Introduction to Non-Linear Optics and its Applications
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Lecture – 07
Basic Linear Optics (Contd.)

So, welcome student to the next class of Introduction to Non- Linear Optics and its Applications. So, in the previous class we have learned the light propagation in the anisotropic system. So, we will continue that because is a very important thing that you need to learn in the context of non-linear optics, because we find that it will going to help us to understand how different kind of phase matching will appear in this kind of crystal where anisotropy is one of the fundamental nature? And by using this fundamental nature how we can use the phase matching. So, that is why the learning of these anisotropic system is very important.

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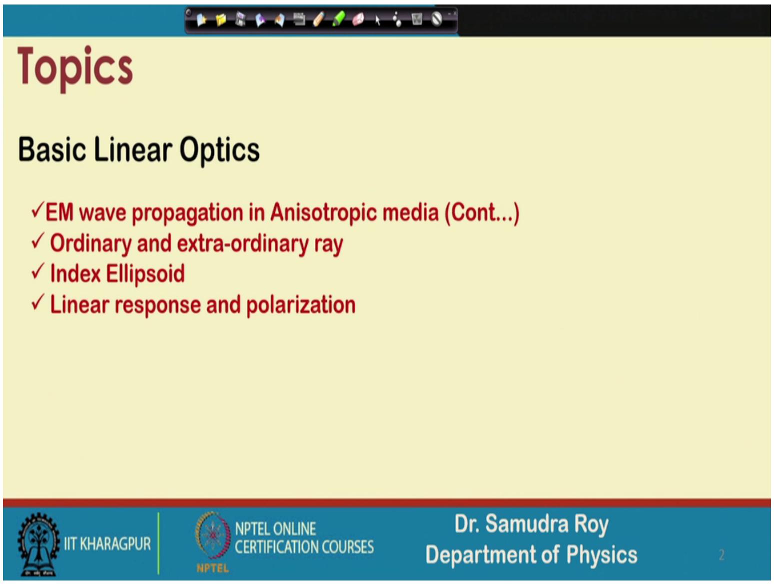

So, let us go back to our topic today. So, today we will going to learn as usual electromagnetic wave propagation in anisotropic media this is a subject. This is a topic that we are covering for last couple of classes or more than that. Then ordinary extraordinary wave or ordinary extraordinary ray that also we have discussed in the last class then index surface we have discussed. So, today we will going to understand what is index ellipsoid and try to calculate how one can generate this index ellipsoid and finally, the linear response and polarization. So, it is very important the polarization because the nonlinearity will be appearing; if we will have a polarization where the response of the system is non-linear. So, we will learn that in detail; so, go back to our slides.

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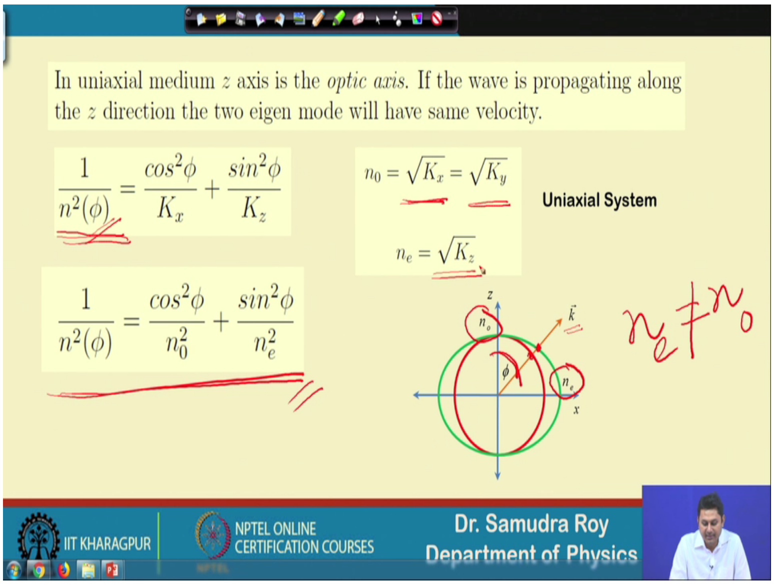

So, this is the old slides I am using this slides we have already used in our last class, that how refractive index n is a function of is the function of theta, here we can see that in this case, that this is a theta or angle phi whatever, what is angle phi? The angle phi is a launching angle. So, K vector is here. So, K vector is making an angle phi with respect to an axis z axis and we mention that z axis is a axis along which the wave is propagating. And if in this directions the wave propagating then what happen? That the refractive index for ordinary and extraordinary ray will be same.

So, now, if I launch any arbitrary direction that K vector any arbitrary direction, for example, here in this figure then what happen? That there is a difference between the refractive index you can see there are two circles are drawn here we called is this circles at index surface. So, these two circle basically represent the amount of the value of the refractive index in particular direction. So, in this 2D presentation if I launch the K vector along whatever the directions; so, 360 degree direction one can have. So, in that case what happened that, the reflective index value here will be a function of phi and how it will be related to n o and n e, where n o and n e is a refractive index along x direction and y direction; x direction and z direction in these case. So, along z direction we called the refractive index you know and along x direction we called this refractive index n e. So, that means, n o and n e are not same.

So, I should write here one important equation that n e is not equal to n o that is why what happened that this shape become ellipsoid so, or ellipse in this case. So, that means, by changing the launching angle we can have different refractive index and one can have the value of refractive index if n 0 and n e is given to him or her then by using the value of phi; that means, at which angle I am launching in this case here this is the angle, I can able to find out what is my n phi, just using this expression.

So, in is also related to our previously defined parameter K big K we have already defined that. So, it is nothing, but root over of K x, K y and K z. For uniaxial crystal the refractive index along x and y is same and it is K x and K y both are same we called it n o and K z is different. So, we called it n e. So, these things we have learned in our previous class. So, I am not going to take much time on discussing this.

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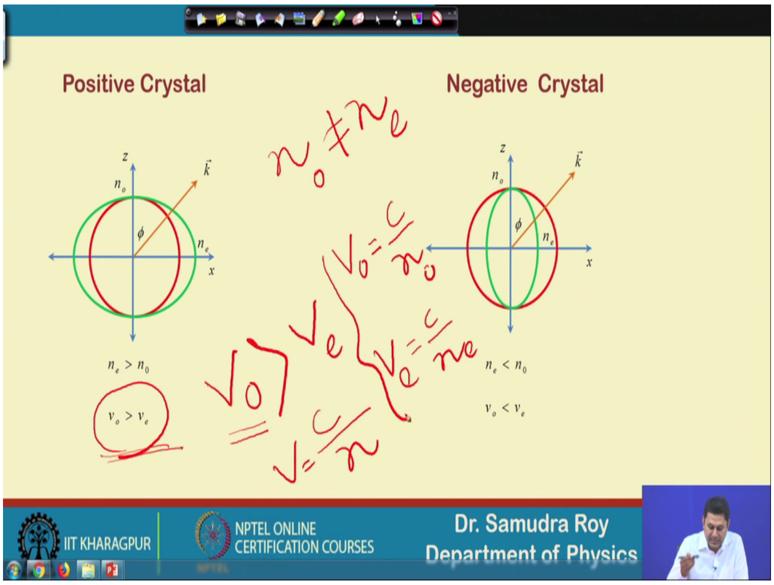

So, go back to our next slide. So, now, it is positive crystal and negative crystal. So, in the last slide I mention an important thing that n o is not equal to n e, that means, refractive index of ordinary wave is not equal to the refractive index of the extraordinary wave and the extraordinary wave and ordinary wave they are propagating a different velocities. Obviously, because their refractive index is different.

So, based on that information I can I can find out two different kind of system; one is positive crystal and another is negative crystal. So, in positive crystal if you see here that ordinary velocity of ordinary wave is greater than velocity of extraordinary waves; that means, refractive index in terms if, what is velocity here inside the medium? We know that it is nothing, but c divided by n, this is the velocity. So, once I say this is v o or the velocity of the ordinary wave then; obviously, I should write v o is equal to c divided by n o. In the similar way I should write v e is equal to c divided by n e.

So, now if the velocity of the ordinary wave inside the medium is greater city of the extraordinary wave as shown in here, then the crystal is the positive crystal. So, now, if I write this equation v e, v o in terms of n e, n o so, let me do that let me do that.

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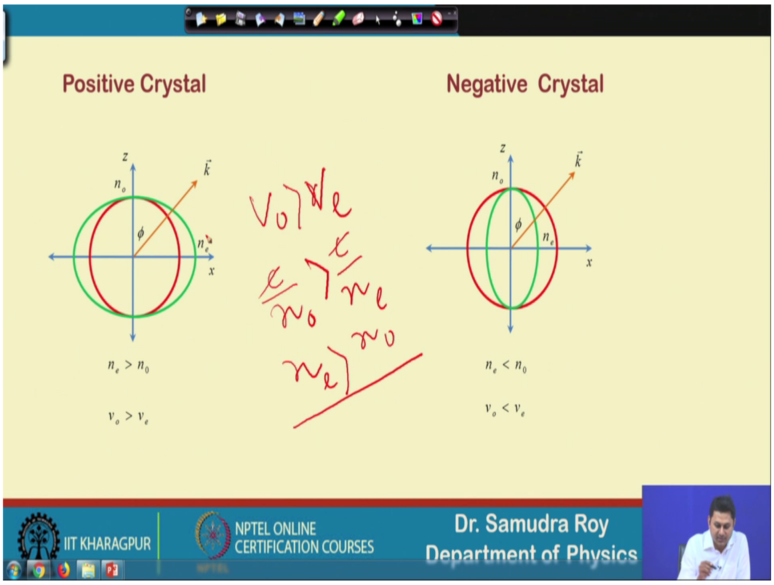

So, I say v o here is greater than v e. So, in another word if I write in terms of refractive indexes so, c divided by n 0 is greater than c divided by n e, c cancels out. So, n e should be greater than n o. So, that means, in the positive crystal what happened the refractive index of the extraordinary wave is greater than the ordinary waves n o.

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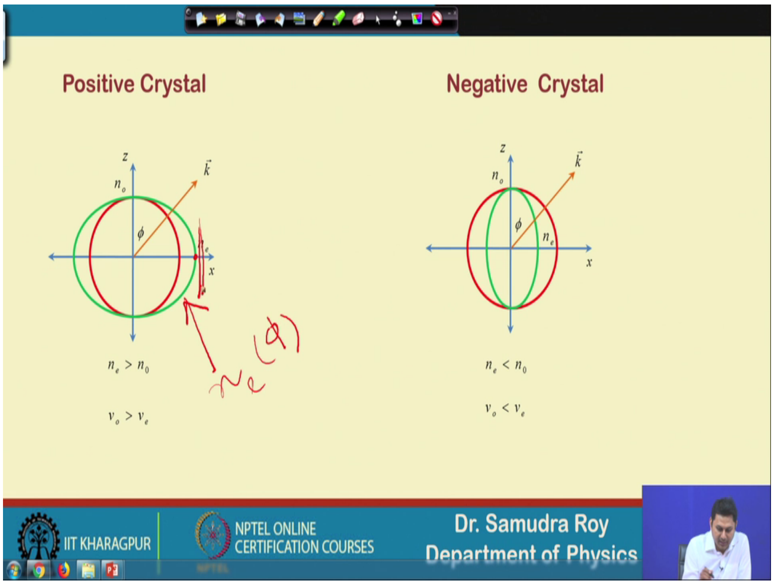

So, if this is the figure of positive crystal. So, let me remove this part so, this is the figure of this positive crystal. So, see you can see here that this green line or the green ellipse corresponds to the refractive index difference or the refractive index variation with respect to the angle phi. So, that means, this is the extraordinary refractive index which has angel phi. When the phi is pi by 2 we know that this has a maximum value and this value we designated as n e. So, here in this point. So, n e is this amount this is the amount from here to here from these value to these value this is my n e.

What is n o? n o is a constant value because this is a circle. The radius of the circle here is constant. So, obviously, what about the value you can choose as radius will be the value of n o. So, n o is there value of this radius of this red circle and this amount is n e you can see that the value of n e gradually change when we change the angle phi. Obviously, that should be because that is the equation we had in our last class. So, when the angle of the phi is 0 then n o and n e will coincide, that means, at this point.

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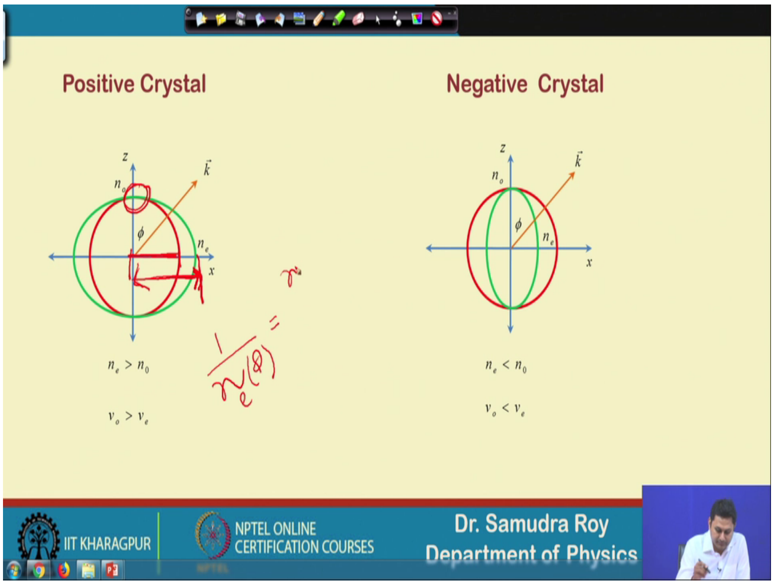

If I again write this equation so, 1 divided by n e I should write it is extraordinary wave this is a function of phi is equal to n, sorry.

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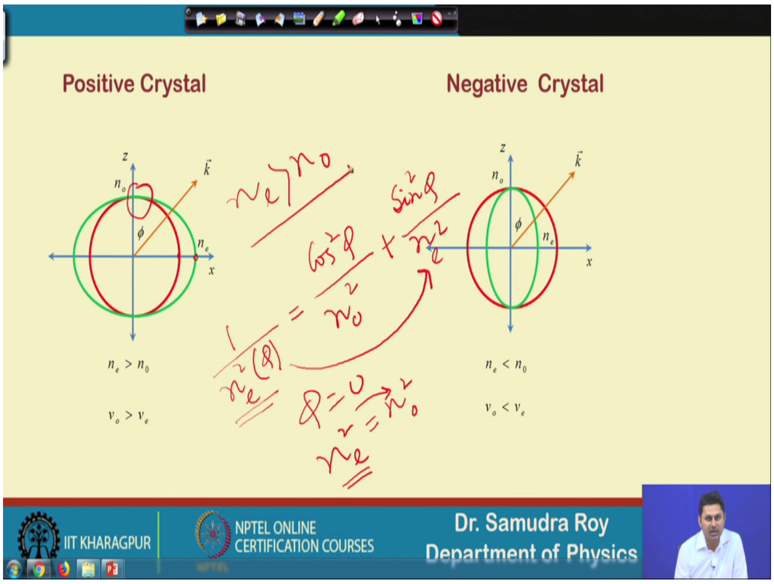

So, let me write once again the equation 1 divided by n square phi e is equal to cos square phi divided by n o square, this is the same equation that was shown sin square phi divided by n e square. So, when phi is equal to 0 then n e square is nothing, but n o square; that means, the value of n e is coinciding with the value of n o; that means, this is the point I am taking about. On the other hand when the value of phi is 90 degree pi by 2 then the value of this quantity will be equal to value of this. So, that means, that is the value of the refractive index along z direction and there is a difference between this and this and also another thing is that n e here greater than n o. So, that is why this kind of crystal is called the positive crystal and this is typically the figure of the index surface for positive crystal.

Well, if there is a positive crystal then the next question is whether there is a possibility to have a negative crystal; obviously, there is a possibility to have a negative crystal because here we find that the velocity v o is greater than v e, but here we will find that the velocity of ordinary wave is less than the velocity of extraordinary wave.

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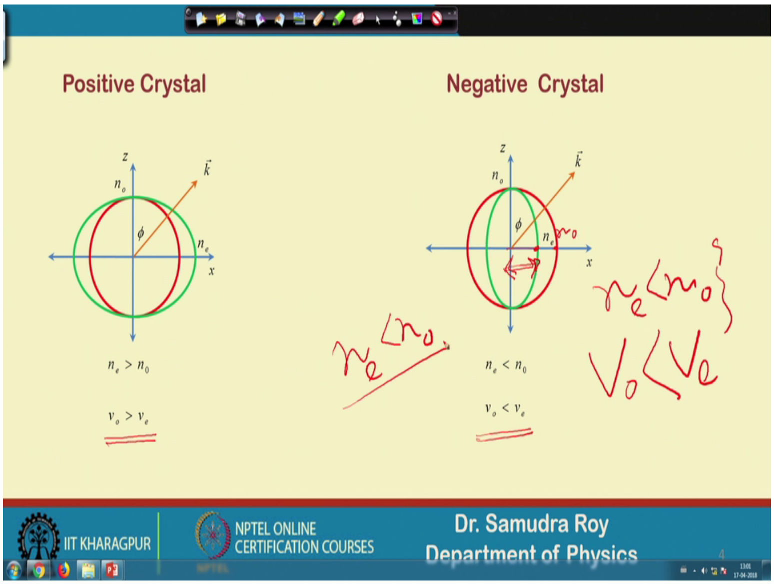

So, here my condition is v o just opposite is less than e and in terms of refractive index I can say that n e is now less than n o if that is the case then the modification here we have is we have a circle as usual and the ellipse that will be the value of the refractive index of n e; that means, extraordinary wave is now inside this circle. Why it is inside? Because like the previous case the value of this portion from here to here is nothing, but the value n e.

And, here from this point this value is n o. So, the lengthwise you can easily find that n e is less than n o. So, the typical index figure or the typical surface or 2 dimensional index surface figure is something like this in negative crystal. So, in the positive crystal the ellipse will be greater than the circle because refractive index of n e is greater than n o and in negative crystal the refraction index of n e is less than the refractive index of a n o. So, that means, the ellipse will be entirely inside, it will never cross the circle. So, this is the feature of negative crystals.

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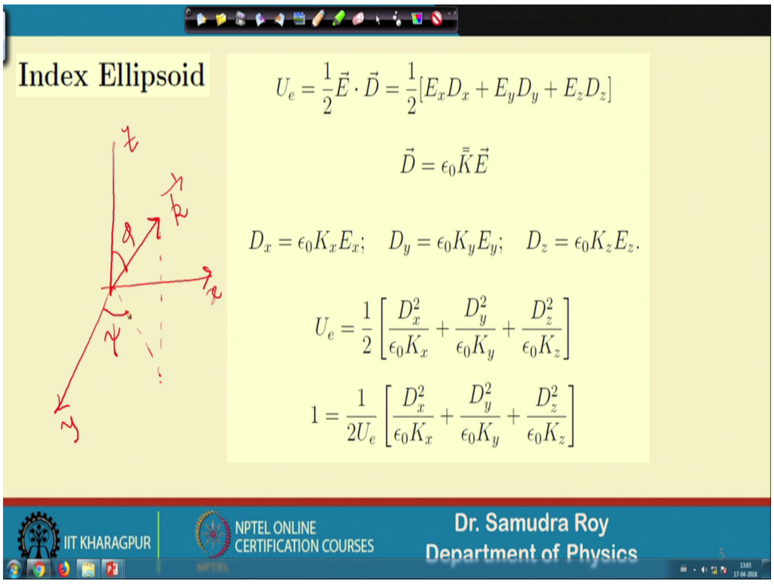

Well, after having the knowledge of negative and positive crystal which will be very important in the later classes because this concept we will going to use in our phase matching. So, let us try to understand index ellipsoid. So, this is the general form of the refractive index variation. So, one can derive very easily the general form of refractive index variation because the material is a 3 dimensional material.

So, that means, if I launch the light the light can be any angle. So, this angle is not restricted to 2 pi rather it will be restricted to entire that is be a cone of angle. So, if I launch the electric field K vector in any angle any angle means any value of azimuthal angle as well as this.

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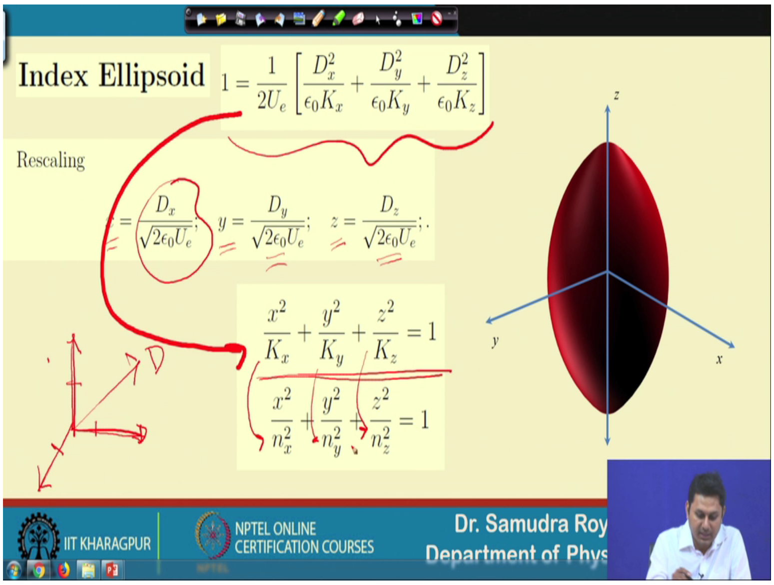

So, in 3 dimensional if I want to find out, ok. So, in 3 dimensional the figure is something like that. So, far I was restricted the K vector in this, but here I try to launch the K vector in some angle here and also some azimuthal angle here. So, this is so, if this is a phi. So, I have another angle psi. So, in 3 dimension this is the K vector. If this is my z axis, this is my x axis, this will be my z axis and this is the figure of general figure of launching in K vector and this K vector is now in here hanging in here having any angle with some azimuthal component here. So, in that case what happened I will have instead of having a surface index surface I will have a index ellipsoid. So, this is a treatment where we can have this equation.

So, ellipsoid equation one can derive. So, let us try to find out how one derive. So, I can write the energy density in this form for electric field. So, E dot D half of E dot D. So, I write half of E dot D then I can expand these as half of E x D x plus E y D y plus E z D z. So, this is the form of the energy stored, the potential energy stored because of the electric field inside the system this is just for electric field the energy is electric field and I am not talking about the magnetic field, but anyway this is the form. So, now, D and E is related to this. Now, if K; K bar is a matrix form. So, I several time I used that. So, if this is diagonalize then D x E x, D y E y and D z K z has a relationship like this. So, D y E x is related to this D y E y is related to this expression and D z E z is related to this expression.

So, now what we will going to do I just replace this E x, E y and E z in terms of D and if I do then we will have something like this.

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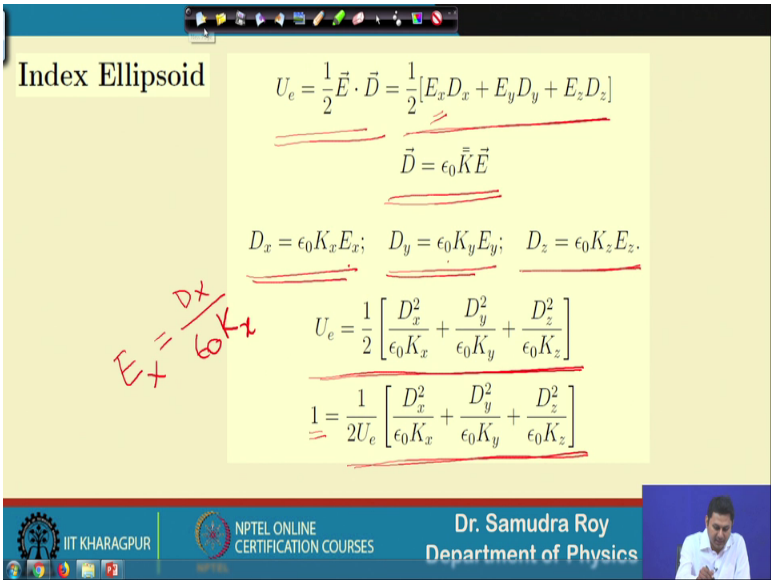

D x square divided by epsilon K x, D y square. Just replace E so, here E x will be how much? D x divided by epsilon 0 K x. So, epsilon 0 K x is D x divided by epsilon 0 K x is E x. So, we will replace these here. So, if I replace this we will have D x square divided epsilon 0 K x in the first term, second terms D y square divided by epsilon 0 K y and third term we have D z square divided by epsilon 0 K z. If we have these three terms and then if I divide this E y to entire equation divided by U U e so, left hand side I have one and right hand side I have this quantity, this entire quantity.

So, now what happened in the next case after having this expression we have already there actually. So, I have a expression something like this. So, now, what we will do we will rescale the thing we will rescale. So, rescale means we will just try to find out what is the value of x y and z in terms of whatever is given here. So, if I write x x is equal to D x divided by root over of 2 epsilon 0 U e, D y divided by this quantity and D z divided by this quantity with respect to I mean if I right in terms of x, y and z.

Then, after rescaling we can have an equation the same equation now have this form. So, try to understand that we have an equation and from that I can change the thing in terms of x, y this is called rescaling after doing the rescaling I have an expression in terms of x, y, z assuming x, y, z is my coordinates system like D x, D y, D z because D x, D y, D z is nothing, but x y and z component of the D vector. So, if a D vector is like this. So, I can divide this D vector into three different components along x, along y and along z and this three different components D x, D y, D z is rescale by root over of 2 epsilon 0 U, U e and we called this segments as our x, y and z, that is all. So, that means, I am rescaling to the component of D x, D y, D z along x y z to x, y and z by dividing that.

So, after doing that I have an expression like this we got this expression x square divided by K x, y square divided by K y plus z square divided by K z and now, if I write the same expression at x square divided by because K x now can be express at n x square n y square and n z square respectively K x, y, K z. Then, I can have an equation which is basically the equation of an ellipsoid. So, this is an equation of ellipsoid. So, we know that in 3 dimension if I have these things. So, this is the equation. So, this equation suggest that here we have a figure and in this figure we try to understand and then 3 dimensional form of this things.

So, when x and y is 0, so, that means, x 0 and y 0 so that means, this point. So, this is the point where the curve is cutting at z axis. So, at this point where the curve is cutting at the z point these value is our n z. From this equation also you can find that x is 0 y is 0.

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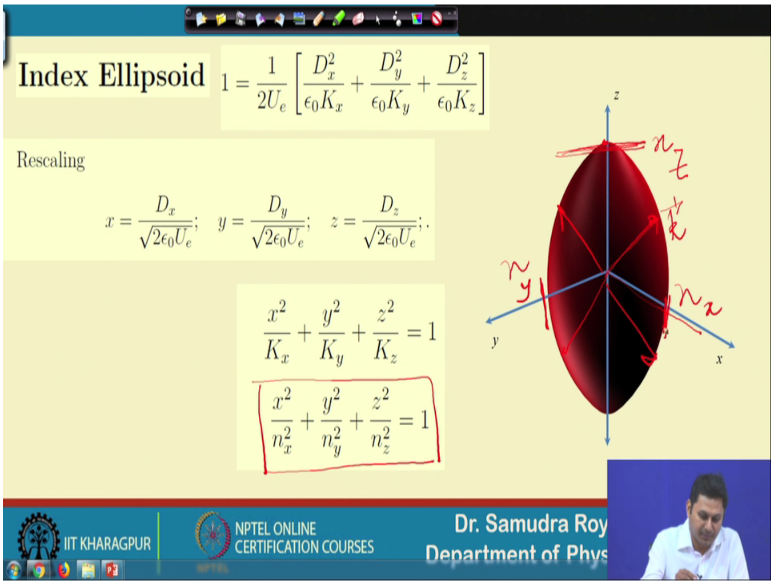

So, z square divided by n z square is equal to 1. So, that means, z value; that means, this amount is nothing, but n z square. So, that means, n z is amount of refractive index and along this direction when x and y is 0. In the similar way if I remove z; that means, at z equal to 0, y equal to 0; that means, z equal to 0 y equal to 0 means this point at this point it is cutting here to the y x axis and this value is essentially are n x refractive index n x.

In the similar way the refractive index of n y is the point cutting at this point. So, one they are cutting x, y and z point then we will have idea how the refractive index is distributed inside the system inside in 3D in 3D how they are distributed. So, all the different points we have a different kind of refractive index here in this point in this point we have different refractive index. So, once we have different refractive indexes so, we can find that what is the angle. So, this is some sought of 3 dimensional ellipsoid and all points all individual points at the individual refractive index for that particular angle or that particular direction.

So, for example, if my K direction is somewhere here so, I will have a direct this point over the ellipsoid I will have the value of n and that will be the value of refractive index along that direction. If my K is on x axis then it is cutting at this point. So, over the surface I have a n x point. So, that will be my refractive index along these direction. So, that is this is the concept of index ellipsoid.

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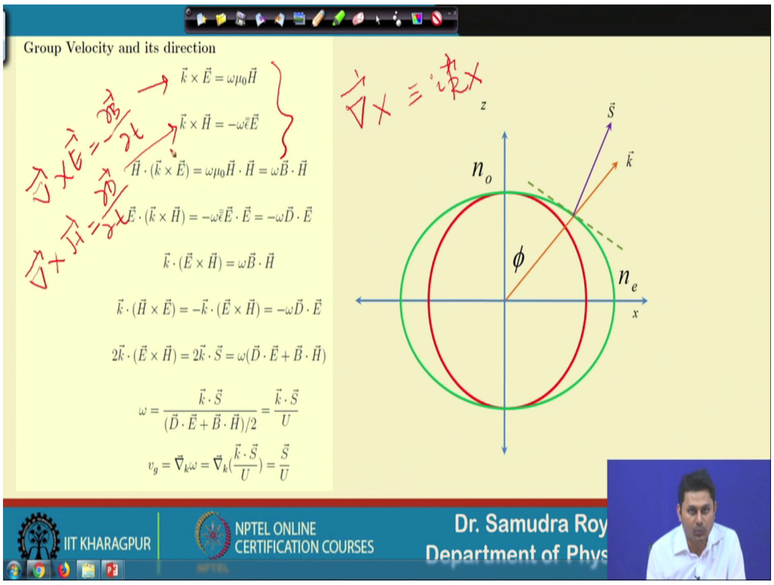

So, after having the concept of index ellipsoid we will try to find out another important thing that there is a group velocity and its directions. So, far we are dealing with direction of k, but now we will try to find out in which direction the S will propagate. We know already that the k and S these two vector will not be parallel if the system is anisotropic, that is the thing we have already figure out, but here we find out that the group velocity is a distant direct the direction of the group velocity is same as the direction of S vector.

So, this treatment is quite straight forward. So, we will going use these two Maxwell's equation. So, this is a Maxwell's equation in different notation in k notation. We know that this cross product this curl thing can be represented as i of k cross. If I do for Maxwell's equations so, two Maxwell's equation here we are going to use or we have use is curl cross E is equal to del B del t with the negative sign that was the equation we use it becomes this and another equation is curl cross H is equal to del D del t this is another equation that we used here. So, this is the corresponding form.

So, these two when these two equations are using terms of k and omega then we have these two expressions these and these and then what happened I will just rearrange this things k dot E, I just make a dot product with respect to H and I will get this for second equation I will get the dot product with respect to E and right hand side I will get this. So, k dot H E cross k dot E cross H is something that we want to find out.

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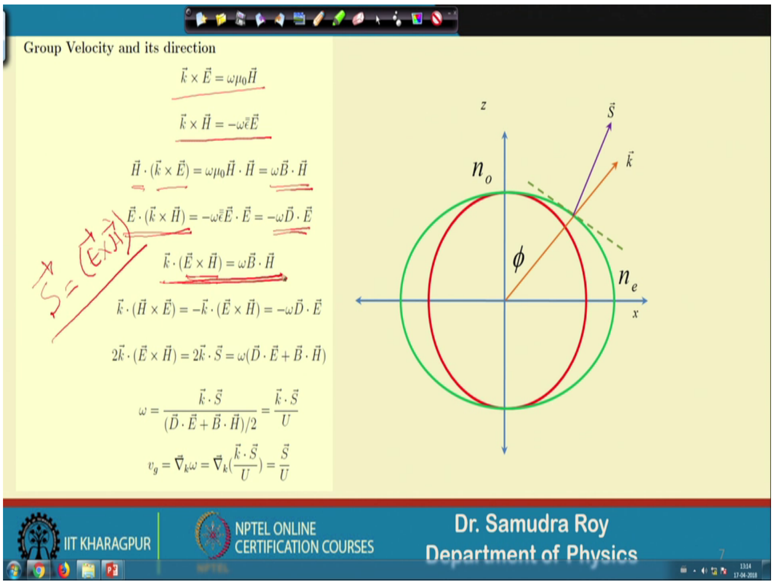

So, E cross H is nothing, but the vector of the vector of S. So, we know that the S vector is E cross H this is my definition of the pointing vector. So, k dot E cross H so, we have this equation. So, we have this equation a dot b cross c. So, a dot b cross c I can write b dot c cross a. If I do b dot c cross a then we have this equation. So, this equation basically one equation, in the similar way in the similar way I have this equation in my hand a dot b cross c. Again I will write it is b dot c cross a if I write it should be this form and if I interchange H and E it should be E cross H with native sign and the right hand side accordingly I will get this.

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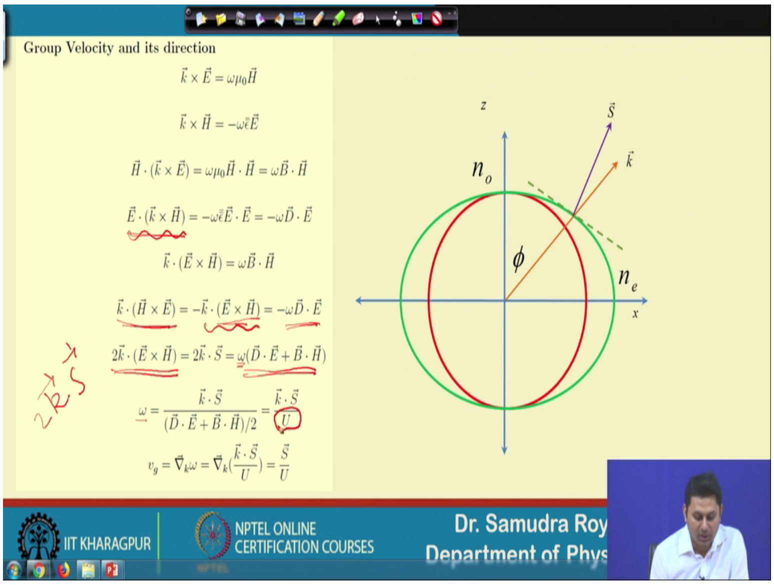

So, now, if I add this equation this and this term here. I will have 2 of E dot E cross H or 2 of 2 of k dot S. Once we have 2 dot E cross if then in the right hand side I have something omega multiplied by D dot E plus B dot H. So, D dot E plus B dot H is nothing, but the total energy stored into the system in using even using this magnetic thing. So, now, what we will do this is the total energy is half of this thing. So, now, omega if I now write omega so, omega will be 2 k dot S divided by this D dot E plus B dot H and this 2 I can write as a denominator by divided by 2 so, here. So, that I can write as a total in terms of total energy U.

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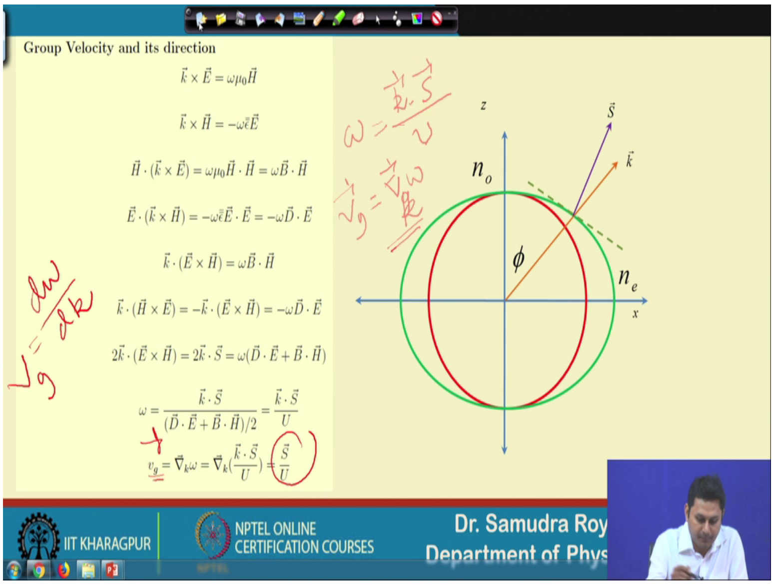

So, it will be so, finally, we will have an expression omega is equal to k dot S divided by U. Now, we know that the group velocity something if I write a vector because velocity is a vector sign it should be it should be sorry k omega. So, d omega normally in the one dimensional we write d omega d k is our group velocity this is a definition of the group velocity d omega d k. So, here we are doing the similar kind of things, but since it is a dot product I need to write this operator form.

So, the del omega is a V g is del omega. So, when we do this del omega then we will find that in the right hand side this K will cancel out because of this operation and eventually we will have S divided by U. So, that means, the V g which should be a vector sign this V g is along this direction.

Now, if I try to understand these things we will find that if k is this direction in anisotropic system the energy is flowing that is perpendicular to the surface.

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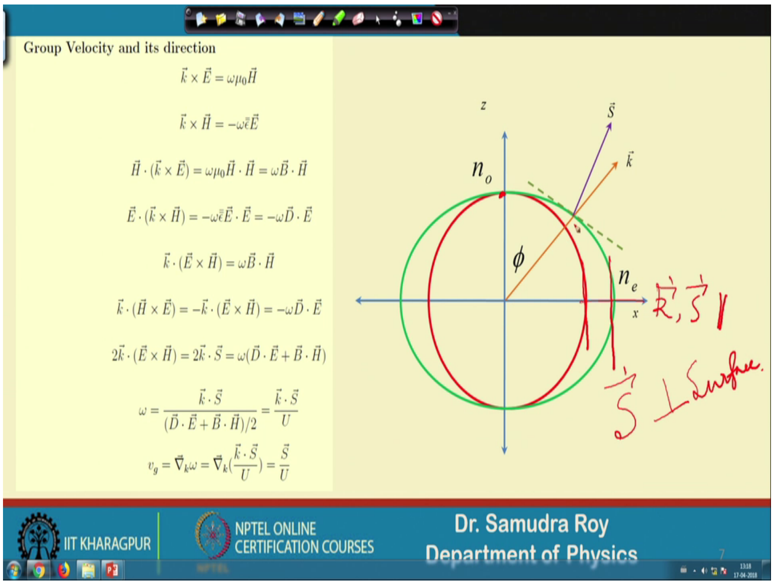

So, S is the energy that is flowing perpendicular to the surface of this index that is a very important thing. So, it will be perpendicular to the surface, but k is not necessarily perpendicular to the surface. k is perpendicular for these system where we have uniform refractive index, that is true, but here for extraordinary case we do not have a refractive index uniform the refractive index is changing. So, k is not perpendicular, but S will always depend perpendicular to this surface. However, at this point S and k both are in same direction and also in these direction in x direction also you will see that here if I make a perpendicular to the surface. Here in make a perpendicular to the surface both are parallel that means, k vector and S vector will be parallel.

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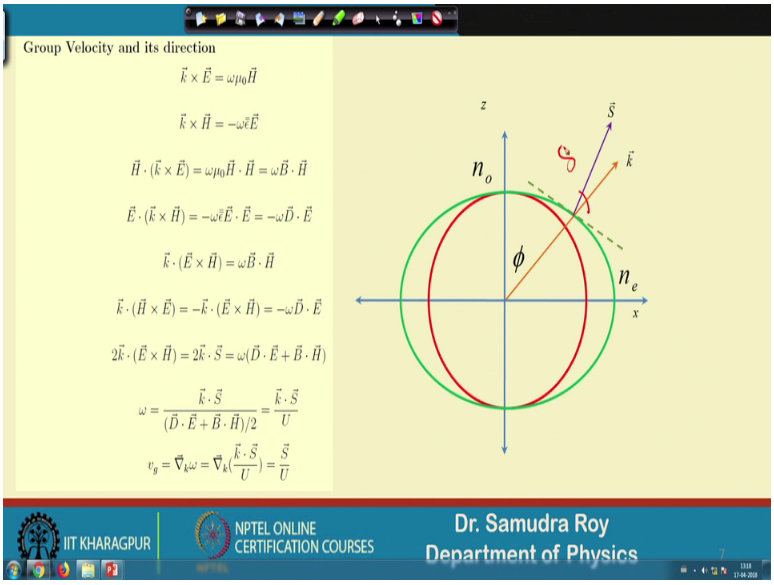

But, any other direction here for example, we will find that S and k are not in same direction. So, there is a angle between if this angle is delta this delta is nothing, but the walk of angle that we have discussed earlier. But, the important thing you need to know that the S vector will be always perpendicular to the index surface, that is a very important information that we have and that is the direction of the group velocity as well.

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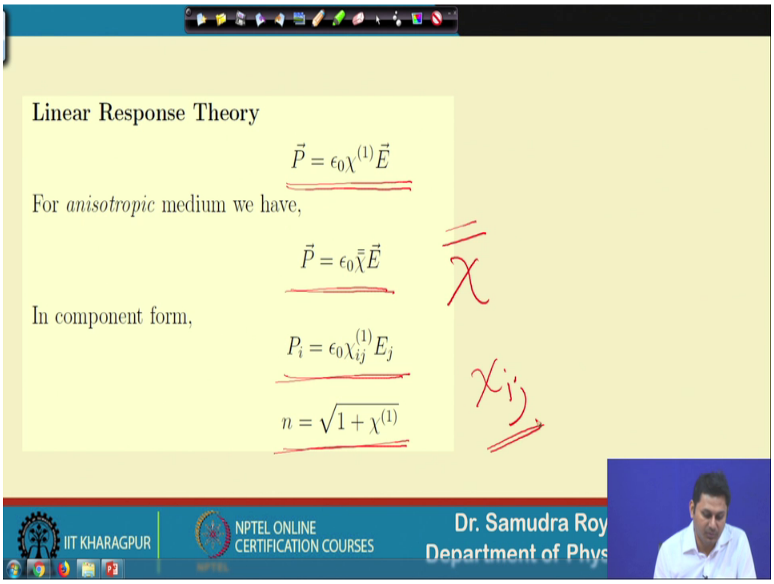

Well, finally, today we will learn an important thing that the linear response polarization. So, far we are dealing with polarization. So, polarization is nothing, but P is equal to so, polarization is something which is proportional to electric field and define like this, but in a anisotropic system we know that polarization can be represented in terms of this chi bar bar. Chi bar bar is something which is not a scalar quantity rather is a tensor quantity. So, we need to write the thing in component form that we have used earlier. So, P you know write in component form and the first component of the P i-th component of the P will be represented by the chi as i z and then E j. So, now, the refractive index can be represented in terms of chi also and it will be 1 plus chi 1.

So, now, if I right this i j and all these different components and obviously, will find the refractive index is also some kind of quantity which is not same for all the values of chi because chi i j chi j chi chi i j is a quantity which can change by i and j. So, there will be 9 components. So, different kind of refractive index one can have and that is a characteristics of the anisotropic system.

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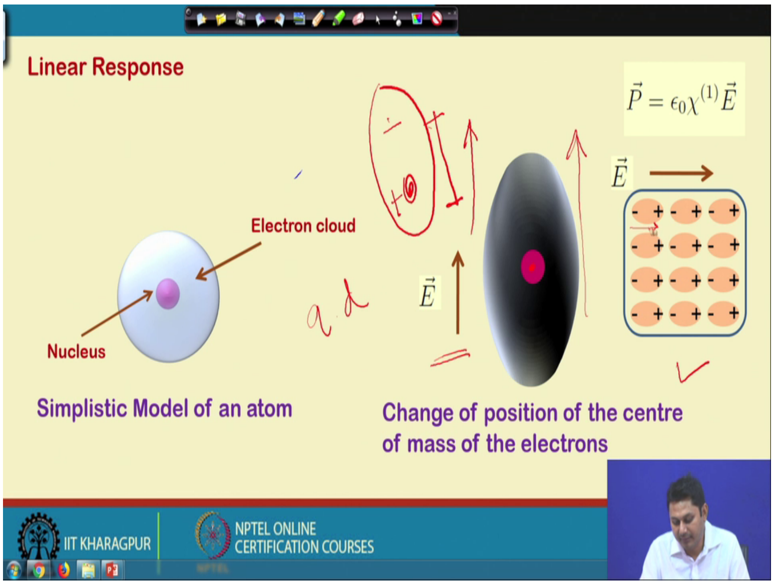

But, the here the linear response means so, we will learn this in the next class linear response means. So, just let me show you this figure. So, this is a system where we have a nucleus and it is surrounded by the electron cloud. So, this is a nucleus surrounded by a electron cloud and what happened I am launching an electric field here. I am launching an electric field here. So, when the electric field is here what happened what happened that we have some kind of shifting. So, there will be some kind of shifting of the entire electron cloud.

So, because of this shifting there is a change of the position of the centre of the mass of the electron. So, electron the nucleus is sitting here and the entire electron cloud because of the application of the electric field will shift from this point to this point. So, there is shifting. So, if I write this the nucleus is sitting here and the negative charge is somewhere here. So, if it is a positive charge, negative charge there is it charge separation and this charge separation is appearing because of the launch on the electric field and we know so, this is more convenient figures so there are many dipoles can be generated we know that dipoles is generated because of this separation of charge, positive and negative.

So, this separation of charge gives a tiny dipoles into the system. There is a series of tiny dipoles and this dipole basically gives the concept of the polarization. We know that the polarization is dipole moment per unit volume. So, dipole moment is nothing, but the charge multiplied the distance we have a distance here and we have amount of charge here. So, charge multiplied by the distance is our dipole moment, so dipole moment per unit volume is the polarization and this polarization is now proportional to E. At least from this equation can see that polarization is now proportional to E. So, that is why it is called the linear response.

So, that means, I am launching an electric field to the material and as a result the electron cloud is shifting. When the electron cloud is shifting we find there is a change of the position of the centre of mass of the electron and when the position is changing of the centre of mass of the electron, there is a dipole tiny dipole will generate and in the entire material we have some series of dipole that will be arranging under this electric field and as a result we will have the polarization, but this polarization is a proportional if I assume this is a proportional electric field then this is linear relation.

So, so far we say something about linear relation in the future class and the next class also we will learn how this is there, what is the meaning of response, what is response theory and all the things in detail. So, here I like to stop this class. So, today we will learn very important concept what is index ellipsoid and how the S vector and E, K vector are not parallel to each other there is a there is a angle deviation and this angel is called the walk off angle, we have already discuss this earlier and also the response and the polarization. So, when we launch an electric field the material will response in terms of the polarization and this polarization is proportional to electric field so far and this is the linear we call this linear response.

So, we though with these note let me conclude the class. Thank you, for your attention. See you in the next class and we will learn more about polarization and response theory.

Thank you.