Introduction to Non-Linear Optics and Its Applications
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Lecture – 58
Linear Pulse Propagation

So, welcome student to the new class of Introduction to Non-Linear Optics and Its Application. So, in the last class, we have started the Raman process, today we will have lecture number 58 where we will going to start a new brand new topic which is optical pulse propagation.

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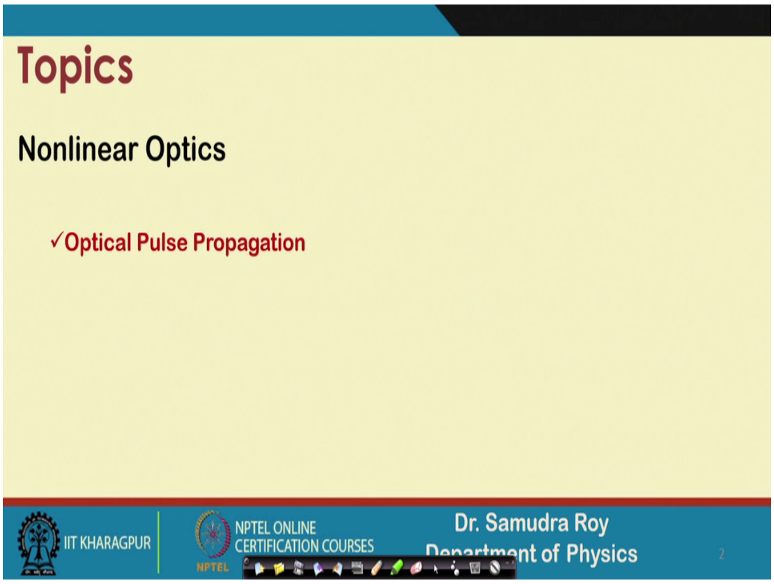

So, this is some sort of advanced topic optical pulse propagation, but we will not going to study in detail, but the overview of a very important concept called optical soliton which is very relevant in non-linear optics. Specially the non-linear optics in fibers where we can generate a structure which is very stable and very robust called optical soliton.

So, how this structure will form and what should be the background mathematics. We will going to learn in this last 3 classes which is a little bit advanced, but I should cover this in this particular course also. So, let us start with this optical pulse propagation. So, before going to start this non-linear pulse propagation, we need to know about the linear pulse propagation, the detail about the linear pulse propagation and the related mathematics.

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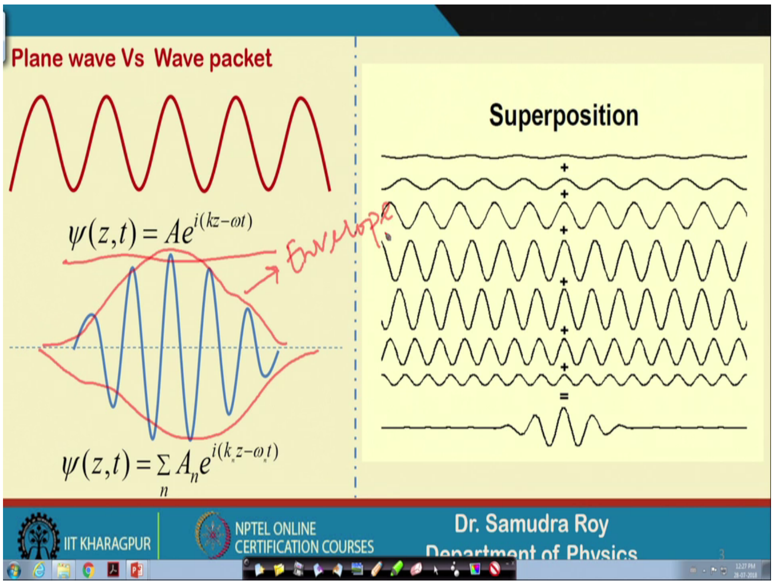

So, let us start with very simple concept and that is the plane wave versus the wave packet. So, there are 2 type of wave we are talking about. So, one is in this particular in this course. So, far we are dealing with mainly plane waves and as the figure suggests the plane wave is in finite distribution of infinite wave with a single frequency. And why it is called plane wave and what is the consequences of this equation whatever the equation is written here why this equation is representing plane wave.

We have already discussed these issues in our initial part of these courses. So, there was a special topic or linear optics and in this linear optics we discussed this issue. So, we will not going to learn again at the same story. So, I believe all of you are now aware of this plane wave because we are now in the almost in the finishing line of our course, but what is important here is the concept of wave packet. I believe many of you have already aware of this wave packet.

So, wave packet is something where you can see the significant difference is we have some sort of distribution over amplitude of what we have. So, there is some sort of amplitude kind of thing that is changing. So, these things are called envelope. So, how a localized kind of things can be generated form plane wave it is in interesting.

So, in the right hand side you can see that there are many plane waves with different frequencies, this is once one sort of plane wave this is another sort of plane wave this is another sort of plane wave all are plane wave with some specific frequencies and so on.

Now, if I start adding we call it the superposition its read in this superposition, if we start adding all this wave together then what happened we will find some kind of beating effect. That mean in some cases we find that some portion of the waves are there and then some beat formation will take place. So, now, this number is very very high and tends to infinity we have only one kind of beat formation like this. So, pulse is very much or the waves are very much localized. So, we call these waves or wave packet a packet of waves.

So, this packet of waves is essentially consists of the superposition of the plane waves have in different frequencies. So, if I now write this mathematically the same equation is slightly modified here you can see that the amplitude is now not a single amplitude because here in the right hand side you can see that, amplitude is also changing and also the frequency is are changing so, as the corresponding wave vectors. So, in both cases we should put some kind of suffix here and since it is a superposition we need to sum it up.

So, when we sum; when we sum it then the resulting thing will give you this kind of waves which is some sort of localized wave or we call it wave packet. So, now, we were dealing with optical pulse is and try to understand how the optical pulse will going to propagate inside the system. That means, there is a localization of the optical field and this localization is essentially mentioned by this envelope thing and it is distributed over a time.

So, the wave the plane waves are distributed it is a infinite kind of thing in time domain normally we called cw or continuous wave, but in time domain if it is localized then this is called the pulse. That means, we have a amount of electric field at particular time which is very high and then gradually decaying in both the side and eventually 0 for very high t value in plus and minus direction.

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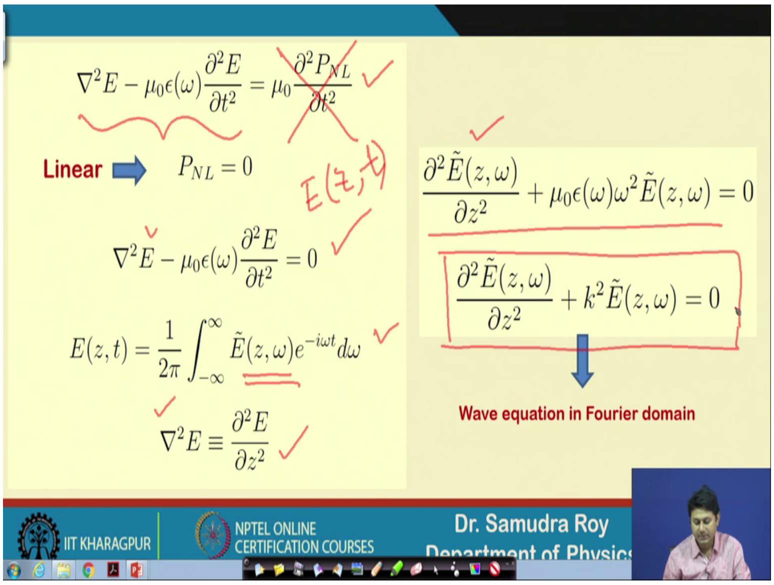

So, with this with this thing we can now, go to our next slide with the knowledge of plane wave and these things we can go to our next slide and try to understand what should be the equation what should be the governing equation of this kind of wave and so. So, the governing equation here is essentially the Maxwell's equation, but since we are very much familiar with this wave, this thing this equation which is a non-linear Maxwell's equation we have been using this equation for long time.

So, we I decided to start with this equation, but since we are dealing with linear case. So, I will not going to take this term into account. So, this term is not there; that means, there is no non-linear polarization term is present. Let us consider this for the time being, but in the later part or in the next class maybe we need to take account this also in order to include the non-linear effects.

So, if I discard the this term P non-linear then we will have only this term in our hand which is simply a Maxwell's wave equation without any non-linear component in it. Now, if I want to solve this equation this kind of differential equation one very standard way that to make this equation in frequency domain.

So, here E is electric field, but this electric field is in time domain. So, it should be a function of z and t if I make a Fourier transform of the electric field that we always can do then we will have a relationship with the Fourier component and electric field like this. Derivative this can be simplified to the z derivative only because e is a function of z.

So, grad square operator is simply the derivative over z partial derivative over z and there should be double derivative because it is a square. So, if I replace this here both the thing, then we find that this equation whatever the equation we have is now converted to a relatively straightforward equation, a homogeneous second order kind of differential equation.

But you should remember that since we are making a Fourier transform over that. So, instead of having a time domain picture now we will basically solve this in frequency domain and finally, we have a equation like this. So, this is some sort of wave equation, but E is in frequency domain, fine. So, once we have this equation in our hand the solution is also readily available because I know what is the form of this solution.

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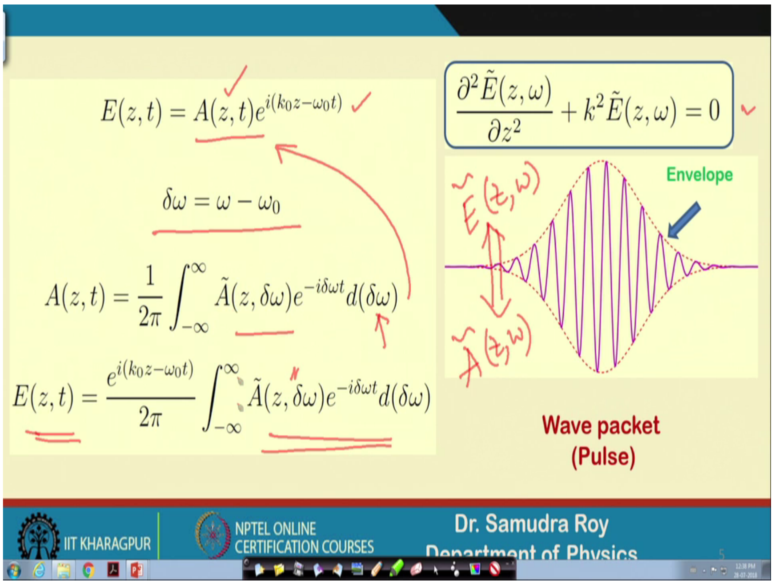

So, before solving these things, now we do one important thing and this important part is to include this envelope stuff into the electric field. As I mentioned that so, far we are dealing with plane waves. So, whenever we are dealing with plane wave we should not bother about this superposition of different plane wave and so. But now we are dealing with optical pulses.

So, when optical pulses are in the picture then what we need to do is to introduce some kind of term here in E and this is A. So, I write the total electric field a total electric E into 2 part amplitude and the as usual phase k 0 and omega 0 is a central frequency of the inter superposition stuff and the corresponding propagation constant. Why I am saying superposition because the A is forming a structure by superposing different kind of waves; different kind of plane waves.

So, if I now try to write this a in a suitable form then basically I will write in this fashion this is some sort of Fourier kind transform thing. So, a is consists of different Fourier components and if I start adding all Fourier components with all frequencies that we have done that we have shown in the first slide where we show that different frequency component of different plane wave are superposed to each other. And generate a wave packet kind of things here exactly we try to do these things and we will have a form of a like this.

Now, what we will do we will put this A here and my E is now in this form my total E is now this form, where A is the envelope this representing the amplitude of this envelope. Inside the envelope we have some frequency distribution and this frequency distribution is omega 0; Another thing is we make this stuff we add all the terms by making omega 0 as the central frequency and around the central frequency we add all the terms from minus infinity to infinity. So, that is why delta omega is a new variable this new variable basically suggests that we are adding everything by making omega 0 as our coordinate omega series our origin or the central frequency.

So, mind it this is the differential equation we had in our hand and if I want to solve this equation I need to put some sort of form of E. So, E now I have taken into 2 into a part where A is introduced which is the envelope thing and then I try to find out what is the relationship between this quantity because in the equation we have the Fourier component of e.

So, I need to find out what is the relationship between E and A, because E and A should be related. So, the recipe is already shown here so, in the next slide we will try to understand or we will find what should be the relationship between E and A.

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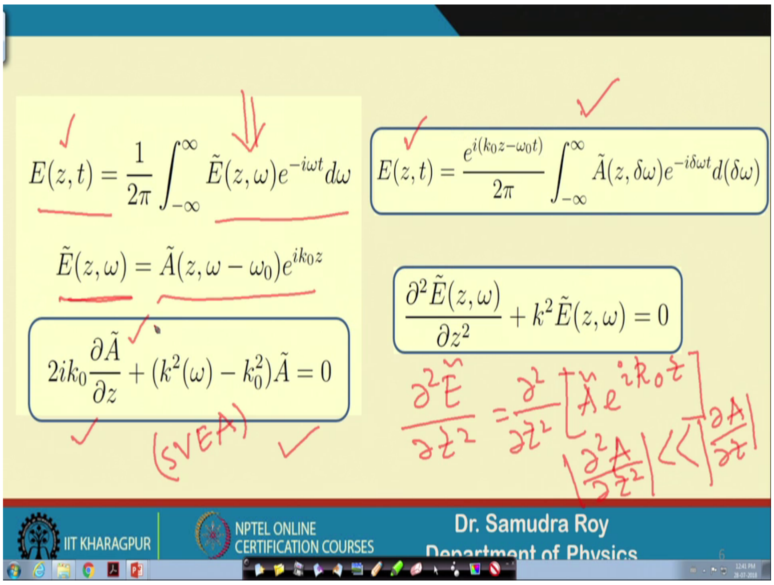

So, E z under Fourier transform these are the Fourier components of this is it can be represented in terms of a like this. So, both the cases this is E z left hand side this is E z. So, now, if I start comparing these 2 equations, if I start comparing these 2 equations we will readily find that e z tilde will be this quantity E z omega minus omega 0 e to the power ik 0 z just comparing this and this because both the cases it is e z.

The wave equation is in my hand and it is in terms of e tilde, but e tilde and a tilde this relationship is now in my hand. Once we have this relationship. So, I can replace this a e into this a thing here in terms of a and if you do that you can readily find out the differential equation for envelope term a which is important because we need to understand how the envelope thing is evolving.

So, in this derivation please note one important thing again I am using this. So, d dz square is eventually d 2 dz square E tilde is A tilde E to the power of I k 0 z a tilde is the function of z and e to the power I k 0 z this is also a function of z. So, this is a multiplication of 2 functions.

So, when we do these things we will have a first order derivative of which is there and also a second order derivative this term is also there. But we know that this term is very less compared to the first order derivative; weightage of the first order derivative because a is varying very slowly.

So, this is again we are using the slowly varying approximation here since it is envelope we called slowly varying envelope approximation; slowly varying envelope approximation in many books it is written as SVEA slowly varying envelope approximation.

So, by applying this slowly varying envelope approximation we can discard this second order derivative term with respect to z and we have if only the first order derivative once we have the first order derivative then things are relatively simpler because now we x can solve this differential equation.

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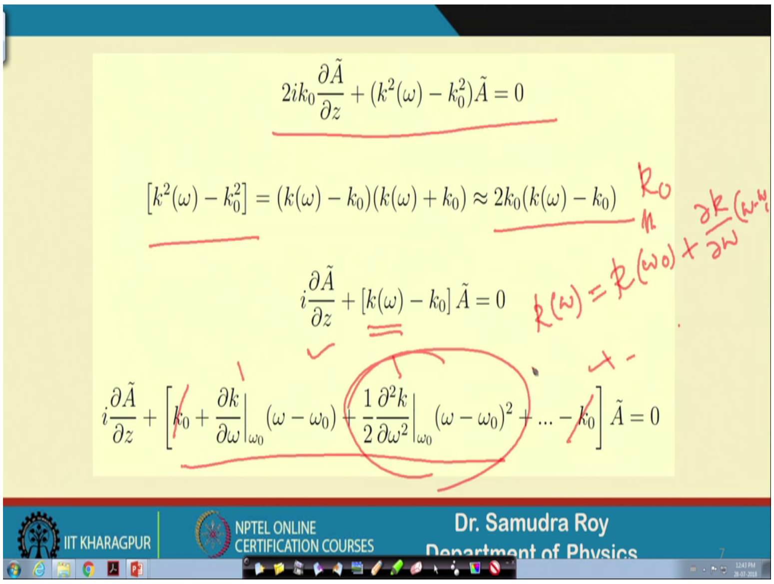

Well let us see how to solve this equation. So, we had our differential equation in our hand which is this is basically the envelope equation, but we will going to simplify little bit.

So, we have k square omega k 0 square this term. So, I can write this k square omega k 0 square as k minus k 0 multiplied by k plus k zero. So, I can write it as 2 k 0 k omega minus k 0 because k omega and k 0 are very close to each other the propagation constant will not going to very much with the propagation constant of the central frequency we assume that.

So, when we assume that this square term will go away and this 2 k 0 term will also going to remove because here we have another 2 k 0. So, these 2 k 0 and these 2 k 0 will cancel out we have I a tilde k omega minus k 0 a after that we simplify the equation further k omega is a propagation constant at frequency omega. So, I can expand this frequency component around the central frequency which is at omega.

So, this Taylor series expansion of k omega is simply k 0. So, if i write k omega I can write it as k at omega equal to 0 which is our notation k 0 and then the higher order term. So, d k d omega multiplied by omega minus omega 0 plus 1 it is written here only then we have minus k 0. So, this minus k 0 and k 0 term will cancel out and we only have this term; this term in our hand.

Obviously, our higher order terms will be there, for the timing will not going to consider all these higher order term only we consider these this first order this second order term and first order is already there.

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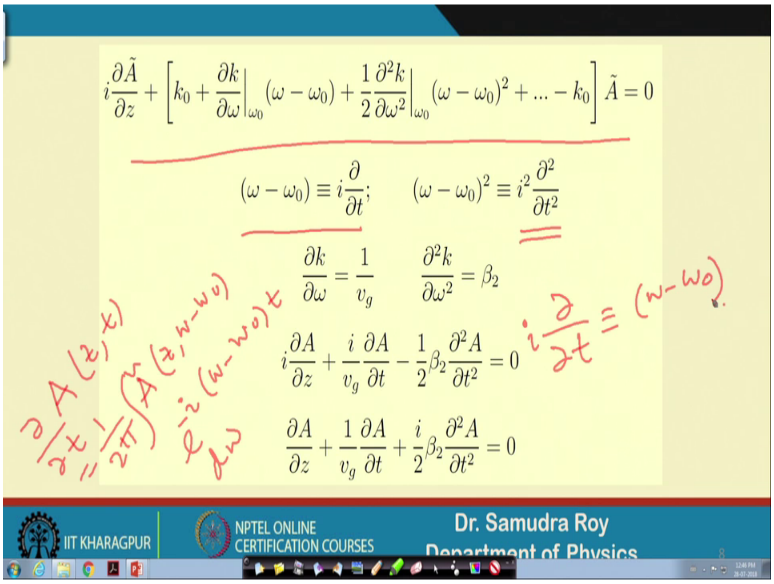

Once we have this term. The next thing the next thing is very important that we have omega 0 and a tilde; so, everything in frequency domain. So, since everything in frequency domain here we can transform this thing into time domain, which we can do very easily by just replacing this operator to i del del t and these things to this.

I can put these things as a small hormone to all of you I would like to put to show that that if I replace this operator then the entire equation will be in time domain, it come to time domain what do you need to do that you just need to use the Fourier transformation.

So, the relationship between A and A tilde. So, A z t was 1 by 2 pi z omega minus omega 0, e to the power i think omega minus omega 0 t d of omega minus omega zero. So, it should be just d omega.

So, now if I make a derivative time derivative with respect to both the side; So, I will have a del del t, when you make del del t you will find that in the left hand side. This I omega minus omega 0 term will come out. So, this I omega 0 term will come out and we will have the same equation in our hand.

So, eventually we get a relationship that this operator is if I might so, there is a I think there is a negative sign here. So, this operator, basically give us these operators basically give us this quantity. So, if I just change this operator then we will have an equation and this equation will be entirely in time domain.

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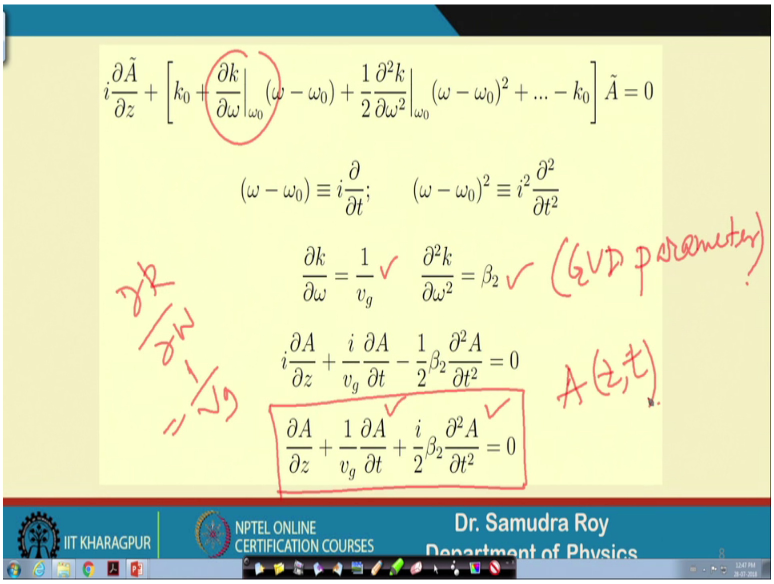

Another important thing that you need to note that this quantity d k d omega is inverse of group velocity. So, this is one by v g and I write d 2 d d 2 kd omega square beta 2 which we call the group velocity dispersion parameter. Normally we call the second order dispersion or simply dispersion.

Now, replacing everything I will ended up with an expression like this. So, one group velocity term is there 1 dispersion group velocity dispersion term is there and we have d

A d z where a is in time domain, mind it because we convert our equation in time domain.

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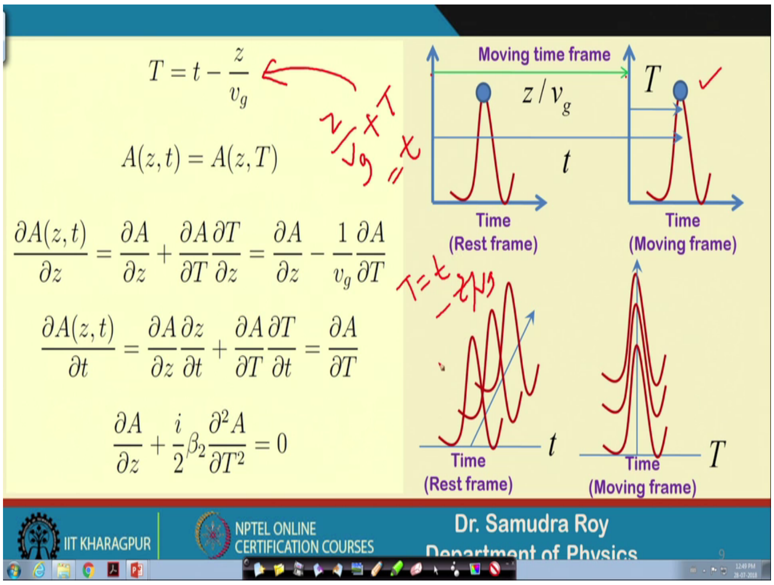

Well, after that what we will do we will just try to understand the consequence of this group velocity term and this group velocity term basically give us. The group velocity term basically gives us a very important thing and which is the moving time frame.

So, moving time frame is in our equation we have one by v g. So, let us try to understand from this picture. So, if the pulse is in distributed in some time domain and if this this is the rest frame and another frame that is moving and we called it big T. The coordinate in the moving frame we called big T, then this particular point this point can be represented in terms of rest frame as T.

But from moving frame it is big T and the moving frame is moving with a velocity v g. So, what is the difference between this and this should be z divided by v j. So, now, if I equate these things try to understand. So, this quantity plus this quantity; that means, z by v g plus T is equal to our small t. So, exactly the same equations written here, only thing I write in terms of t; so, T is equal to t minus z by v j.

So, in moving frame I can write a new coordinate T and this t is the move time coordinates of the moving frame. So now, if I understand here so, in moving frame what happened in rest frame what happened the pulse we move with a group velocity v g. But if I change this frame and if I start move with the same velocity that of the pulse then I will see always the pulse is standing in a same position. So, this is in the moving frame.

So, I can convert this entire thing into moving frame and if I convert this my equation become simply this. So, I can absorb this one by v g eta how these things works it is shown here, I will not going to discuss because this is a elementary calculus. So, you can just change this variable to this variable by using chain rule and you will readily get the result, but the important thing is that. Now, the entire stuff is coming in terms of big T which is a time coordinate in moving frame.

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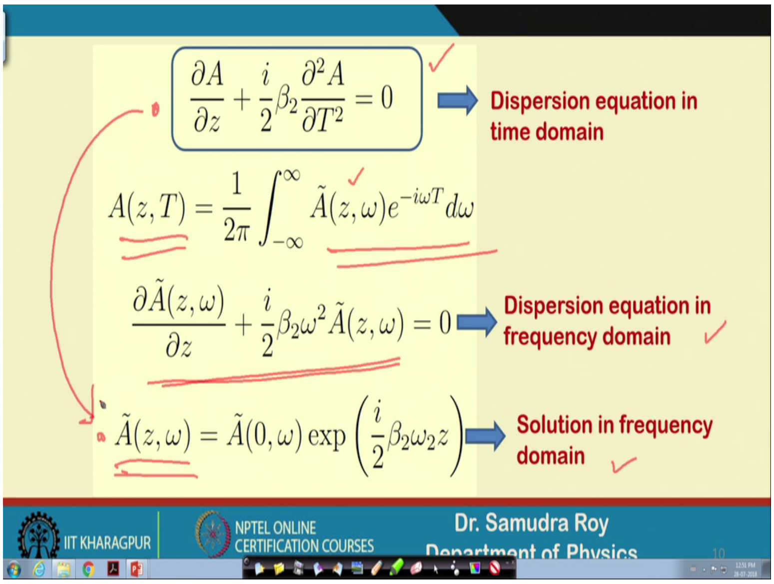

Well so, equation is further reduced by reducing v g. So, next thing is just to solve this equation it is called the dispersion equation. So, this dispersion equation we can solve again by putting this A 2 a Fourier transformed like something like this and when we make a Fourier transform of A; A is in Fourier domain.

I can go back the same equation in Fourier domain and I can have an differential equation which is this and this is basically the dispersion equation in Fourier domain which can be solvable and avidly you can solve this is the first order differential equation, its first order differential equation can be solved easily. So, we will have the value of a z tilde at omega a z omega is this quantity it is a solution in the frequency domain.

So, I have this equation in my hand and I have a solution, but kindly note that this equation in time domain, but the solution in Fourier domain. Anyway even if I have a solution in Fourier domain we have the facility to make it, make it the transformation and I can do that o find out what is the value of these things in time domain.

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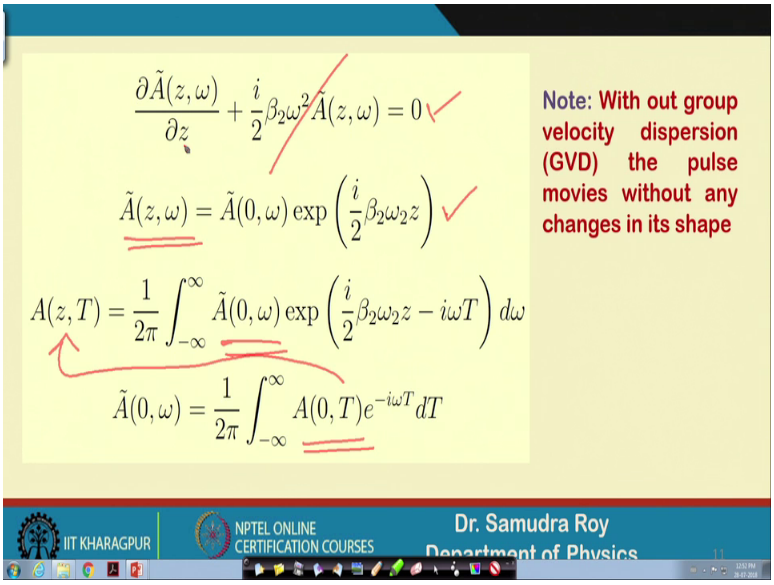

So, this is the solution this is the solution and I have this is the equation and this is the solution and once we have the solution I can in frequency domain I can know what should be the solution in time domain because I will again make a Fourier transform that.

One once I make a Fourier transform of that I will have the initial value of a at frequency domain again initial value of a or the frequency domain can be given by the initial condition that is the initial form of the pulse at time domain. So, initial form at z equal to 0 the time domain if it is given I can readily from here I can readily say what should be the value of the pulse at some other point z in time domain.

Another thing should note here so, without group velocity dispersion term if it is not there you can see that del A del z is simply 0; that means, the pulse it move the pulse width move without any distortion. So, which is quite trivial, but it is important to note that it can move without any distortion.

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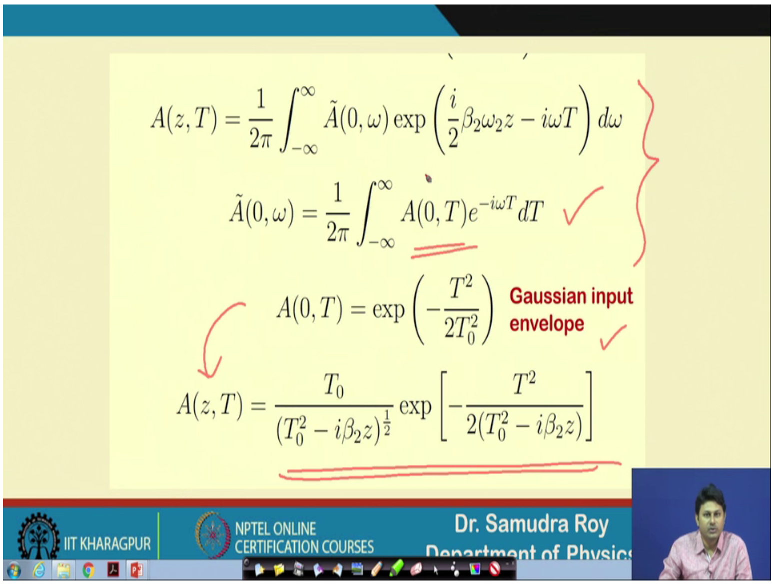

Well this is the solution form that is shown here this is the set of solution. So, what we are getting? We are getting a 0 t this is the thing it is unknown if I know this is what is the value of the input value input calls then I can find out his output pulse.

If we take a Gaussian pulse here then readily you will get the solution. So, I strongly suggest you to please do this exercise use a Gaussian pulse and then try to find out just make a Fourier transform and just try to find out whether you are getting this result or not. So, these are quite standard result given in many books, but I intentionally did not do all the detail calculation because I want you people to do the calculation and find out if the initial pulse is given under dispersion what should be the form of the pulse. And you just need to solve this Fourier transform thing and once you solve this Fourier transform thing you readily get the result the consequence of the result I will going to discuss in the next class.

So, here I like to stop my class. So, thank you for your kind attention and in the next class we will again start from this point and we introduce the non-linear effect and try to understand another non-linear effect what should be the evolution of an optical pulse.

Thank you very much and thank you for your attention.