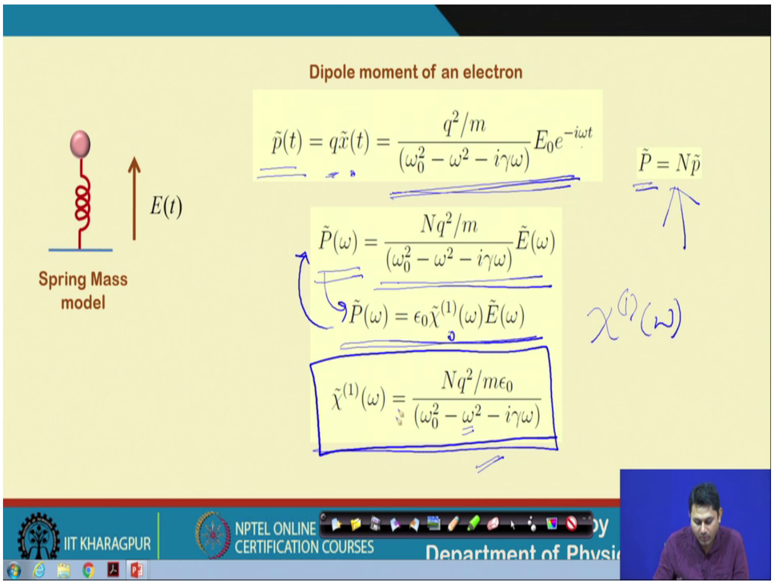

After that what happened that my I want to find out what is my susceptibility. So, here what we will, I will try to find out what is my polarization or the dipole moment of an electron. So, dipole moment of an electron is q charge multiplied by the distance because of the applied electric field what happened this things will now change so, there is a distance or there is a separation this separation I figure out in the last line in the previous slide and if I multiplied the entacting with q I will have my polarization or the dipole moment of electron like this.

So, polarization is now dipole moment number of dipole moment per unit volume. So, if N is a number of dipoles per in it volume if I multiplied with that is an if this is a number and if I multiplied these with the p, then the big P will be the dipole moment per unit volume. So, dipole moment per unit volume, if I multiply the number of dipole per unit volume N with the individual dipole moment p then I have the total dipole total polarization which is dipole moment per unit volume. So, now, P is a function of frequency as I mentioned because this omega is sitting here. So, this is the form because P is multiplied by N into p.

So, N into this quantity I will have and again in the previous calculation I find P and E omega is related to this where we have our function chi i omega sitting here if I compare these two things I will finally, have an expression of finally, I will have expression of the susceptibility. So, this is the very very simple model to find out the susceptibility and we find that now the supportability is a not a real quantity rather this is a imaginary quantity. So, in the next class we will try to find out what is the consequence of the susceptibility if it is an imaginary part is associated with that and omega 0 is a resonance frequency.

So, we can find out that if omega is omega 0 then this entire quantity become P only complex so, when a P only imaginary. So, when this things P only imaginary some kind of absorption will be there and if omega is far away from omega 0 and if I have only the real term of that so, that gives is to the refractive index of the system. So, we will come back to these things in our next class.

So, so far we learned that susceptibility is a function of omega and how the susceptibility can be modelled by simply this spring mass system inside the inside a material we find that. So, the dependence of the susceptibility with respect to the frequency omega is there, we find out the expression. In the next class we start from this expression and try to find out more of the susceptibility this is important because susceptibility play an very important role in non-linear optics and we find how these frequency dependence susceptibility will give rise different kind of frequency when the responses is non-linear.

With this note let me conclude this course, this class. So, see you in the next class and thank you very much for your attention.