Introduction to Non-Linear Optics and Its Applications
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Lecture – 59
Nonlinear Pulse Propagation

So welcome student to the next class of Introduction to Non-Linear Optics and Its Applications. So, we are almost in the finishing line of our course. So, today we have lecture number 59.

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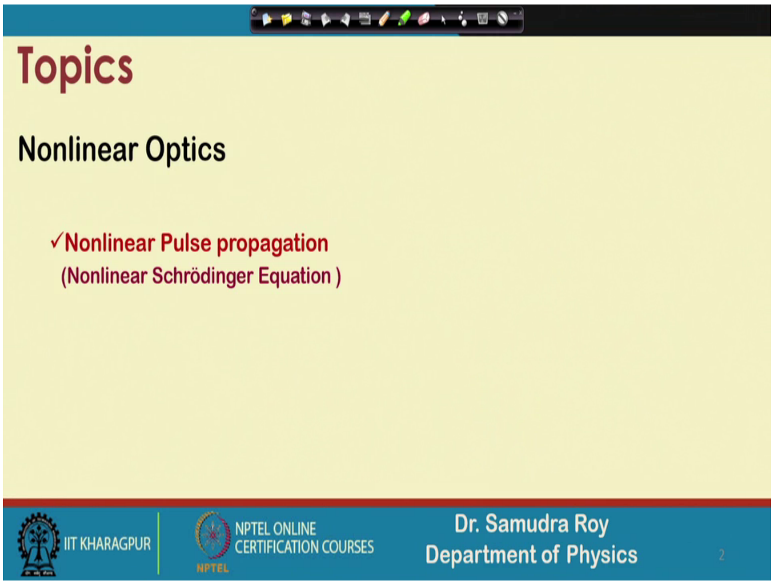

And today we will going to learn the Non-Linear Pulse Propagation. And this non-linear pulse propagation mainly governed by one very important equation or well known equation called non-linear Schrodinger equation. So, today we will going to derive this equation to understand exactly what is going on when a wave is propagating in a non-linear medium.

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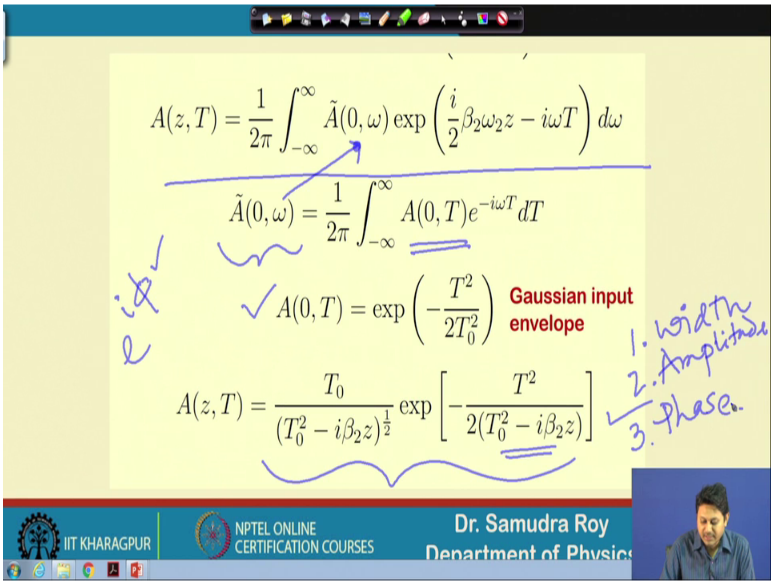

So, let us start with the previous classes result which is the Gaussian wave that is propagating in a linear medium, the linear medium contain the dispersion term. And if we see this is the Gaussian envelope we had and the solution is of the form this. So, at is the solution of the pulse envelope, at time domain and in order to find the solution in the time domain, we need to put the information here at this point A 0 omega which is the frequency of the wave or the frequency component of the wave at z equal to 0 point.

Again that information one can transform with the fact that if the input pulse shape is known in time domain, this is the input pulse shape A 0 T. In the time domain and if I make a Fourier transform of this, then we will have the value of A 0 omega and if I put this value of A 0 omega here, then we can readily have the solution which will be in this form if the input pulse is Gaussian. So, that was the small exercise I already mentioned in the last class that we should do, this exercise and if you do carefully and correctly, then you will have this kind of expression.

So, if you look very carefully this expression if I compare this input and output expressions, then you will find there are certain changes, one change is here which is width so, first width is changing. Secondly, amplitude part is also changing if you carefully see if beta 2 is equal to 0; that means, there is no dispersion then the amplitude becomes 1, but if beta 2 is not equal to 0, then we have amplitude and this amplitude has a real and imaginary part or some sort of complex amplitude, not only that the amplitude is also reducing because of this part.

So, amplitude is affecting and finally, and finally this entire pulse is having some sort of phase here, in the input we find that only envelope term is there and everything is real there is no phase term, phase means there is nothing relate e to the power i phi kind of term. Only thing is that we have a real amplitude here, but if I write this equation properly this expression properly then we should have a phase term here so, phase is evolving.

So, we find that even the pulse is propagating in linear region, the width amplitude and phase all are changing for a Gaussian input pulse.

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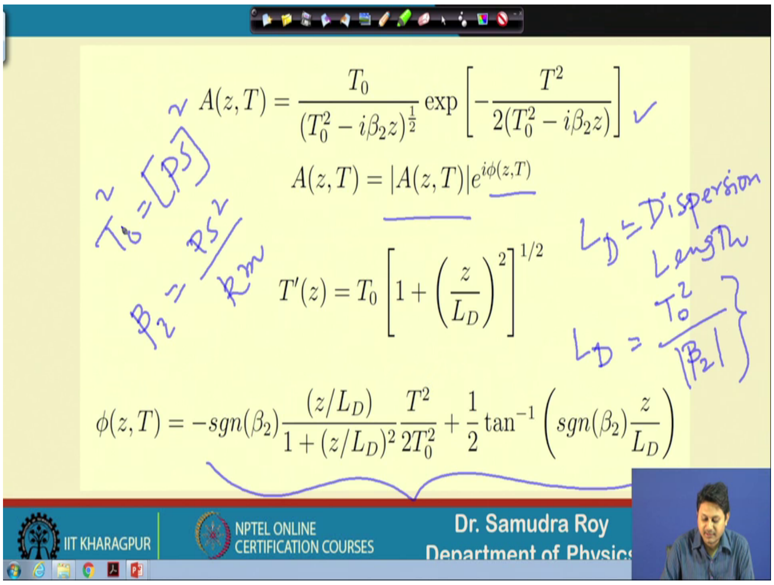

Now, if I find this try to find this thing, then I can have some kind of expression of the pulse width and the phase which give us important informations, this is the pulse shape Gaussian pulse shape that we had so, we had the Gaussian pulse shape and then this Gaussian pulse shape is written in terms of amplitude and phase as I mentioned.

If I calculate this phase then I will have this term this big term, if you calculate properly then you just you should have this kind of term, where we introduce a new term L D which is called dispersion length dispersion length, L D is nothing, but T 0 square divided by mod of beta 2, this is the quantity we called L D and everything is written in terms of L D, this is called the dispersion length T 0 square divided by beta 2 T 0 is the pulse width at z equal to 0 point and beta 2 is a dispersion coefficient.

If you carefully look the unit of these things, then L D is a since it is a unit of length because it is representing some sort of distance, then right hand side should be the unit of length also. So, here t 0 will be unit of say Pico second and T 0 square will be of the this will give you Pico second square the unit and beta 2, we know that is the dispersion parameter and its units is Pico second square per kilometer normally, this is the way we represent the dispersion coefficient Pico second square per kilometer or Pico second square per meter.

So, now if I make T 0 divided by mod of beta 2, you can see that the unit of this quantity become kilometer so; that means, it is a unit of length that is why L D basically represent a dispersion length. And this length become very very high, when beta 2 is very small that means, if the dispersion effect or dispersion coefficient is small, we should have the dispersion effect at very long distance that is the physical meaning of these things well.

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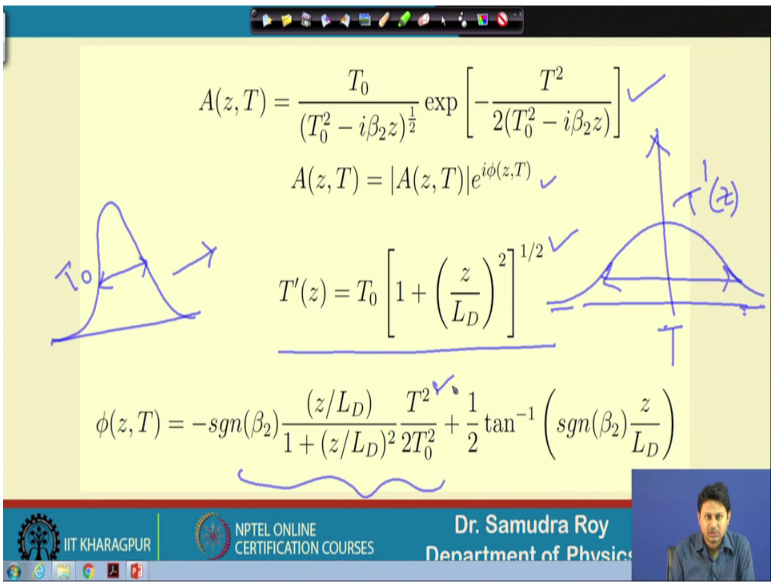

If I now look this term which basically suggests the evolution of the width pulse width, then you can see that if I increase z, then the pulse width will going to increase. So, if this is my input pulse which is some sort of Gaussian shape, having some initial width according to our notation it is T 0, then after the propagation distance it should be something like this, amplitude will decay and width will going to increase this is in time.

So, this quantity is nothing, but T prime which is a function of z and how, T prime that is the width of the pulse will going to increase is given by this expression.

You can readily find this expression if you write this equation in amplitude and phase form that is written. So, as I mentioned this is given to you as a home task. So, you should do that and just find the wave in this particular form and you will get this width. After having the knowledge of the width the next thing is to find out the phase which is written here and, also we can see that phase is changing and not only that phase is a function of time. And now here we find that phase is a function of time and it is a T square.

We know that the frequency instantaneous frequency we mentioned that in a earlier classes.

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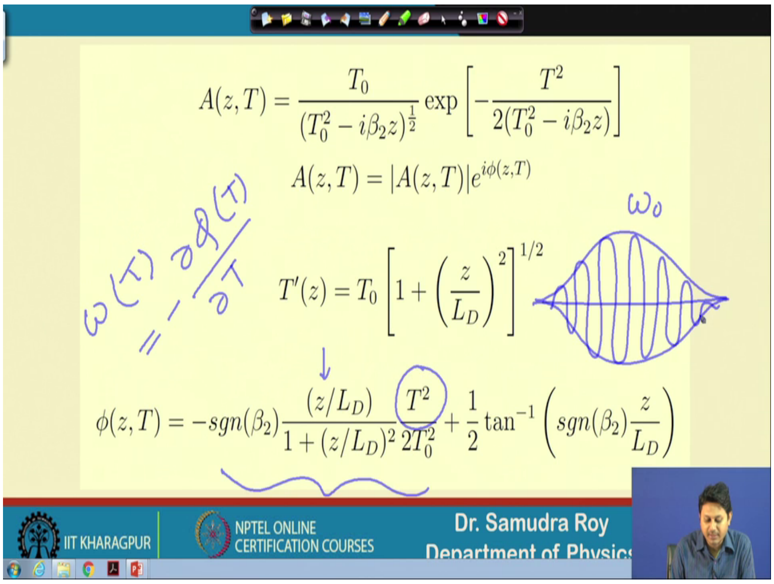

Instantaneous frequency which may be a function of time is represented by minus del phi which is a function of time del T, if I make a derivative with respect to T of a phase phi, then we have instantaneous frequency. But if you make a double derivative of that quantity, then we have something called chirp, here you can see that this quantity has T square so; that means, I can essentially make a double derivative and if I make a double derivative of this quantity I should have something called chirp.

So, the chirp basically a function of z here and what is the meaning of chirp; that means, the frequency distribution is not uniform. So, if this is my wave this is the envelope and this is the wave this is the distribution of the frequency inside the wave, it is a wave packet lastly we mentioned that. So, this is a some sort of uniform frequency distribution. So, if I write this frequency at omega 0 inside the wave packet, we have a frequency distribution omega 0.

But the same thing if it is chirped, if I draw that this is the freq envelope and for the chirping case, what happened this frequency distribution is not uniform some part of the frequency may be small and some part of the frequency may be large, when I plot this in time say T and it is changing linearly over the time, which basically gives this form. So that means, when the pulse is propagating in a dispersive medium what happened that the amplitude of the pulse will go down width of the pulse will go will increase and at the same time the pulse will be chirped.

Now, if we remember that what happened in self phase modulation case, then similar kind of effect was there. And there also we find if you remember that was the distribution of intensity. And if I make a derivative of distribution of the intensity, then it was something like this. And based on that we find there is a chirping of the pulse. So, self phase modulation is the effect where also we have chirping you should keep it mind that dispersion effect individually gives something, which is chirping and self phase modulation also give something which is chirping.

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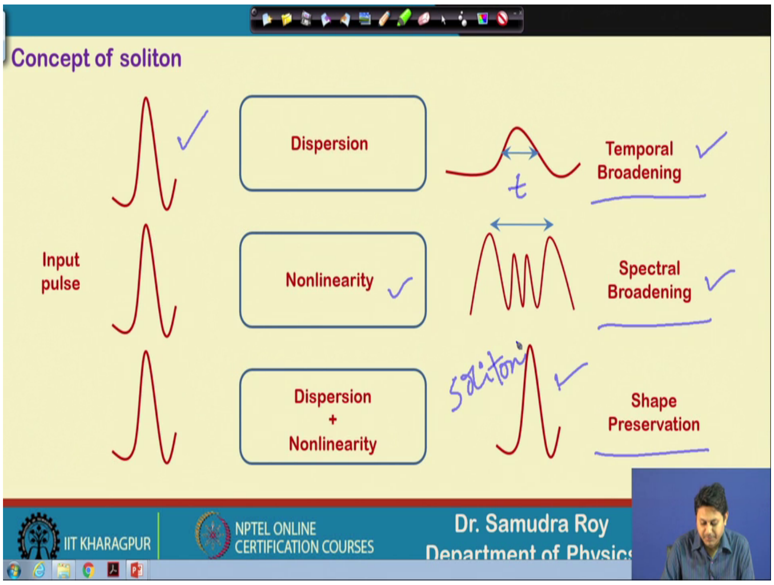

Now, with this knowledge we can now have a concept of soliton. So, what happened when the pulse is moving individually in dispersive medium, individually in non-linear medium and dispersive plus non-linear medium both. So, what happened let us try to summarize this. So, this is the input pulse in time domain, when it is moving through a dispersive medium only dispersion take places and when the dispersion takes place then what happened, there will be a temporal broadening as I mentioned the pulse will broaden in time domain.

So, this is time similarly if the same pulse is moving in a medium, where dispersion is not there only nonlinearity is there we should have a spectral broadening the specter will broaden, whatever the specter you have it will broaden. So, here we have temporal broadening here, we have spectral broadening. In this case we have chirping here also we have some sort of chirping. So, now, the idea is if the dispersion on non-linearity is put in such a way that, they can counterbalance each other, that there is a possibility that these temporal broadening can be restricted as well as this spectral broadening can be restricted and as a result, we will have something where there is no change of pulse shape this is called the shape preservations.

So, I can preserve the pulse shape by adjusting the dispersion and non-linear parameters suitably. So, that there is no change in pulse shape in time domain as well as frequency domain. So, this thing is called soliton. So, soliton is something where soliton is something where the shape of the pulse width will remain conserved during the propagation.

So, this is the robust structure so, when a optical pulse is moving in a medium, where dispersion and nonlinearity is there. And the value of dispersion and nonlinearity is maintained in such a way, that the effects of dispersion and nonlinearity are counterbalanced by each other, then we have a pulse which does not change it is shape. So, this is the basically the concept of optical soliton, we will learn more about this soliton maybe in the next class.

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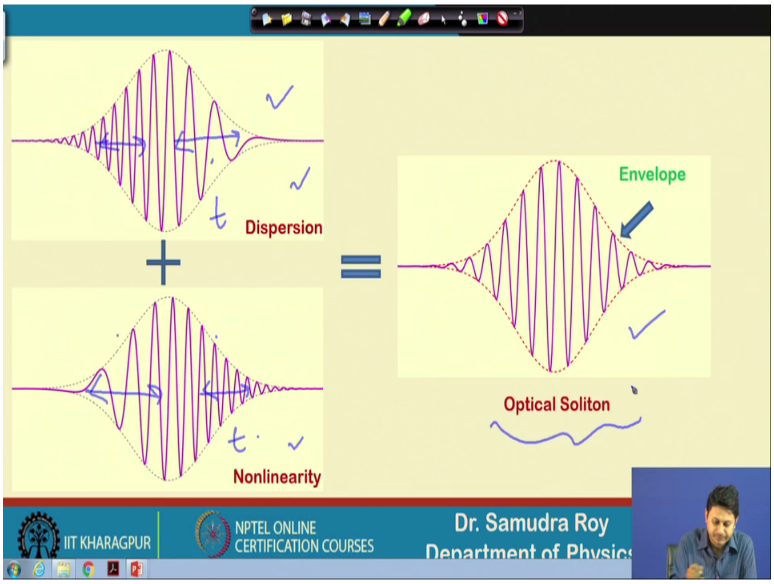

But try to understand what will happen inside the pulse. So, as I mentioned during the dispersion, we have some sort of chirping and due to the nonlinearity the self phase modulation, we have also some sort of chirping. So, here in this schematic plot I have shown that this is a pulse which chirping you can see that, this portion of the pulse is very fact; that means, the frequency is decreased here, but here the frequency is increased linearly there is a change of frequency inside this pulse. So, that is called the chirp.

But for nonlinearity what one can do that they can have a similar kind of pattern of chirping, but in the opposite direction so, if it is t. So, here in the leading part of the pulse, we have small frequencies and the frequency get higher and higher in the later part of the pulse. So, exactly the opposite that we have in dispersion. So, if I add these two things then what happened these things will be counter balanced by this one, this part of the pulse will be counter balanced of this part. And as a result we have something where the frequency distribution is uniform so; that means, there is no chirping and the shape of the pulse remain conserved, this is essentially the optical soliton.

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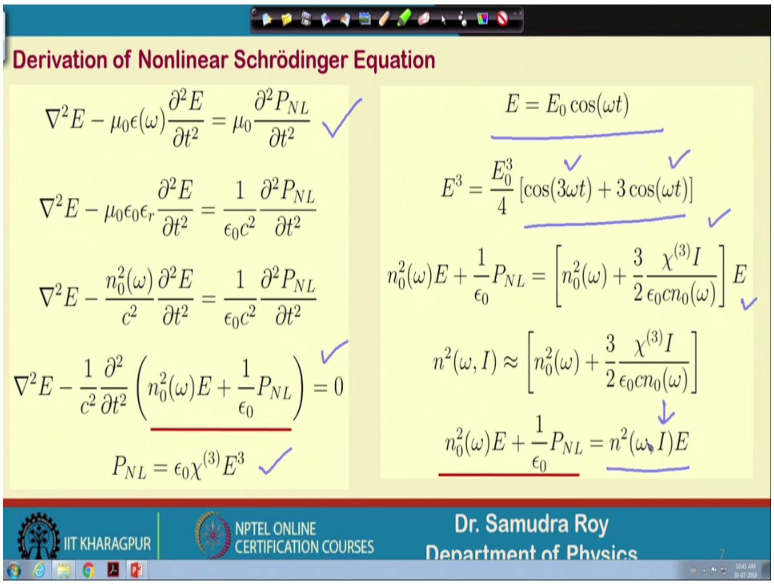

Well, once we have the idea optical soliton now, it is time basically to understand how this optical soliton is evolving and what should be is a governing equation, this governing equation is essentially the non-linear Schrodinger equation, it is a very famous equation. So, in non-linear Schrodinger equation what happened that if I take this equation which is the Maxwell's equation. From this Maxwell's equation non-linear Maxwell's equation, we can derive our a non-linear Schrodinger equation.

So, previous in the previous class base, we have we start with this Maxwell's equation non-linear Maxwell's equation and mentioned that, if P non-linear is 0 I have the equation of motion equation of this electric field. And this equation of electric field was governed by these two parts. Now, we include the non-linear part so, when we include the non-linear part what happened so, this is mu 0 mu 0 epsilon. So, I can write this epsilon omega as epsilon 0 E r and mu 0 as 1 by epsilon c square.

And then I can put this side and I can write here in this case 1 by c square d 2 E and then n 0 square E plus 1 by epsilon P non-linear. So, just manipulating whatever the equation is written very simple, then P non-linear is epsilon 0 chi three E cube. Now, E is the electric field so, if E is the elliptic field. So, I can write this electric field really electric field in this form E cube, I can write by making a cube and then I can write it is as 3 omega omega component that, we have already done when we are dealing with the self phase modulation.

Next if I put this here in this equation, then what happened that I should have a frequency component 3 omega and the frequency component omega. So, frequency of 3 omega component is basically give us the third harmonic generation part. So, third harmonic generation part we may neglect it at this point, because third harmonic generation requires some sort of phase matching to evolve. So, if the phase matching is not there, which is the case in general then only this three omega part is there. So, E cube is essentially E 0 cube by 4 into 3 cos omega T.

So, this 3 cos omega T I can write E 0 cos omega T which is E and I can take this E outside. So, another term is there which is E 0 square so, E 0 square can be represented in terms of intensity so, if I do all this manipulation, then this portion which is given in the red line can be simply written as n 2 square E, where n n square E where n is some sort of intensity some sort of refractive index depending on intensity that is important. So, the entire equation become simplified with the fact that now, whatever the refractive index we have it should have an component of intensity well.

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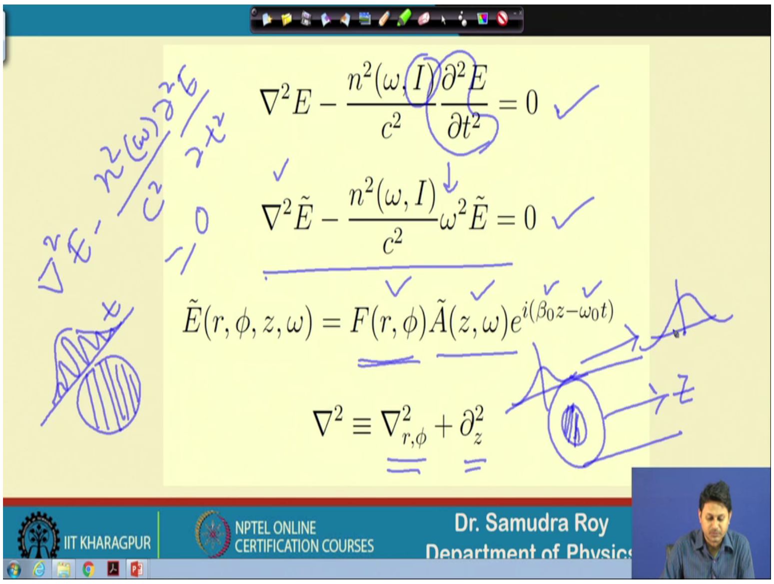

Next we will write the equation whatever we derived and this equation suggest that, this will be my equation now to solve this is the non-linear equation and the nonlinearity is introduced here. In linear case the form of the equation looks quite same the form of the equation was this, it was just omega and when refractive indices represented in terms of omega only; that means, this is the linear case. So, we were doing the same thing, we are having a same kind of expression even by introducing nonlinearity.

But here you should remember that n is now function of intensity also. Now, we know that whenever we have this kind of equation in our hand it is easier to solve it in Fourier domain and, if I try to solve this in Fourier domain, which we have done in the previous class also the technique we discuss in the previous class. If we apply the similar kind of technique we will find that this equation can be simply written in Fourier domain like this. So, this derivative d 2 d square derivative can be simply represent replaced by omega square and the electric field is now in the Fourier domain. So, it should be E tilde.

So, this equation is relatively easy to solve. So, that is why we write this in Fourier domain. Now, if I want to solve completely, then the I need to write the electric field in total form. The electric field in Fourier domain can represent it total form, this is the envelope part and this is the field distribution, when we write the electric field so; that means, electric field should have two component, say this is my laser light this is that, this is the laser light that is falling on some wall or something.

So, there is a distribution over space. So, this space distribution is represented by this r phi coordinate and also there is a temporal distribution of this pulse if it is from two second pulse this pulse is distributed in time domain so; that means, some sort of component should be time component should be there here it is written in Fourier domain. So, we have the frequency component and it has some sort of frequency and this frequency and the propagation constant this component should also be there.

So, there is a complete form of electric field, in terms of spatial distribution and temporal distribution. So, now, this grad square operator can be divided into two part one is the transverse part and another is the longitudinal. So, if this is the waveguide or the fiber, if I consider this is a fiber. So, it is moving like this is the core of the fiber and if the electric field is launched here it will move along this direction z. So, there will be a spatial distribution, we call it more and along the z it will going to evolve and, this evolution along z can also contain the modal part and the temporal part.

So, the pulse which is distribution in time domain can move along z and we should have some equation for this pulse distribution also which is a entanglement.

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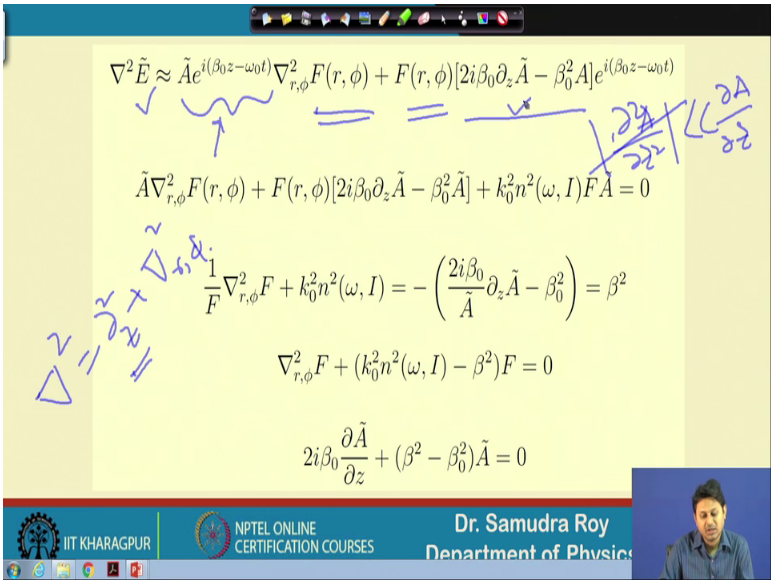

So, if I now extract the these two parts separately, then grad square E I can write simply the derivative. So, grad square is first the derivative with respect to z square plus the transfer component which is r phi component. So, when we when we apply that over E. So, separation of variables suggest that only the F, this we will operate only the F, because F is a function of r phi. So, this quantity will remain outside of the operator.

In the similar way when we make a derivative with respect to z, we should have F r phi outside and all these things here. So, here you should note that when I make a derivative of the quantity, this I should have a double derivative with respect to z, but this quantity is very very less than the first order derivative of the envelope. So, we can neglect that this is essentially the slowly varying envelope approximation.

So, when I make a slowly varying envelope approximation, we just neglect this term and we have this term in our hand. So, this is again something we have done before. So, this is not a very new thing only thing is that here we have we are dealing with two different part of the equation one is related to F. And another is related to a tilde. So, now, if I complete right completely due to this equation I will ended up with this and if I make a separation of variable thing, then I can put this one side and the envelope term in other side.

This is a function of r phi this is a function of z only. So, when these two are equal then I can write this as a function as a constant, we call this constant beta, beta is essentially the propagation constant of a system when the nonlinearity is there. So, eventually we have two equation in our hand one is the field equation this and another is the envelope equation. We will write now we will not bother about this field equation rather, we will try to find out how these envelope equation basically give us this non-linear Schrodinger equation.

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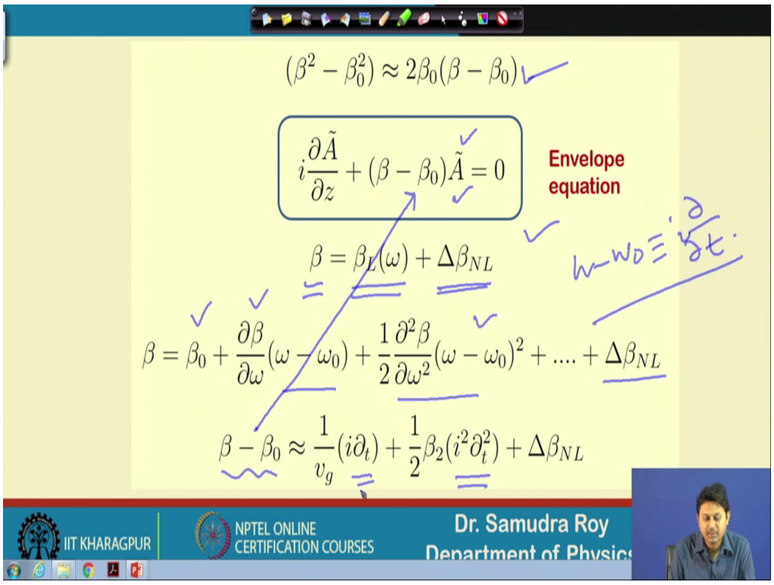

So, beta 2 this quantity I can simply write this which we had done in our previous class. So, this basically simply become the envelope equation. So, 2 omega 2 beta 0 term will cancel out both the case both the side. And we will have that, now the beta which is a propagation constant, I can write it in the two part which is the linear propagation part and due to the nonlinearity the propagation constant may change. And I write this propagation changing propagation constant at delta B N L.

Next what we will do we just expand this linear part. So, if I expand this linear part it will be beta 0 first order derivative of beta omega minus omega 0 second order derivative and so, on plus delta beta N L, now beta minus beta 0 if I take up to this will be simply this quantity. And now I replace omega minus omega 0 to the derivative so, omega minus omega 0 I can always rip the last class, we have done this we have shown this, but we can change it to this operator.

So, when you change this operator; that means, these things I put here. So, this operator is basically will operate over A which is in frequency domain so; that means, I need to change this A to time domain, because now I change the entire part in time.

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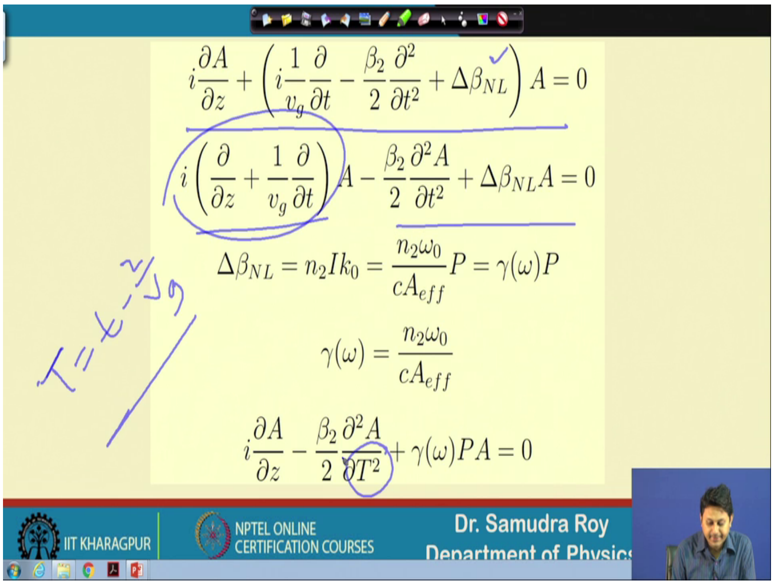

So, if we do that if we do that then we will have this expression, this is a derivative first derivative and this is a second derivative corresponds to omega minus omega 0 and omega minus omega 0 square term and delta beta N N L will be there.

So, now here I have this part and the rest of the part I write this we know this, because this is basically the transformation that we are looking for and this transformation is t minus z by v z, when we have this kind of Galilean transformation, we are moving basically the group velocity of the pulse. So, this coordinate is now changing to t, what about delta beta delta beta N L is the non-linear propagation constant which is the N 2 I k 0 and I can write it as n 2 omega c effective P. And essentially we will have this expression in our hand.

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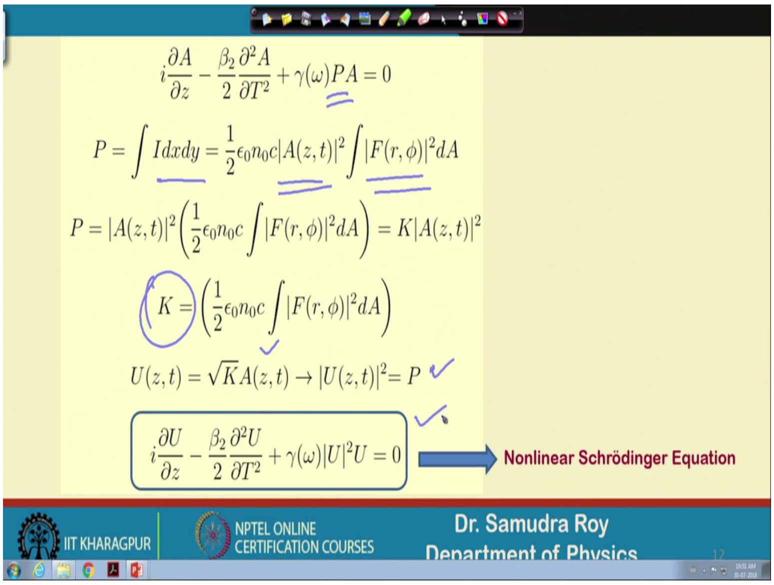

So, let us see what else so, this basically expression can be now written by putting some kind of rescaling and, when you put this rescaling stuff, we have this P I can write it as I integration of dx dy. And I again I write in terms of a square mod of a square because intensity can be represented in the field squared term. So, field square is a square and also this F and if I write this entire stuff as a constant K, then a my rescaling is this.

So, if I rescaling make a rescaling of this equation, then I will ended up intercept which is a well known non-linear Schrodinger equation. So, this is the equation which is a non-linear Schrodinger equation. So, in the next class we will again discuss about the non-linear Schrodinger equation and try to find out what should be the solution of this non-linear Schrodinger equation and how this solution leads to optical soliton. So, with this note let me conclude here.

Thank you for your attention and see you in the next class.