Introduction to Non-Linear Optics and it is Applications
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Lecture – 06
Basic Linear Optics (Contd.)

So, welcome students to the next class of Non-Linear Optics, and it is Application. So, we have already studied in future classes, the basic optics or basic linear optics. We will continue that.

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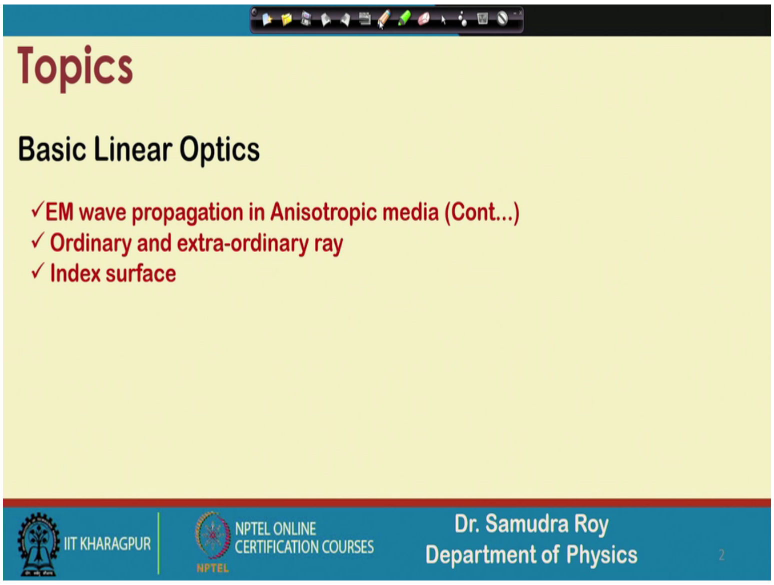

So, in the previous classes we were going to learn, I mean we were studying that how the electromagnetic wave is propagating inside a an isotropic medium and then we will understand how for different orientation of the k vector, what is the direction of k vector and associated s vector; that means, the pointing vector.

So, we will extend this and try to find out different kind of solution under the launching of different direction of the light. So, today we will going to learn in the basic optics. So, E M wave propagating in a anisotropic medium and then we will going to learn at the ordinary and extraordinary ray; what is the meaning of ordinary and extraordinary ray, we will going to learn that and then the index surface what is index surface, we will study.

So, these are the three basic topics then that we will going to cover in this particular class ok.

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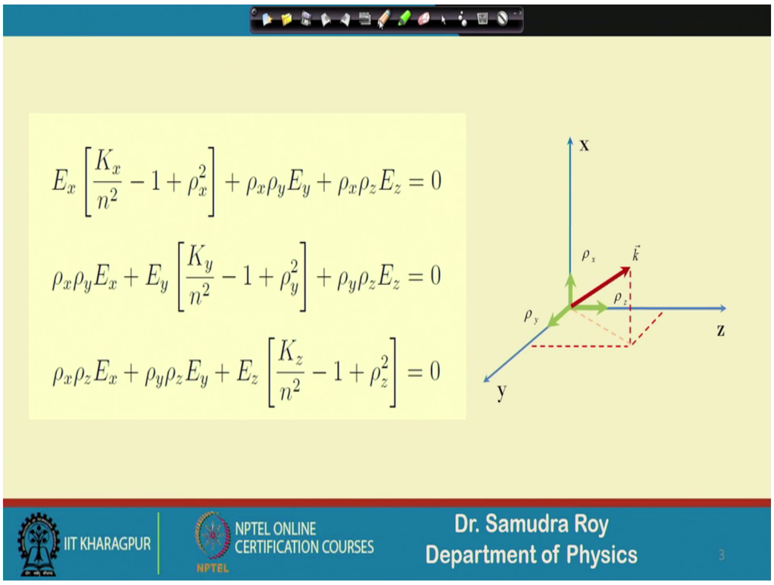

So, this is a very old equation. We have used this equation earlier. Since this equation is very important. So, I thought, again I should give you the picture what is going on. So, if the k vector is launch in a particular direction where the direction cosines are rho x, rho y, rho z given in the green line here, then the from the Maxwell's equation, we have the relationship between E and rho and k are like this. So, this equation was already derived in the previous class. So, I am not going to derive it once again. But this is the basic equation that we have and we need to find out the solution of E x, E y and E z with the condition that the rho x rho y is given.

So, that was the equation we used earlier.

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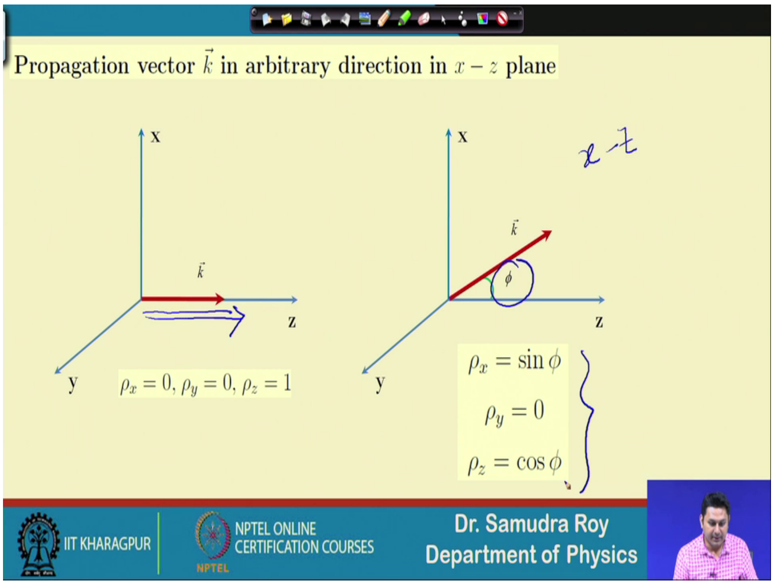

So, now in the next case, what happened that we put some kind of condition. So, what kind of condition we put? Just to put the value of rho x rho y rho z. In the first problem, we put rho x equal to 0, rho y equal to 0 and rho z equal to 1; that means, that the electric field the is launched in z directions. So, this was the direction along which the electric field was launch. So, k vector is along this direction and try to find out what was the solutions and we figure out there are solution in x and y direction. These are the x polarized solution and y polarized solution. We called it the, we called it the polarized state and Eigen state.

So; that means, this polarized state will remain conserved, you also find that. Next, in the next part, we try to find out what happened if the k vector is launched with an angle with an angle phi with the z axis, whatever the axis is there is x, y, z. So, with the z axis, it is making an angle phi and also k, vector k is in x, z plane. It is in x, z plane; it is in x, z plane. If that is the case, then rho x, rho y, rho z will have simply the form like this, so rho x will be sine phi, rho y will be 0 and rho z will be cos phi as shown here.

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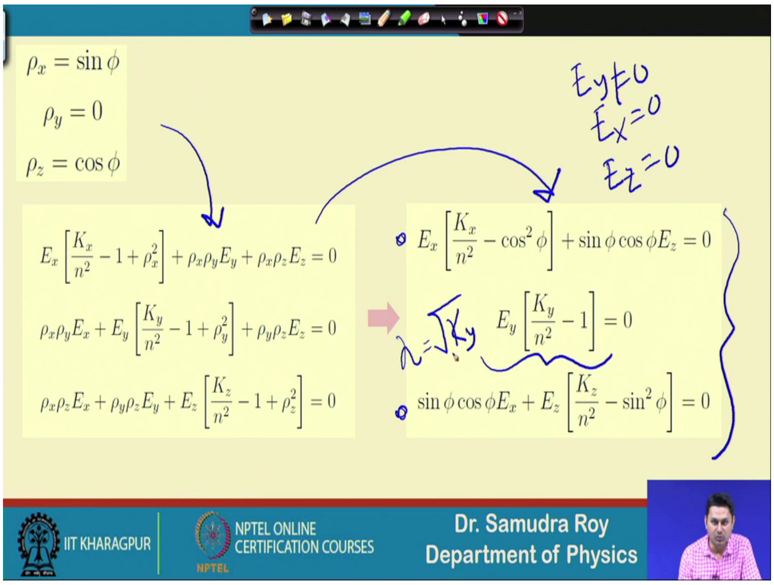

So, if rho x, rho y, rho z is given, then we can use these things into our old equation as shown here. This is also a old slide. This is not a new slide but I want to recap what once again what is happening here. And then, if you put rho x, rho y, rho z into the equation, these things into the equation, this equation will be converted to equation like this and you find that there are three equation related to E x, E z, E y and E z; one equation here is containing only E x term; that means, that one solution, we can take which is some sort of trivial solution here that E x equal to 0, E y, E z equal to 0 and E y is not equal to 0; that means, in order to satisfy these state of equation for a given rho x, rho y, rho z, I try to find out the solution of E x, E y and E z.

Here E x and E z is considered to be 0. If that is the case, this equation and this equation satisfy naturally or this are the trivial solution, if you put E x and E z 0, both the equation will satisfy and if E y not equal to 0, from this equation, we can say that my refractive index n should be equal to root over of K of y; that means, my refractive index along y direction is root over of K y and E y is not equal to 0; that means, E y is along y direction that is a non-zero component.

All other components are 0 and if I launch the E, k vector in one particular direction which is making an angle phi with respect to z axis and it is in x and z axis, then I can have a solution with E E y not equal to 0 and their corresponding refractive index is also given in this equation.

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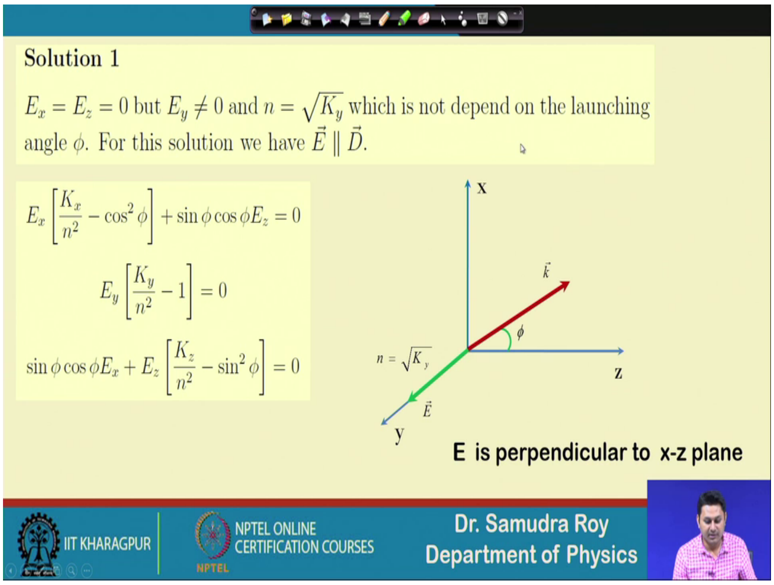

This is we have discussed earlier. So, these are the solution 1 as I mentioned in the earliest slide that in solution 1, we find that even if I launched k vector in one direction which is making an angle phi and if it is in x and z plane, then we have a solution of E which is moving without any kind of change and it will be polarized along y direction. And these are the solutions that E z is not, E z is 0, E y is 0 but E x is not equal to 0. So, E is perpendicular to x, z plane so; that means, x, z plane so; that means, E will be perpendicular to K as well.

So, in an isotropic system, mind it this equation is strictly valid for anisotropic system where E and D is not supposed to be parallel but we here we have, even in an isotropic system, we can have a solution where E and D can be parallel if I launch k vector in x, z plane with an angle phi and one trivial solution is this, well. Let us try to find out what are the others solutions one can have.

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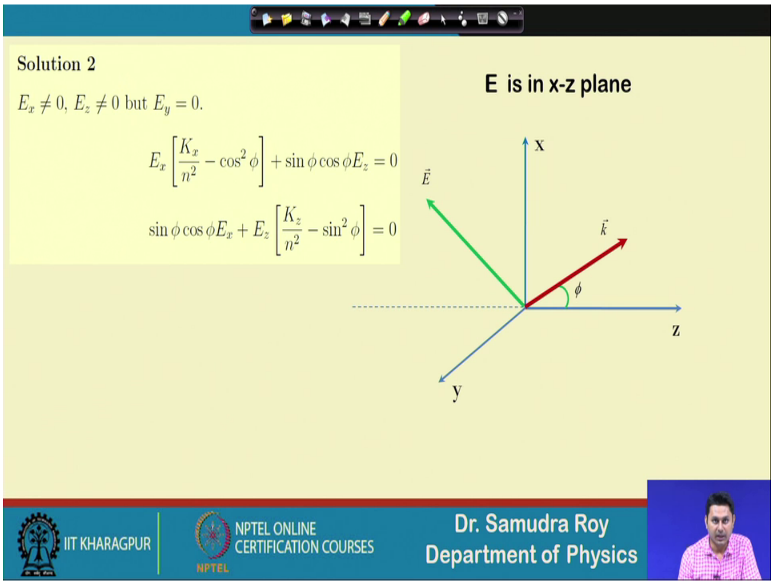

And this is the solution 2. And the solution 2 is interesting because in solution 2, what happened that I launch the k vector as usual in a direction which is making an angle phi with z axis but here we are taking E x and E z not equal to 0 which in the previous case was 0. Here, we consider this is not equal to 0, but E y is 0; that means, essentially the E vector is in x and z plane.

So, I put E vector in x and z plane. If that is the case, x, z plane is the plane of these paper or slide whatever and in this particular plane, if the E vector is there, then the solutions what we have was this. So now, this is two equation. So, from these two equation, we can solve that is one method, two equation, two variables are there. So, we can solve that if the phi is given to you, then you can solve but we will try to do something different here and try to find out some condition what kind of condition we try to find out, let see.

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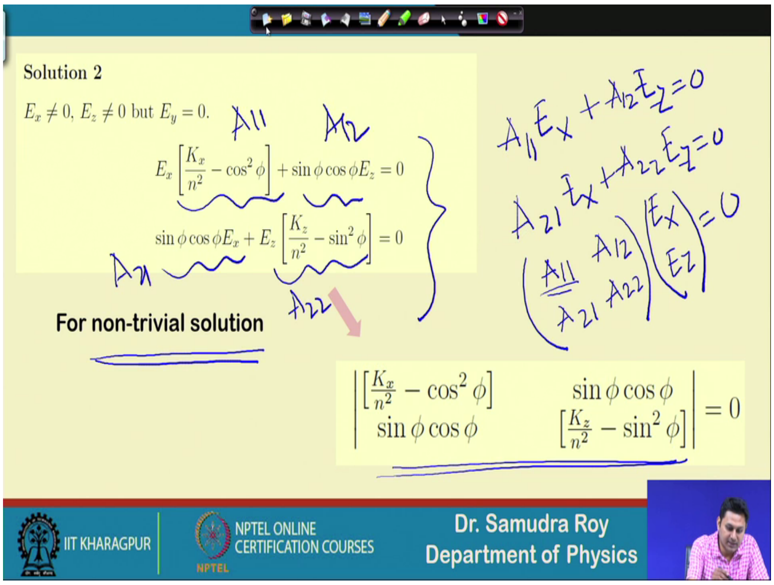

So, if in the solution 2, E x and E z is 0, then as in the previous slide, we find we have two solutions in the our hand and that is this two equation in our hand that is this. So, considering that, we are now for the time being not bothering about to find out the value of E x and E z, rather try to find out what should be the condition for non-trivial solution. So, here for non-trivial solution means, for non-zero value of E x and E z, what we should have, what is the condition and we know that this is a matrix form and this matrix form if I have. So, if I write this matrix from as say A 1 1 E x plus A 1 2 E z is equal to 0 and then I can write A 2 1 E x plus A 2 2 E z equal to 0.

So, this is a matrix form kind of thing. So, I can write the entire thing as A 1 1, A 1 2, A 2 1 and A 2 2 multiplied by E x, E z equal to 0. So, that is the equation. We know that if we have this kind of form, then what happened that for non-trivial solution E x E E x equal to 0, E z equal to 0 is also a solution of this kind of equation but this is a trivial solution because I am putting both E x and E z is equal to 0, but for non-trivial solution what happened, the determinant of this thing has to be 0, then only have a non-zero value of E x and E E y, E E z.

So, in this case, so I have A 1 1 is this quantity. This is my A 1 1, this quantity is A 1 2, this quantity is A 2 1 and this quantity is A 2 2. So now, if I put all this A 1 1, A 2 2, A 2 1 and a all this value here, then I have a matrix form and this is a matrix form of this A 1 1, A 2 A 1 2, A 2 1, A 2 2 and the determinant of this thing is 0. Once we have the determinant of this quantity 0, we will have an equation out of that.

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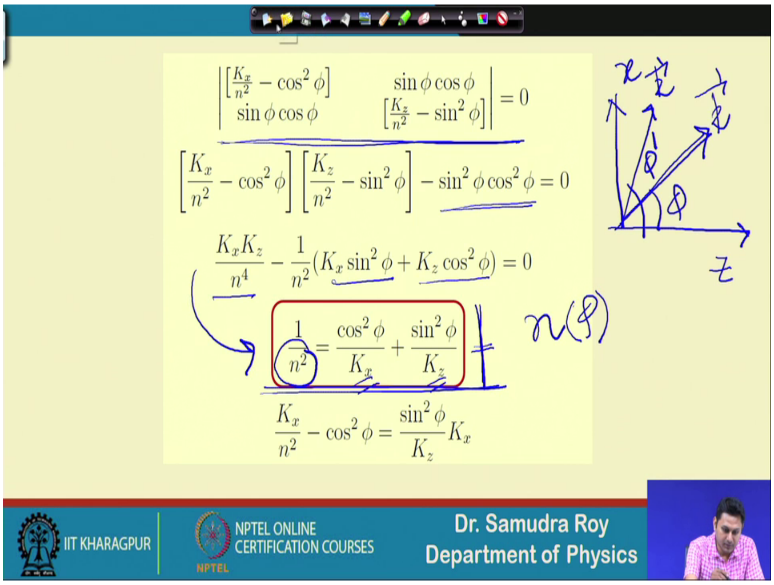

So, this equation, so let me explain that. So, this is my equation here. So, determinant equal to 0. So, determinant if I now decomposed is determinant, then these cross product and this cross product with a negative sign will be 0 that we know. And then if I simplify this things multiplied by this things, we what we have? So K z, K x divided by into the power 4 this then K z n square cos theta, cos square theta and 1 K z, K x n square sin square theta.

So, if I take 1 by n square common, then we have K x sin square theta andK z cos square theta this. So, once I have this term, then another time will be here. So, this are the first. So, 1, 2, 3, 4; four terms are there. From this four term, if I multiply, then we will have 1 term, 2 term, 3 term and four terms.

So, three time is written here. So, fourth term will be plus cos square phi sin square phi. So, these term will be cancel out with this additional cos square phi sin square phi. So, it will be cancel out. So, eventually we will have one equation like this. From here, we have a very important equations, very important equation and we will going to learn about this important equation and that is why it is in the brackets.

So, 1 by n square is equal to cos square phi K x plus sin square phi K z. Why it is important because this equation suggest that the value of refractive index for non-trivial solution of K x and K y; obviously, k z. So, from that equation leads to one very important equation and this equation suggest that 1 by n square is equal to cos square phi divided by K x plus sin square phi divided by K z. Very easily one can derived from this determinant to this equations. All the steps are given.

So, I will like to, I will, I will be happy if the students were taking this course can do that by their own hand, then they will understand how this but the calculation is very very simple but all the steps are down here. So, nothing a very new here. So, 1 by anyways, so 1 by n square is cosec square phi K x plus sin square by K z and these suggest that this now whatever the n, I am talking about is a function of phi. That is a very important because in the right hand side, we have phi and this K x and K z are constant term.

That means, what is phi, what is phi? If you remember that, if you remember that the file is nothing but the angle that is making by the k vector with z axis. So, this was the phi. If I go back to the previous slide or if you notice carefully my k vector was in x, z plane and it is making an angle phi with the z, z axis. If it is making an angle phi, then this ray will experience a refractive index; obviously, this refractive index will be not very simple refractive index. It will be a complicated kind of refractive index and here we find an expression of that.

Again we will come back to this point but you should appreciate, does this refractive index, value of this refractive index will be now function of phi; that means, this ray and whatever the ray is here, suppose I am launching the k vector in this direction having a new phi, say phi 1 or phi dash, then from this, this equation I can say that this refractive index is not changed which is which is very very important thing that now my refractive index is direction dependent and in anisotropic system, this is one of the important thing that if you go in different direction, different physical property will be there. So, physical property is direction dependent.

And physical property means here is nothing but the refractive index. So, if you go in different direction, you will have your different kind of refractive index ok. So, so what, so far we have, we have an expression like this and if I now simplify this things slightly that making a K x because I need to find out what is the condition. So, in order to find out the condition or the relationship between E x and E y, so what we will do that using these things, I will have one expression like this. If you look carefully these things is same which is here.

So, I just replace now, what I will do I just replace this things here with this term and then find out what is going on ok. Go back to our slides and this is the next slides.

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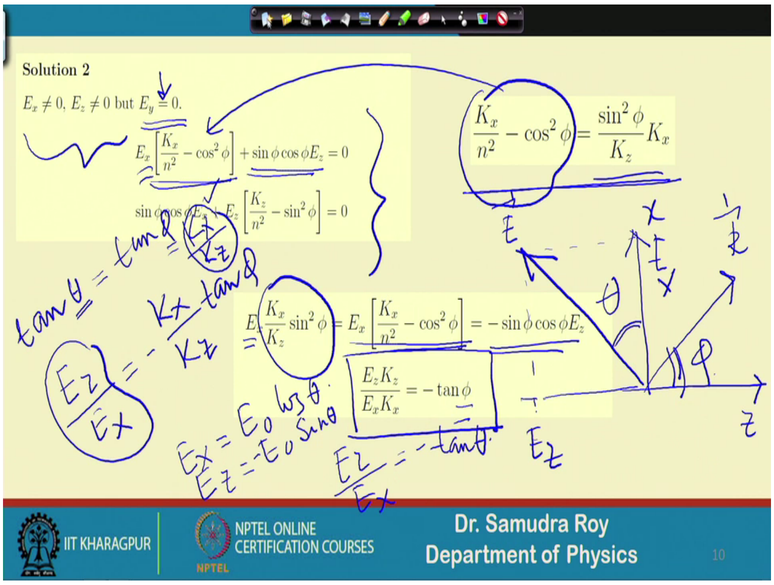

So, in the next slide, what happened as I mentioned, I just replace this things to whatever the value we had there and if I replace this. So, this was the equation earlier. So, this is the value we derive and these were the, these solutions. From here I have the determinant form.

So, determinant form, this term was there. So, in this two equation, what we are doing? I am just replacing this quantity here with this one, sin square phi K z K x. If I do, then I will have E x which is this one is E x. Then it is multiplied by this quantity sin square phi this, we can see that this is multiplied, that will be equal to this because I am replacing this things and this things is nothing but minus of this things from this equations because if it goes this side, there will be a negative sign. So, this.

And from here, we have a relationship between E x, E x K z K z K x and this relationship suggest that E z K z divided by E x K x is equal to minus of tan phi. So, try to find out what is going on. This is my k vector which is making an angle phi here and then my E vector because this is a non trivial solution reaction E x and E z was equal to 0 but E y was not equal to 0 and sorry E, E y is 0 but these things was not equal to 0; that means, mu E vector is in this is my z direction exhibition somewhere here in exit plane, this was my E vector.

Now, these two components are related to this angle phi. So, the component I know that this is, if I make these two component. So, this component is the E z component and this component will be the E x component. So, E z and E x component will be related to tan phi and if I do that, my E z E x will be minus of K x K z, then tan of phi; what is the meaning of that? That means, this is how much if I say this angle E z, so if these angle, say ok.

If this angle is say theta, then E x is nothing but E 0 cos theta and E z will be e 0 of sin theta and since this is a negative side, I can put a negative value here. If I make a ratio out of that, then E z, E x will be minus of tan theta. So that means, tan theta which is the angle making by E x E with x is equal to tan of phi multiplied by K x divided by K z.

So, K x K divided by k z if this terms is equal to 1, then what happened, tan phi is was equal to theta and that eventually means that k vector and E vector are perpendicular to each other; that means, for anisotropic non and for isotropic system, they are perpendicular to each other and we know that. But here this quantity, since K x and K z are not equal to each other then that means some multiplication factor are there and that means, theta and phi are not equal.

So, that means, what happened that the, angle between these two things are not 90 degree. So, there will be some angle difference ok. That is the physical thing that you can say.

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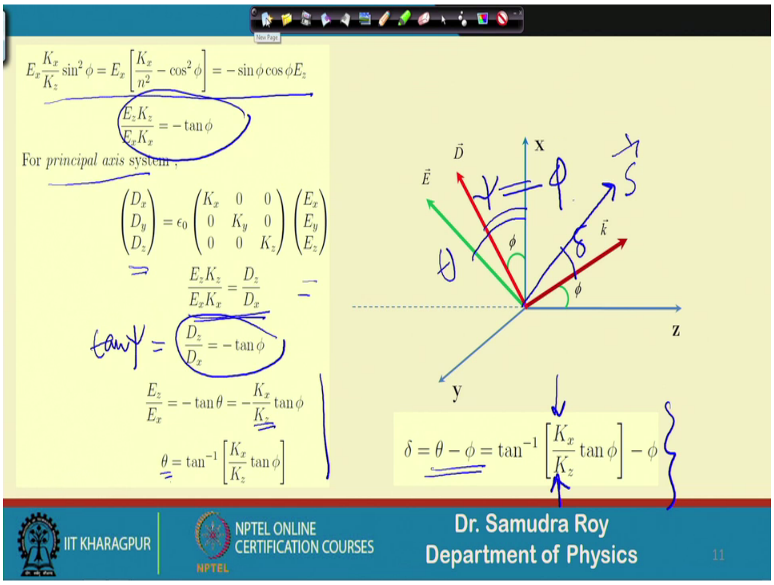

So, again we derive the entire thing here and you can understand now clearly whatever we have done in the previous slides. So, this was our equations and we find this is the thing and now we can modify the equations because in principal axis system, these are the relationship between D and E. So, that means, if I do, then we find that E z K z is nothing but D z and E x K x is nothing but D x.

So, we can replace these things and here now we will find that D z divided by D x is equal to tan phi. So, now, this is nothing but if I making an angle say, in the like the previous way so, this angle is say tan psi, if this angle was psi and we find that this psi is equal to phi. If this psi and phi are same and we can say that K and D are perpendicular to each other because this angle and these angle 90 degree and this angle plus these angle is 90 degree.

So, we will have a relationship between phi and psi and this phi and psi are same; that means, D and K are really perpendicular to each other. But for E vector, it is a different story. And for E vector, we find this ratio is present here and because of this ratio what happened that there is a different angle. So, here I say this is theta. So, from this figure, I can say this is my theta. So, here this phi was equal to this psi was equal to phi because D and K are perpendicular with this treatment. We can understand but theta, we can find and this theta is nothing but whatever the angle we get, tan inverse of that. This is the old equation, the previous equation that we derive.

So, tan theta is equal to K x K z tan phi. So, theta is equal to tan inverse of that that thing. So, I have the value. So that means, K x, K z value and phi value is given to you, then you can have what is the value of theta. So now, the interesting thing, I want to find out the deviation between E and D. This deviation is nothing but these deviation is nothing but the angle between S and K; that means, S vector if you remember is perpendicular to E vector.

So, S and E, they are making their difference is delta. So, if delta is there, so delta is nothing but theta minus phi. So, this theta minus phi is called the work of angle. So, work of angle, one can easily find out. So that means, if I launch an k vector one direction, so s vector will be in different direction. If somebody want to know that what should be the direction of S vector with respect to K factor; that means, what is the angle between k vector and s vector, here in this treatment we call it delta.

So, delta will be equal to this amount. So, K x K z is given. If K x K z is given to you and if phi is given to you, then you will find out what is the work of angle between these two; well I think we are almost in the last part.

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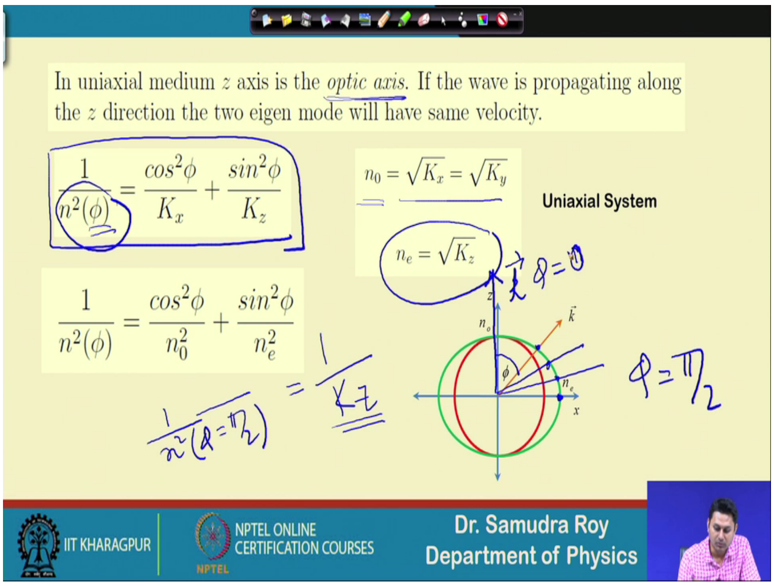

So, let me give you the hint, some idea about what is, what was the equation you may remember, this equation I, I have shown your earlier. That this a very important equation. The next class again, we will deal with that.

So, these equations suggest that n is a function of 5. So, in the uniaxial system, what happened that there is excess called optics axis and it is normally considered along the z direction where the velocity of two way we say; that means, the refractive index is same along with this direction and the refractive index for ordinary and extraordinary wave are same means ordinary wave refractive index is same in all the direction but extraordinary wave is related to this refractive index where it depends on the launching on the light. That means, if a launch a k vector in different direction, if the medium is an isotropic in nature, we have a refractive index structure shown in here which is ellipse in nature.

This is nothing but a structure of ellipse with respect to psi. So, these circles suggest the refractive and this index refractive index is same in all the direction. This is corresponding to the refractive index for isotropic system; that means, there is no refractive index difference in all other direction. This is the same for all launching angle, but here, if I launch an k vector with a particular angle say, then what happened that at this point, at this point, at this point, if I change my angle, you will find the refractive index is also changing and when the phi is here; that means, when the phi is phi by 2, according to this equation what happened, sin cos 5 by 2 is 0 this term will cancel out.

So, 1 divided by n square, if I write phi equal to pi by 2 at this point, what will be the refractive index of the system; sin square phi by 2 is nothing but 1 and it will be K z. So, the refractive index will be K z and K z is a refractive index of n. So, these refractive index will be the extraordinary refractive index here and that point. Now, if I reduce and go launch the electric field along this, so k is along this direction; that means, phi is equal to phi is equal to 0.

So, what happened from this equation? So, let me erase whatever we have. So, what we are doing, we are launching the k along this direction so that my phi is equal to 0.

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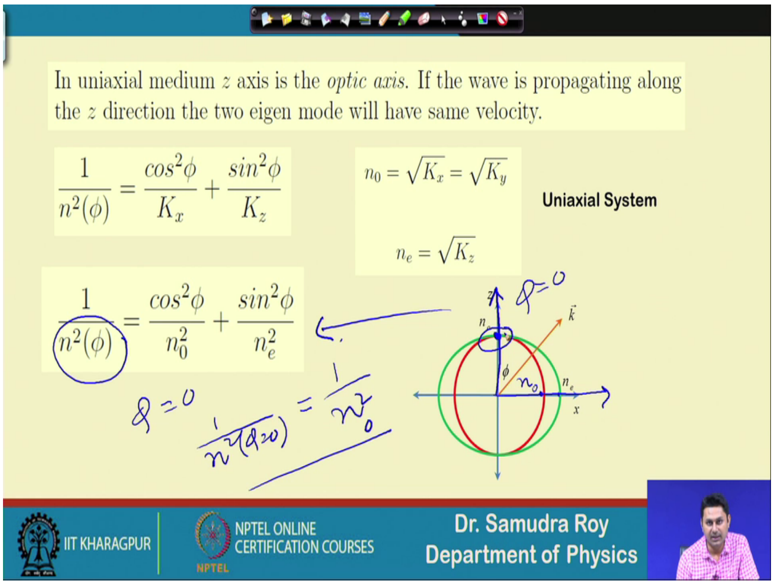

If my phi is equal to 0, if I put this things here in this equation what happened that phi is equal to 0 condition, 1 divided by n square phi is equal to 0, in this condition mention and refractive index, 0 mean this quantities is not there. 1 divided by n o square refractive index will be equal to a n o and you can see that at that point, the refractive index is n o. That means, along this direction the refractive index is same for O ray and E ray and that is why this ellipse just touches here because n value, the n value which is the function of 5 will be coincide with the value of a no at that point and if I now gradually changing the direction, then we find this value is my n o value and this value is my any value.

So that means, in this direction the refractive index is completely different for two different rays. The ray which is following the ordinary refractive index is called O ray, the ray that is following extraordinary refractive index that means, the refractive index that is depending on the direction is called the extraordinary ray. So, we will discuss in our next class. So, we will like to stop here. So, so far we will learnt that there are two kind of refractive index in one direction. We have one refractive index and perpendicular direction, this is a different kind of refractive index and there is a mismatch between these two refractive index.

So, this will help us understanding few things in non-linear optics; why it is important we will going to learn in our future classes. But in an isotropic system, it is interesting that we have two different kind of refractive index; one we call the ordinary refractive index in another we called extraordinary refractive index. And extraordinary refractive index is a function of direction; that means, it depends on in which direction I am launching the light.

Based on the direction of the launching of our light, we have different refractive index. That is interesting things. And we will learn more in our next class, what is refractive index ellipsoid and all these things, how these things look like in real picture, we are going to learn. With this note, we will like to stop our class here. So, see you in the next class and

Thank you for your attention.