Introduction to Non-Linear Optics and Its Applications
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Lecture – 60
Optical Soliton

So, welcome student to the next class of Introduction to Non-Linear Optics and Its Application Essentially, this is the last class we have lecture number 60. So, let us see, what we have in the last class. So, we have Optical Soliton.

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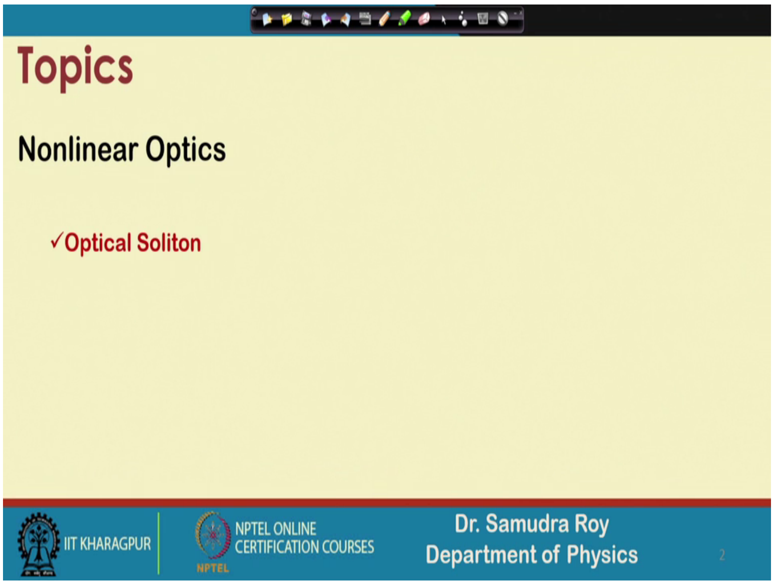

So, in the previous class, what we studied is, how from the Maxwell's Equation we can able to find out the equation which is called the Nonlinear Schrodinger equation, and the derivation was, if all the derivation part is given in the slides. And I just give you the overview and how this equation the Non-Linear Schrodinger Equation can be derived from the Maxwell's Equation.

And I want the student to please do that by your own hand, because if you do not do this by your own hand then it will be very difficult for you to understand all the derivation and all the things are there in the slides. When you have the study material with you, you will readily find that how this equation one can derive from the basics non-linear Maxwell's equation.

I also give you the outline that how this equation can be derived, but I want all the students. So, please do this calculation by your own if you are interested and then you will readily find the form of the equation which is very important. So, let us go back to today's lectures.

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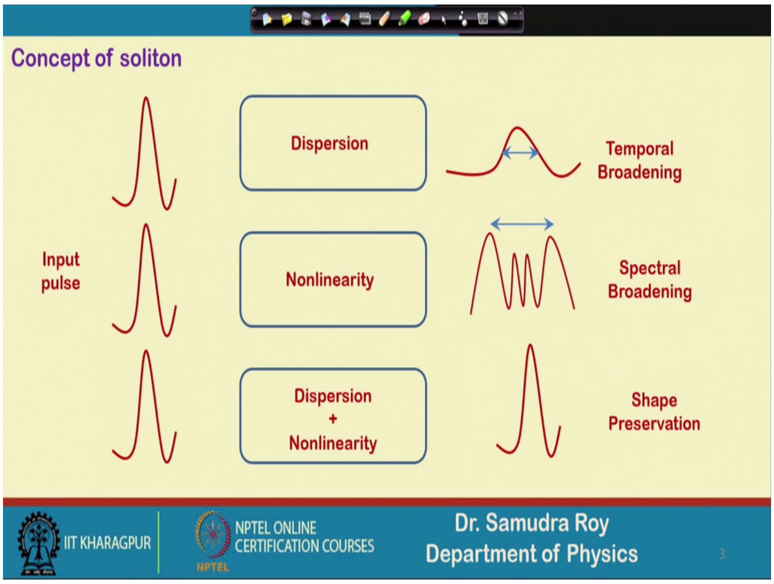

So, the concept of soliton this figure we have already shown to you in the previous class where we find that when the pulse is propagating is the dispersion medium, there will be temporal broadening. When the pulse is moving in a medium which is non-linear in nature, there will be a spectral broadening. But if the pulse is moving in a medium where dispersion and nonlinearity both are there, then there is a possibility that dispersion and non-linear effect may counterbalance to each other. And as a result, we will have, a something which preserved is shape and this is basically the concept of soliton.

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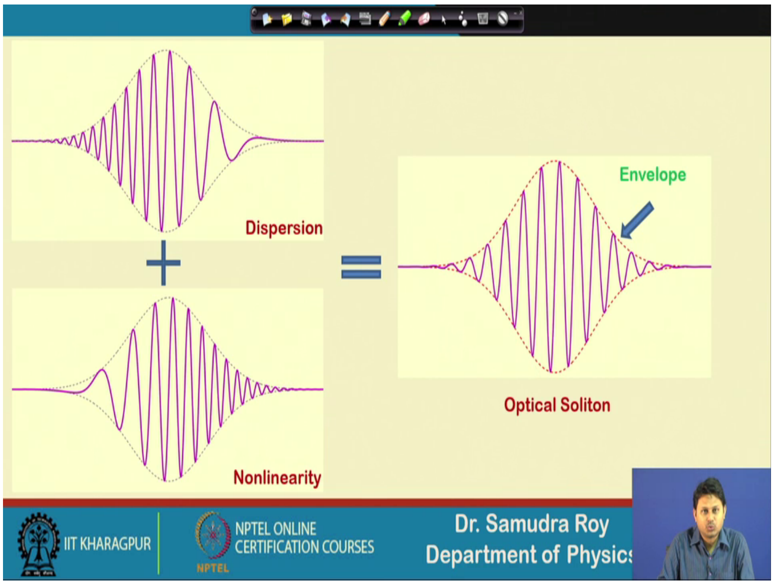

Also, one figure is shown to you and dispersion and nonlinearity can produce some sort of chart wave packets. This chart wave packet can be added and if the dispersion and nonlinearity, the amount of dispersion and nonlinearity is fixed in such a way that they can counterbalance. Then we can have an optical pulse whose frequency distribution is not chart and we called it optical soliton.

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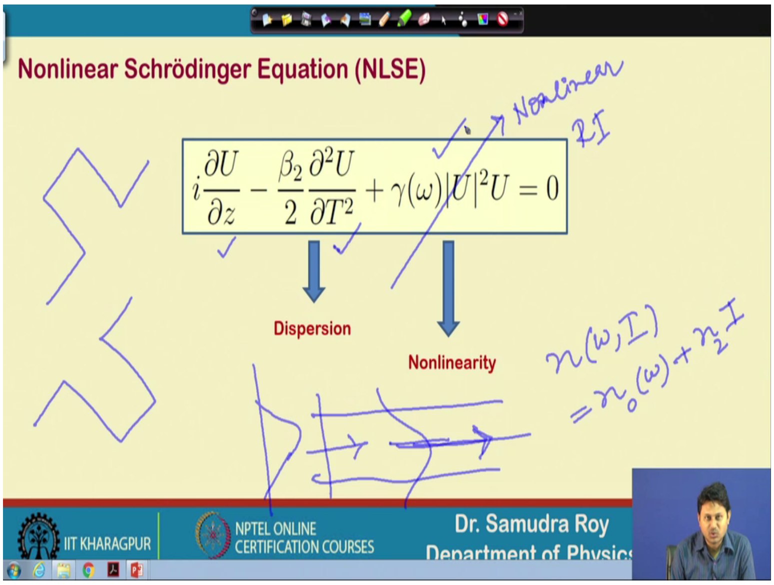

So, this is the very basic concept of optical soliton, but in order to understand in detail we need to solve this equation. That is now shown in your display board that we have an equation which contain two terms, let us try to. So, last day we basically derive this equation to the Maxwell's Equation. Again, I should mention that, you should be careful about deriving this equation and if you do that once in your hand, then you will readily understand that, how this equation 1 can extract from simply the well-known Maxwell's Equation.

Well, let us now concentrate in this equation, this is the equation if I remove this part this non-linear part, then the rest of this equation will be similar to the equation, that we have in our quantum mechanics class which we called the Schrodinger Equation. In Schrodinger equation we have a single derivative of time, in time dependent Schrodinger Equation and a second order derivative of space.

Here, we have exactly the same thing, but the space and the time coordinate is now interchanged. Here, we have first order derivative with respect to z which is space and the second order derivative with respect to time. And these things is normally opposite in quantum mechanics when we deal with the Schrodinger Equation

So, these two are quite same, only difference here in these optics that, we have an additional term here, this is nothing, but a potential term in Quantum Mechanical Schrodinger Equation. And this potential term is some sort of non-linear in nature; that means, if you say U is the wave function, then the potential is generated by the wave function itself.

So, if you tally these two-equation side by side, you will find that, the first order derivative, second order derivative both are there in Non-Linear Schrodinger Equation in optics which is same as the quantum mechanics. And another term is here, which is basically a non-linear term and this non-linear term is nothing, but the non-linear potential term in terms of quantum mechanics, if I consider this equation and try to tally this equation with quantum mechanics.

If I try to understand more this term basically, give rise to non-linear refractive index. This is coming, because of this non-linear refractive index. So, when an optical pulse is moving to a system. This is optical pulse that is moving system, what it does is, it changed the refractive index here, with the law that n of omega I is n 0 of omega, this is the frequency component plus into I the car effect. Because of this car effect, what happened the pulse is basically when it is moving inside the medium it basically change the refractive index.

Now, in quantum mechanics and in optics, there is a similarity between the potential and refractive index. You should remember this fact that in quantum mechanics, whatever I say, potential is equivalent to the optics and this equivalent thing is refractive index. In quantum mechanics we have, quantum will like this; in optics we have the refractive index profile something like this.

So, refractive index behaves exactly in the same way the way potential behave in quantum mechanics. So, these two things are analogous to each other. And here, we can see when the pulse is moving, it is basically changing the refractive index or in quantum mechanical term, we can say, that the pulse or the wave packet itself changing its potential.

If the pulse is changing its potential by itself then, this kind of potential is called the non-linear potential and exactly we have this term here, in optics. And that is why this equation is basically called non-linear Schrodinger Equation, because some sort of non-linear potential is associated with that.

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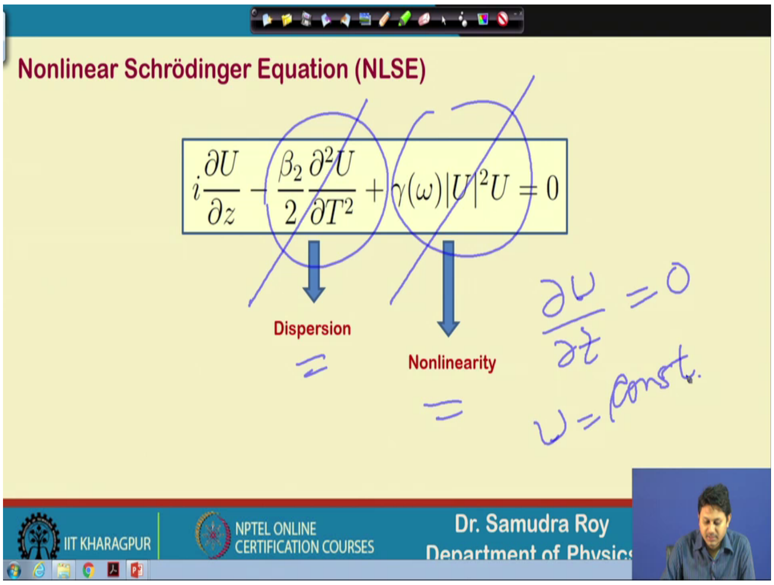

Now, come back to our optics domain. So, in optics this term the second term, basically give us dispersion and the third term give us nonlinearity. If these two terms somehow counterbalance each other, then we have an equation simply this. It suggests that U is constant or in other word there is no change of the input pulse, if it is propagating inside a medium.

So that means, dispersion and nonlinearity, if I somehow able to counterbalance these two terms, then we can have a stable propagation. So, these things so we try to find out here.

Before that, one thing we need to do that is the normalization of non-linear Schrodinger Equation.

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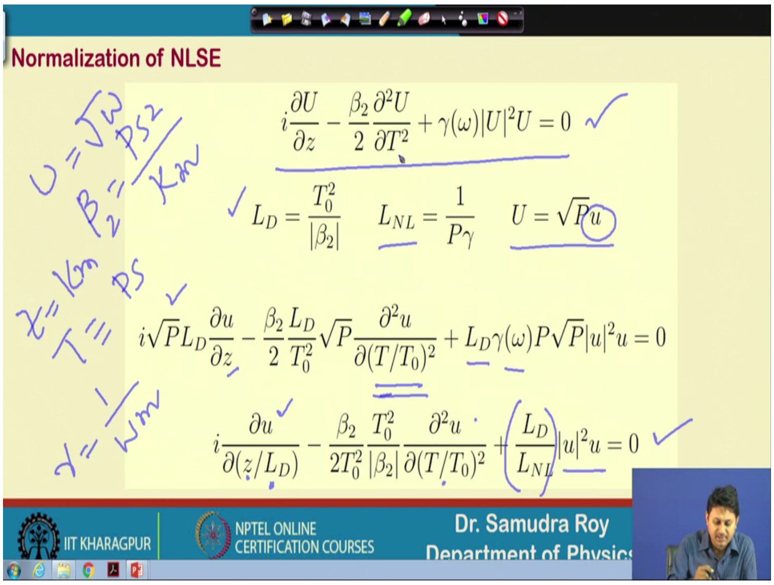

So, this is a non-linear Schrodinger Equation, which is having some sort of dimension. Here what is the dimension beta 2 is picosecond square per kilometer, T is picosecond gamma is 1 by watt meter U is root over of watt and z is kilo meter. These are the units here, in this equation all the terms if I write in terms of units, it will be something like this. If you put all this unit you will find that they are matching. This is root over of watt by kilometer. So, this term also be root over of watt per kilometer and this term also be root over of watt per kilometer. If you calculate carefully then you will find that, units are matching.

But, it is convenient to write this equation in normalized unit which we always do as far as the non-linear Schrodinger Equation is concerned and in order to normalize we make some kind of rescaling. So, we introduce L D, as I already mentioned that L D is T 0 square divided by mod of beta 2, this is called dispersion length. Also, we introduce something which is called non-linear length which is 1 by p gamma. Again, gamma is the unit of 1 by meter watt meter and p is a unit of watt. So, 1 by p gamma should be unit of meter; that means, the unit of length U I write root over of P and this small u; note that is dimensionless.

And after having L D L N and U, if I start putting this thing into the equation then, I put u as root over of P u. So, root over of P, I can take it out and u is there and then multiply L D to entire equation if I multiply L D, it should be beta 2 by 2. l d T 0 square which is already there and I write this T as T divided by T 0 square; that means, some sort of normalization I am making and this normalization is with the input pulse, pulse width. Then l d is here gamma is here, I write u in terms of small u then it should be P root over of P mod of u square. u equal to 0 and now this L D, if I divide this L D to z.

So, I have something called z by L D and then if I replace this beta L D to t 0 square then, this term will cancel out we have beta 2 divided by mod of beta 2 with a half term and here we have L D divided by ln l; that means, the ratio of these two; mind it when I make a ratio of these two, these become dimensionless. u was already dimensionless T by T 0 is dimensionless, u is dimensionless. Here, everything is canceling out. So, these quantities are dimensionless, u is dimensionless z is dimension with the same of L D. So, when you make a ratio of z by L D again this make dimensionless.

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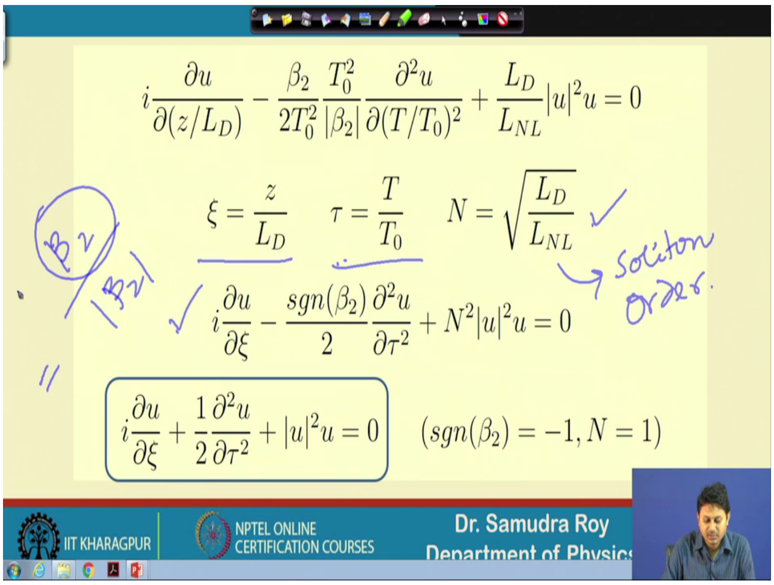

So, already by making some kind of rescaling, I can make this equation which has some sort of dimension. Now, we can land it up with the equation which is dimensionless in nature. So, if I now write this equation and put some dimensionless parameter like this term, which is the dimensionless propagation constant tau which is T by T 0, which is the dimensionless time and N is a ratio of L D by N L and root over of that, which is called the soliton order; soliton order.

Now, please note that if L D is equal to l N L, then we have N equal to 1. So that means, we have soliton order one or sometime it is called the fundamental soliton or the fundamental case. Now, I replace this quantity here and if I replace; I will have these terms in our hand del u del xi minus of signum of beta 2 signum of beta 2, essentially, means the sign of binodium talking about. So, here we had a term beta 2 divided by mod of beta 2, which is nothing but the sign of beta 2; that means, second order dispersion may have positive or negative, if beta 2 is positive, we called is a normal dispersion and if beta 2 is negative, we called it is anomalous dispersion. And mind it optical soliton will evolve. If my dispersion is anomalous in nature then only we can counterbalance the dispersion with the nonlinearity.

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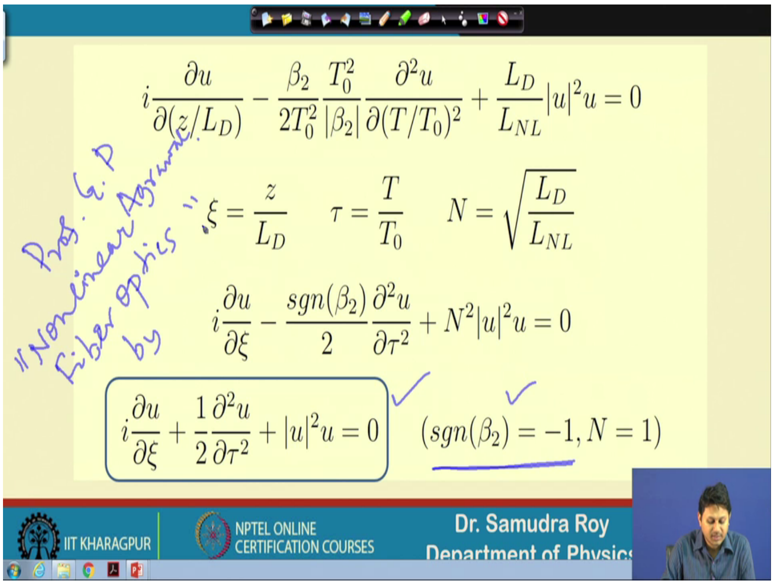

So, sign of beta 2 is minus 1 is one of the essential criteria to have optical solitons. Mind it, when we say, beta 2 is negative, so there is a possibility also beta 2 is positive. For beta 2 is positive also, we have some kind of stable structure which is called dark soliton, that we also going to discuss briefly in this very class.

Well, after putting all these things N equal to 1 and all these things, I will have an equation in my hand which is a Non-Linear Schrodinger Equation. In normalized form. This is a very well-known form in books and literatures. You will have this form and it is easier to solve, because I am making all the dimensions out and if I try to solve this numerically, then, it will be very easy to write the code of this equation and those who are interested, they can please look the book called Non-Linear Fiber Optics by Professor GP Agarwal.

So, this book, you can follow and if you follow this book, there they have a solution of this equation, numerical solution of this equation. So, if you are interested you can go with that and write the code in computer and solve that. Otherwise it is very difficult to solve this equation in general and you need to use something called inverse scattering method, which is a very extensive method to solve.

So, I am not going to discuss in this class. In this class you just need to know few things that what should be the solution of these things, I am not going to solve this equation but what we do that, we will find some sort of solution. And since we know, what is the solution I just directly write its mathematical form.

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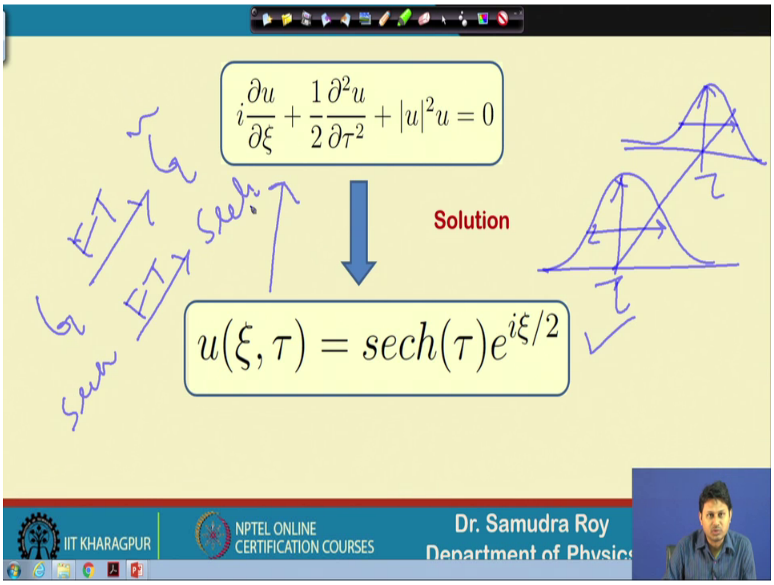

So, if you solve this equation then you will going to find that you have a solution and this solution should be in this form i du z plus half d 2 u d tau square plus mod of u square u. This is a non-linear differential equation and for this non-linear differential equation, since, it is evolving with xi, we have one boundary condition here and this boundary condition is what should be the value of xi at z equal to 0.

So, here we have a solution and the solution suggests that, if I launch an optical pulse having a sech hyperbolic tau kind of form, then what happened that, it will be going to evolve, and it will remain, it is a preserve. How to know that it will remain? It is a preserve and why this is a solution we will do that by putting this U here in this equation. So, I have the explicit form of U, I will put this here and check the left-hand side and right-hand side are matching or not here in the right-hand side it is 0.

So, if I put this solution that is given to us here, then I will find that in the left-hand side we have 0. If really this is a solution that is one thing. Second thing that you should note that the phase is changing here, with respect to xi which is changing in a linear fashion, but the temporal part remains unchanged. So, whatever we launched in time domain over the distance there will be no change in the shape.

So, if this is tau this is tau. So, the shape will be something like this the width will not be going to change the amplitude is not going to change. So, that means, the pulse will remain the pulse shape will remain conserved. So, this is the solution typical solution of optical soliton and you should note that the typical solution is of the form sech hyperbolic tau.

If I plot the Sech; Sech hyperbolic tau you will find this kind of structure it will look very close to the Gaussian structure, but it is not typically Gaussian structure it looks something like that, but it is different it is sech hyperbolic. Next thing is that in frequency domain also this shape should preserve, because that is the criteria of soliton that it will not going to preserve the shape in time domain, but in frequency domain also.

Now I have a solution here, if you make a Fourier Transform of that you will get the distribution of these things in frequency domain and you should know that there are few functions. If you make a Fourier Transform of that particular function this function is written back one very well-known example is the Gaussian function. If you make a Fourier Transform of a Gaussian function then you will be written back Gaussian function. Here also sech hyperbolic tau is some sort of function if you make a Fourier Transform of that that the Fourier Transform also give you sech hyperbolic.

So, Gaussian function g if I make a Fourier Transform, I will get a Gaussian function in Fourier domain sech hyperbolic function. If I make a Fourier Transform; I will also get sech hyperbolic function; this there are few typical functions whose Fourier Transform gives you the similar form. So, this basically tells you that. If sech hyperbolic pulse is launched then it follows if it governed by this equation, the frequency domain also, it will its shape will be preserved which is basically a condition of optical soliton.

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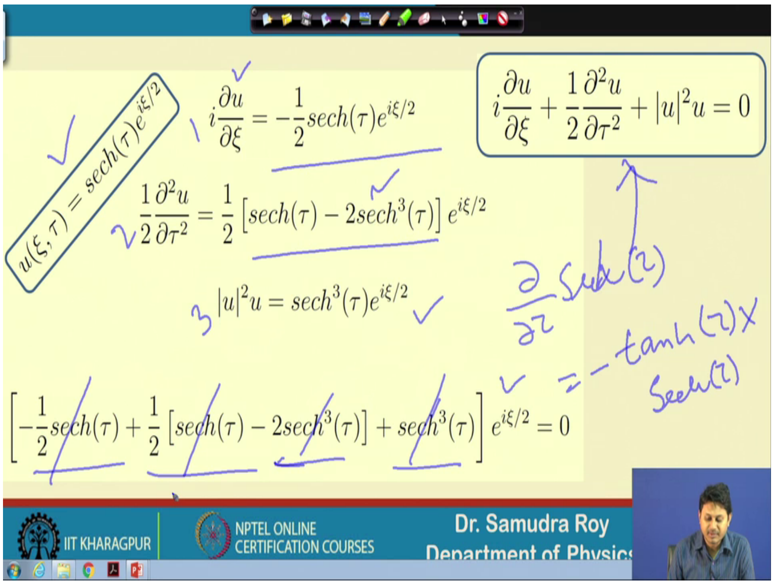

So, now we have the solution in our hand, once we have the solution in our hand then the next thing is that, we should put this solution into the equation and check whether they are valid or not. So, in this particular slide we are doing this. So, here we have the solution U once we have the solution U, it is sech hyperbolic e to the power i xi by 2. So, I need to put this, here, in this equation this is my governing equation and check. So, first term is i del u del xi. So, if I put this first term then i will have minus half sech hyperbolic tau e to the power i xi two quite easy

Next term is the derivative double derivative with respect to tau. So, when I have a sech hyperbolic, I need to make a derivative of this quantity sech hyperbolic tau. So, we know that the sech hyperbolic tau. If I make a derivative it should be minus of tan hyperbolic tau multiplied by sech hyperbolic tau.

Again, I need to make a derivative and we know that tan hyperbolic tau, if I make a derivative it will be sech hyperbolic tau square and sech hyperbolic tau. Again, minus tan hyperbolic sech hyperbolic. So, if you do this calculation, then you will find that this value is simply this. One thing you need to do that and just replace this tan hyperbolic kind term and you just change it to sech hyperbolic, then, everything will be in sech hyperbolic. So, you will be landed up with this term and finally, mode of U square, U is simply sech hyperbolic cube tau e to the power i xi 2.

So, all the three terms this, this and this is now in our hand what we will do that, we will put all these three term together here and if I do the first term is half sech hyperbolic tau, second term is half sech hyperbolic tau minus 2 of sech hyperbolic cube tau and finally, sech hyperbolic cube.

So, you can see that, this term and this term will cancel out there is a half. So, this term and this term will cancel out and eventually we have 0 in all cases e to the power i xi 2 i xi 2 i xi 2 is there. So, I can take i xi 2 common and this rest part will give you 0.

So, we can see quite easily that if the solution is of the form sech hyperbolic tau indeed. This basically is a solution of Non-Linear Schrodinger Equation; that means, Non-Linear Schrodinger Equation optical soliton should have a typical form which is sech hyperbolic tau.

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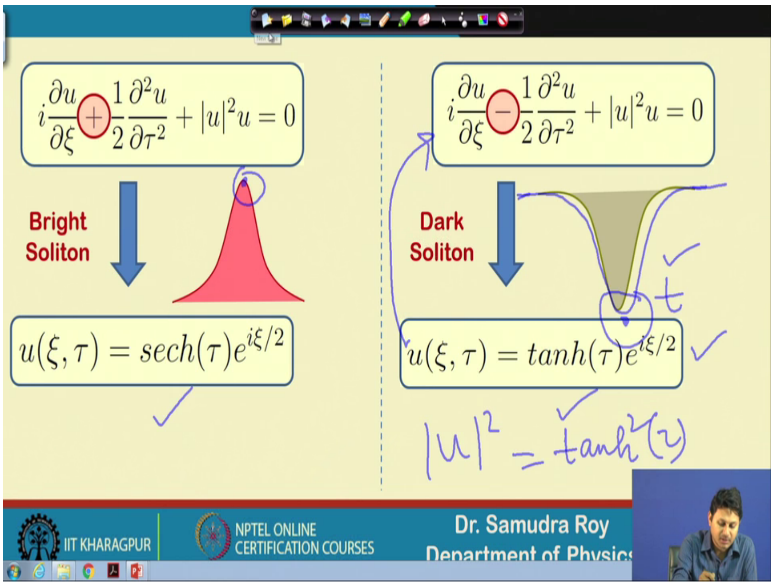

Here finally, we like to compare these two things. So, in this particular equation, we can see that there are two kind of possibilities. One is positive dispersion and another is negative dispersion, if the dispersion is negative. Since, there was a negative sign here, so we have plus sign here and once we have a plus sign here, we have something called bright soliton solution, which we have already discussed.

So, sech hyperbolic is a solution, but there is a possibility that I have a normal dispersion and in normal dispersion beta 2 value is positive. If beta 2 is positive, we have a negative term here, if we have a negative term then still we have a solution and the solution is of the form of tan hyperbolic tau.

Now, if I plot tan hyperbolic tau square which is basically the intensity. So, if I make these things, which will be equal to tan hyperbolic square tau and if we plot that, it will look like this here, we have the value and this value will go down and this is basically 0, it is over T and this term will give you 1.

Since, the intensity vanishes at T equal to 0 point this kind of solution is called dark solution or dark soliton. This dark soliton is a some sort of soliton; that means, the shape will remain persevered, but it is dark, because at T equal to 0, the value is minimum here, exactly opposite to that we have in sech hyperbolic case at CT equal to 0, it is maxima here.

So, the solution is again there. So, if somebody is interested he or she can put this value of u here and check that really, it is a solution or not, the way we have done for sech hyperbolic. One can do that for also tan hyperbolic to check, whether this is a solution or not.

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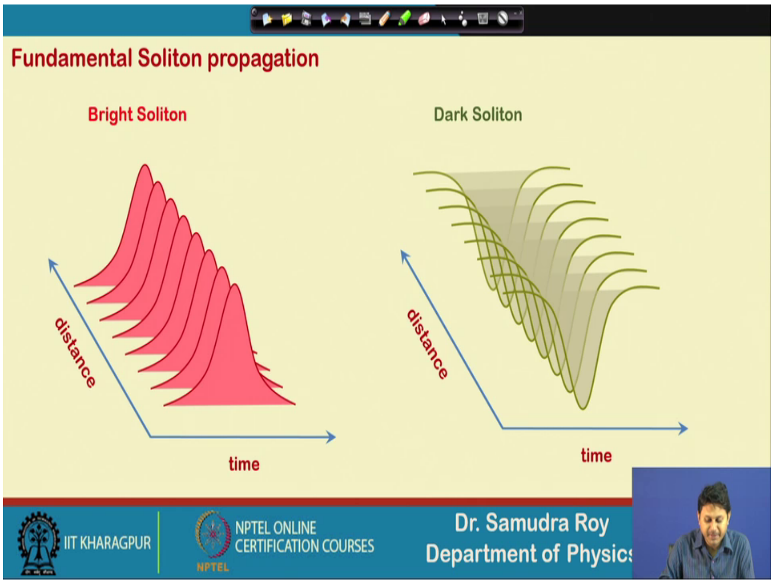

So, this is a typical picture, how these two waves are moving for bright soliton. In time domain, if it is distributed, this distribution remains conserved and it is moving along the distance. In the same way for dark soliton, also one can have a pulse shape and it will move throughout the distance and it will something it will look something like this is schematic diagram. If you do a numerical solution, you will get exactly the same results.

So, here these two is basically, the stable structure, one is called the bright soliton which is a much interest and another is the dark soliton depending on the value of dispersion, if the dispersion is anomalous then we have the bright soliton. If the dispersion is normal, we have dark soliton, but normally we do not deal with the dark soliton we, because its intensity is vanishing.

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Well finally, we have some sort of application of optical soliton. There are a few fields where optical solitons are applied, mainly in optical communication, where soliton beats are there. So, soliton beats basically gives you, because it is a stable structure, it will propagate without any distortion.

So, we have an application in optical communication. Then in pulse compression, because optical soliton for femtosecond laser with different mechanism, we can reshape the pulse and even we can compress the pulse with different mechanism. If the optical soliton is evolved then we can manipulate these things by putting some.

Some sort of external effect and reshaping these things, which is also useful for different application and for femtosecond laser, we can do that super continuum. Generation is something very important in these days, where we can generate an optical spectra, which is very wide and this can be generated by generating optical soliton.

So, if I generate optical soliton they can move and under some perturbation, they can break to several solitons and as a result we can have super continuums which is basically some sort of spectra ranging from say 400 nanometers, typically 400 nanometers to 2000 1800 nanometer and so on. So, this wide spectrum we called super continuum and then finally, soliton logic gets are something where we can use this soliton, and these are the few applications we have.

So, well now, we are almost in the end part of these things. So, today is the last class as I mentioned, I do hope you people enjoyed this course. So, I tried my best to put all the results, all the derivation as much as possible, but I strongly suggest you to please do all this calculation by your own hand. In many books you find the calculations are not there.

In this particular course one of my emphasis to put all these calculations there in the slides. So, that you can understand how these things are happening; what is the physics? So, I believe it will be helpful, this course will be helpful for all of you and hope for the best and thank you, for your attention and your support with this note let me conclude here.

Thank you and good luck for your exams.