Introduction to Non-Linear Optics and Its Applications
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Lecture – 57
Raman Amplification (Contd.)

So welcome back students to the next class of Non-Linear Optics. So, today we have lecture number 57 and the last class we started Raman Amplification process. We will continue with this thing.

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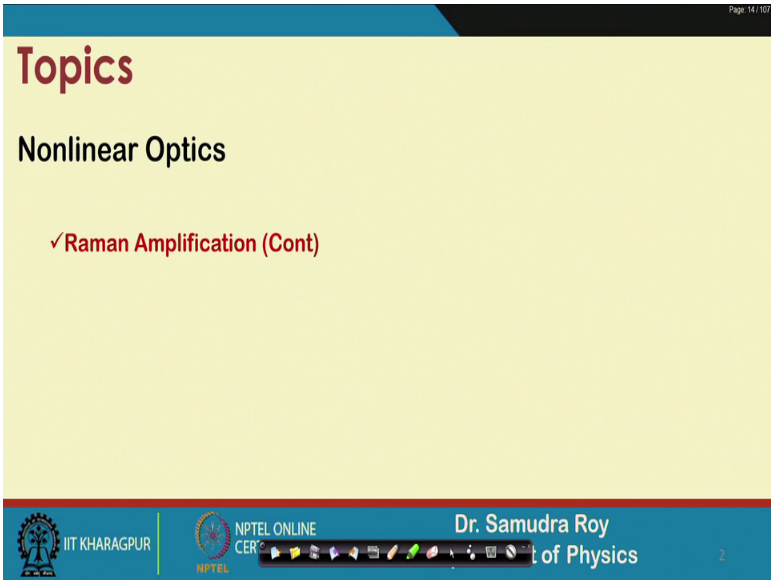

So, let us quickly go back to our original problem.

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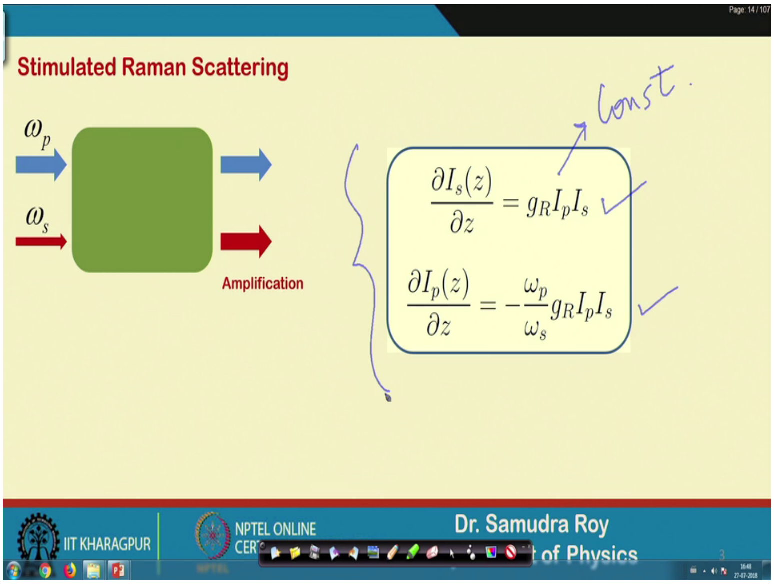

That we want to find out how the stoke wave will going to evolve and in order to find out how the stoke wave will be going to evolve. We need to solve the differential equation that we derived in the last class.

So, this differential equation is coupled, and this coupled differential equation tell us that is the how the intensity of the signal and intensity of the pump will going to change and if the pump is constant if the pump is constant then in these two equation I can consider I p is constant. So, life will be simpler and I can solve it seems that I can solve this coupled equation by decoupling some of the equations. So, I will do that today's class to we will try to solve that and try to understand the physics behind these things.

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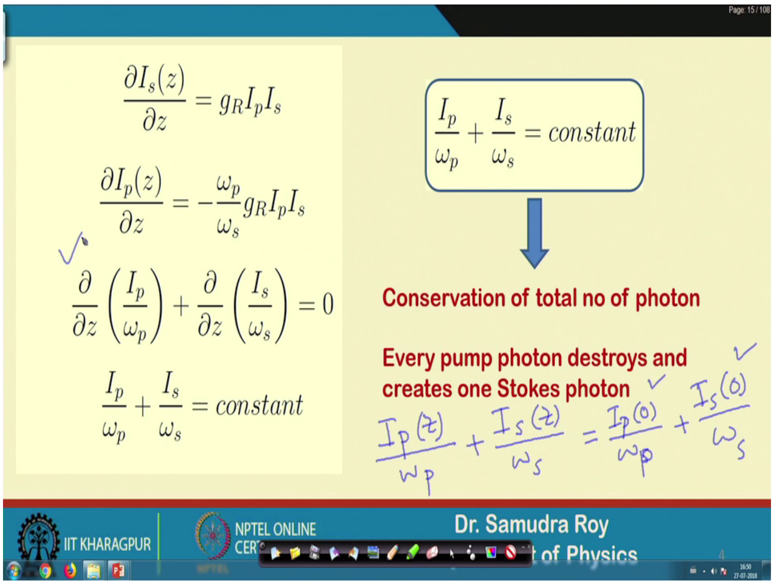

But before going to the solution, we would like to remind once again that what these two coupled equations basically suggest which is very very important. Then it suggests that one particular component I p divided by omega s plus I s divided by omega s is constant.

So, if I now carefully understand what is going on, then this equation I p is a function of z it is omega p plus I s a function of z which is omega s. This is constant means I can put the boundary condition here and I can write it I p 0 omega s plus I s 0 omega p sorry and it is omega s. So, this equation I can write because this is a constant. So, even if I change the z this things will not going to change. So, initially if I have I p value and I s value, and calculate I p divided by omega p plus is divided by omega s whatever the value, I will have at any z point even though please note that even though I p and Is both are changing.

It is not that I p and I s are constant. They are changing but the important thing is that even though they are changing. If I add this quantity, then they are conserved. And what is the meaning what is the physical meaning of that? I p divided by omega s and is divided by omega s is the total number of measurement of some sort of measurement of the total number of photon. So, this total number of photon before the operation and total number of photon after the operation before the operation means at the input and at the output should remain same.

So, the total number of photon remains conserved. Another thing important thing we find from this equation this equation suggest that the rate the change of pump photon is equivalent to the gain of signal photon.

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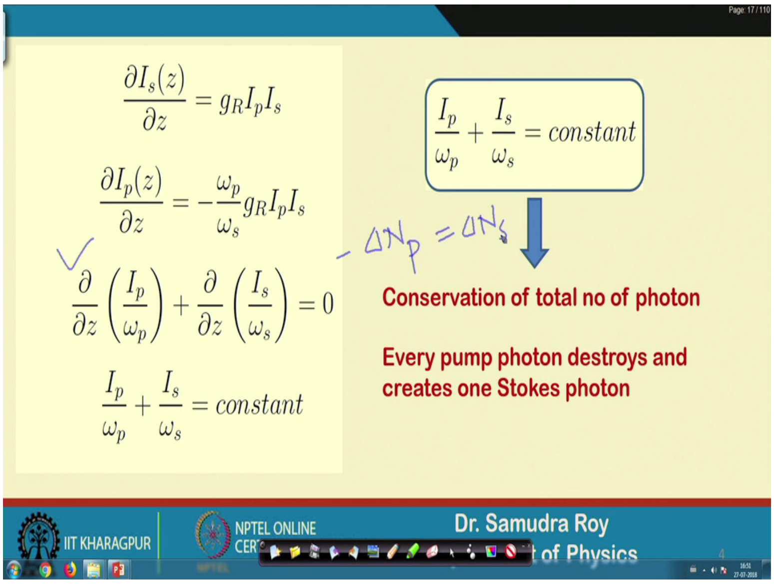

So, the number of pump photon that will destroy we are going to create the same number of signal photon. So, this relation is basically a (Refer Time: 04:33) relation and it basically tells you the conservation of the energy take that is all. So, we have already discussed these things in the previous class it is just a recap.

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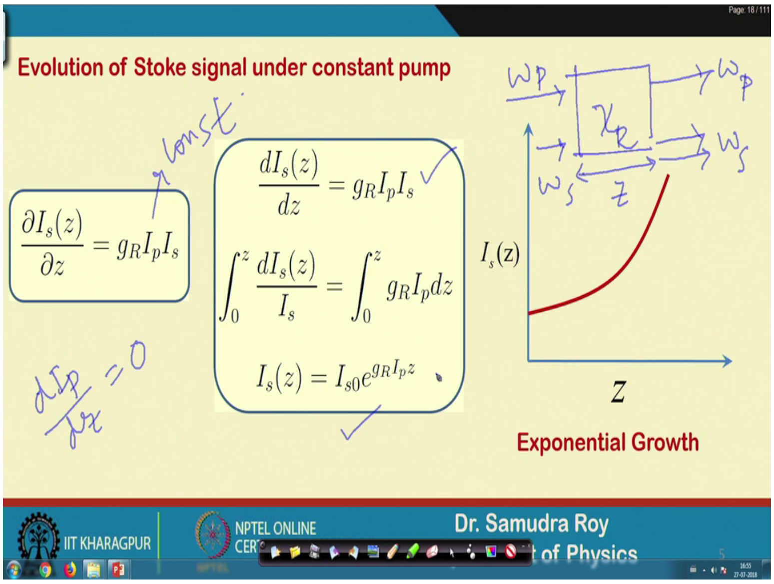

So, now we are in a position to solve the coupled differential equation. So, if I solve the coupled differential equation, we will have this equation in our hand and this coupled equation its now not remain coupled because I p is constant. If I p is a constant. So, the second equation related to I p the second differential equation related to I p is 0 or; that means, this equation this coupled equation is not remained any coupled anymore.

So, if I p is constant then the solution is very quick one can solve this very easily, that this is my equation where g R I p is a constant, I can put is here and integrate it over 0 to z right hand side also I integrate over 0 to z and the solution is something like that where we find that is will going to increase exponentially, which is expected because we are talking about gain. So, if we solve this differential equation, we must have something in our hand which is having an increment and in this case this increment is exponential in nature.

So, this is some sort of exponential growth. So, once we have the solution we can really understand that we have exponential growth in our hand, when I p is a constant. That means, physically this is our system, I launch pump omega p and small signal omega s in the output I have omega p and also omega s, but this omega s is amplified and this amplification is now exponential in nature. So, this exponential thing depends on z. So, if I increase the length of this material where this Raman process is happening this material, where chi R is there chi R is the Raman susceptibility term which is inside this gr mind it.

So, then what happened that we will have enormous amount of signal, but you can really understand that this is a non physical kind of solution, because this extra energy from where we will get the extra energy is that is from omega p. So, omega the field related to omega p or the pump. So, pump has to decay. So, in realistic consideration, so, pump will not remain constant. So, it will going to decay. So, this structure will not remain same. So, there will be some difference. So, only for initial case when z is very small or the efficiency is small, then only maybe this exponential expression is acceptable.

But if we have long distance or the efficiency is large; that means, we are generating omega s very efficiently, then this exponential modeling is completely a failure because physically it will not going to give you anything, but the important thing to understand this exponential model is that. So, with this expression you can really understand that the signal will going to increase. Without doing anything without calculation without much calculation rather if we have the equation and if we put the condition that I p is equal to constant, then you can really say that there will be some exponential growth of the signal or in this case the stoke wave.

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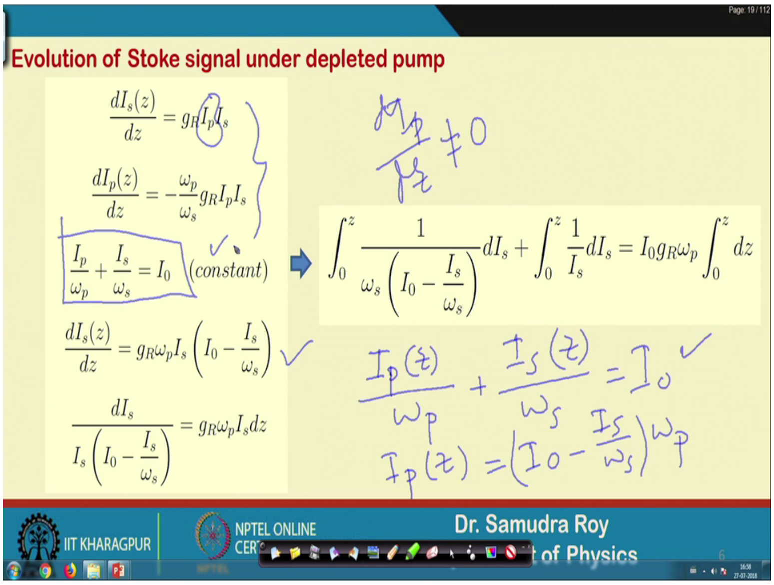

Well now, also it is my duty to show you the real case and this real case happens when we have the depleted pump situation. The depleted pump situation means the pump is now not remain conserved and dI p d z dIp d z is not equal to 0. If this quantity is not equal to 0 then; that means, I need to solve both the equations simultaneously and I need to decouple this equation and then try to solve.

Again this is not a very tough task to decouple these two equations, because we have an additional equation in our hand and we will go into this additional equation to solve this. What was the additional equation? The additional equation is this relation which tell us tells us that I p divided by omega s plus is divided by omega s, I p divided by omega p plus is divided by omega s this quantity is conserved throughout the process. That means, if I write very clearly it should be I p function of z divided by omega p plus I s function of z divided by omega s is equal to constant; that means, I 0. When this things is constant what additional advantage you will have we can replace I p and is in terms of I 0 which is a constant.

So, this decoupling thing becomes quite easy. Because, in order to decouple, what we need to do? We need to just replace this I p in terms of I s which we can do very easily with this equation. So, from here I pz is equal to I 0 minus is omega s multiplied by omega p. So, now, if I replace this I p here which we have done in this line you can see that the equation is solely depend on I s the left hand side we have is in the right hand side we have I s. So, now, we need to solve these simple differential equations.

So, I just decouple this by using this equation which suggests that the total photon will remain conserved. So, from that we can decouple one equation which is related to signal, and then we can now in a position to solve the equation. So, if I solve this equation this differential equation as dx divided by I s multiplied by I 0 minus is divided by omega s. So, this is the twelve class problem how to solve this differential equation and now if you believe you can do quite easily. These things I can just divide these two things in this way as shown in the right hand side in this way.

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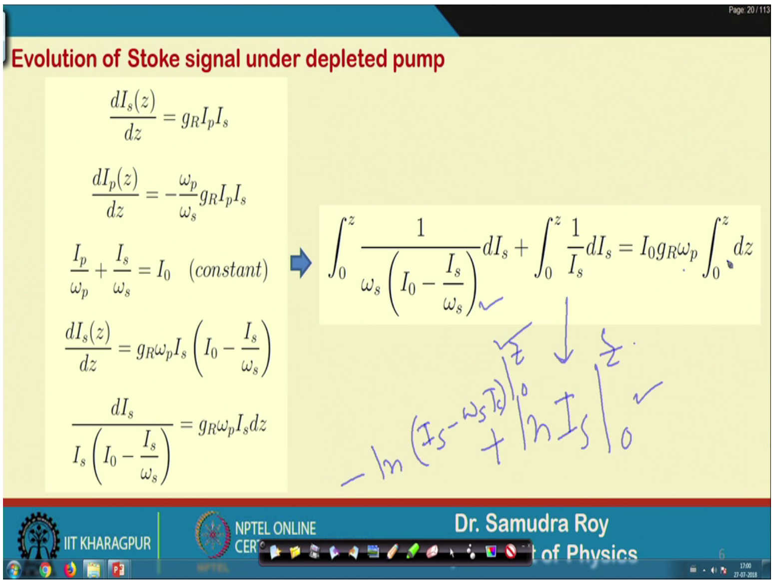

And in one part we have 1 by I s and another part 1 by I s, but slightly different in different way.

So, if I do this separation, then it will be easier for us to integrate and this integration can simply give me log of ln of I s with the boundary condition 0 to z with a plus sign, and this basically give us minus of ln I s minus omega s I 0 with the limit 0 to z these are the two integration we have. So, we can evaluate this integration quite easily in the right hand side it will be I 0 g R omega p multiplied by z.

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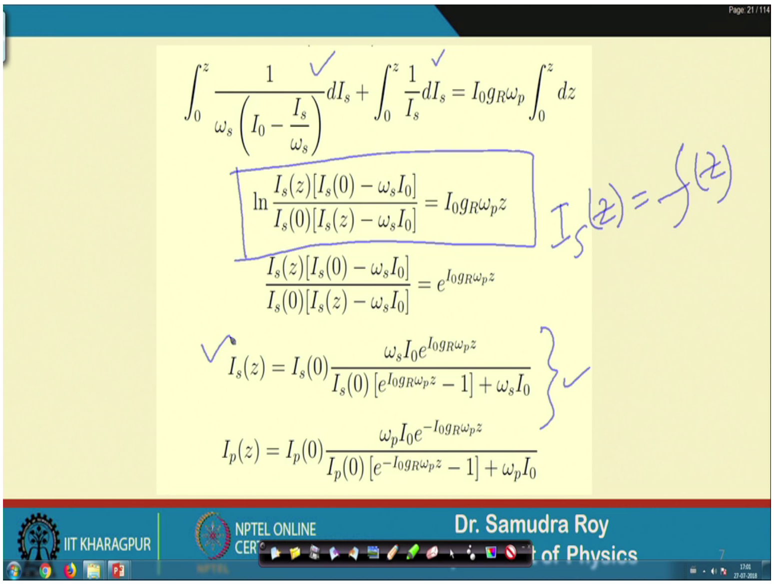

So, we can very quickly integrate these two thing and if I do this integration quickly, then we have and combine these two together, then we have the expression something like this, integrate this integrate this, combine these two you will really get this expression.

And now the next thing is to write this equation in the form I s function of z is equal to some function of z. So, I need to find the value of the evolution of I z. So, I can put I z one side and the rest of the part in the other side, which is again not a very big deal if you do the calculations the algebraic calculation write in the proper way. So, I can I can put it as a small work homework. So, just extract is from this equation and if you extract the I s z from this equation, you will landed to this expression. This is the expression of is under the condition that the pump is not remained conserved rather it is decaying.

Now, after solving this kind of differential equation, the very first thing that we always check is a boundary condition whether this boundary condition is whatever the expression we have, the boundary condition is really valid for this expression or not.

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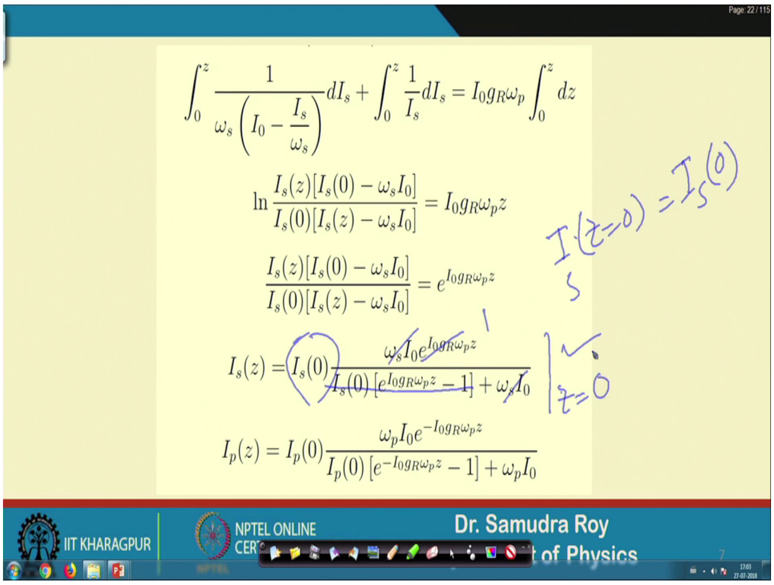

So, in order to do that what we do, that we put this equation try to find out the value of this this expression at z equal to 0. So obviously, we know that at z equal to 0 the left hand side will be I z equal to 0 I s z equal to 0 or is 0 which is already in the equation. So, this term will simply be is 0. So, if you do these things carefully you will find that if you put z equal to 0 this term will remain 1 will be one this term is 1 minus 1. So, this term will cancel out we have omega s in the denominator and omega s I in the numerator these two will cancel out and you will simply have is 0.

So, the boundary condition is basically give us the correct result so; that means, it seems that whatever the expression I find is correct, but you need to check that by yourself I strongly suggest you to please check that by yourself. In the similar way if I put calculate I p I will get the similar kind of results and it is easy if you calculate I p then you can also calculate is there are two way to calculate, I p you can use the fundamental differential equation.

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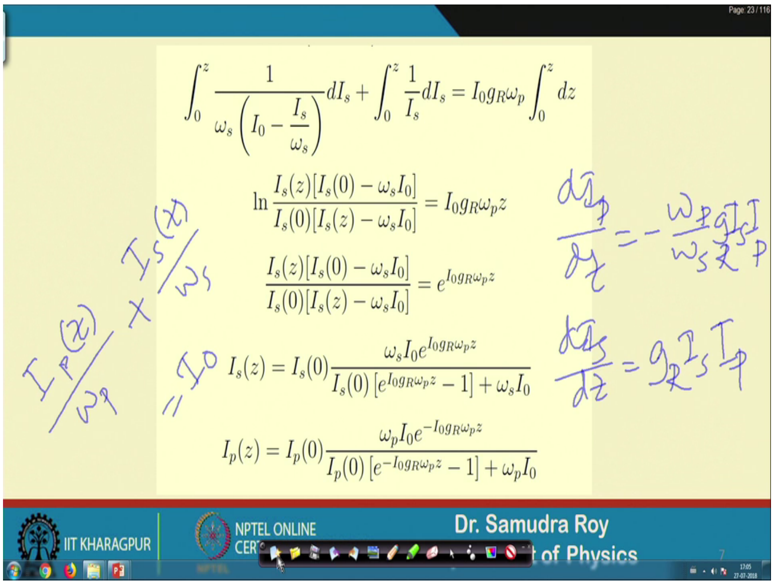

That solving I p dz is equal to minus of omega p omega s g R I s I p and another equation d is d z g R is I p.

So, you can solve again in the similar way these two coupled equation or you have I s in your hand. So, you just make a derivative of these things and you can have I p. So, I p will be derivative of is divided by g R I is this is another way, but these things will be will be complicated. So, it is better to use another process and that process is I p z you have another equation in our hand mind it omega p plus I s z omega s I s equal to I 0 is use is known. So, I p will be I 0 minus of these things multiplied by omega p. So, this is another way to find out this equation of I p well.

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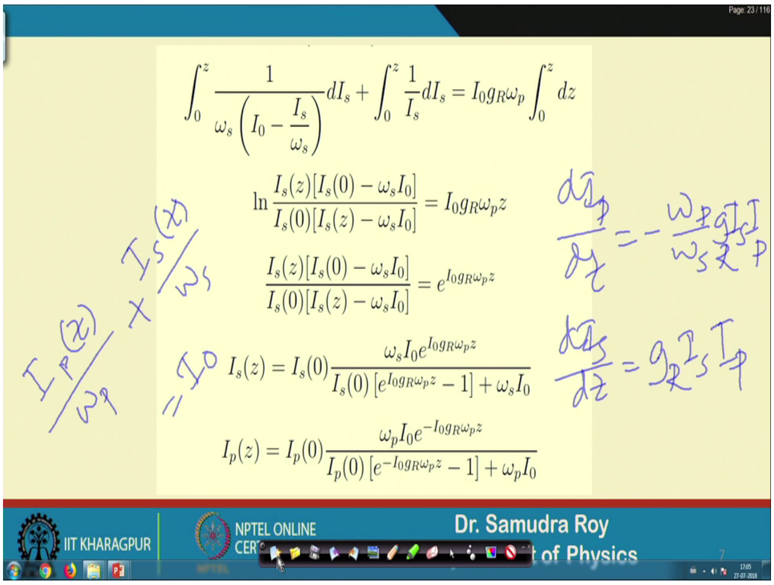

I believe you people can do that quite efficiently. So, now, the next thing to plot these two things if we plot these things you can find that the plot is completely different from the exponential growth, and it is more logical or more realistic plot and you can see that I is I s gradually this is basically is. So, is gradually increasing and goes to some saturated value which should be because it is not that if you increase the z length every time it will increase. So, there will be some kind of saturation. So, with the increment of this length only thing that will increase the initial part and then it will go to saturate, and I p will go down because it is basically gives the corresponding energy to amplify.

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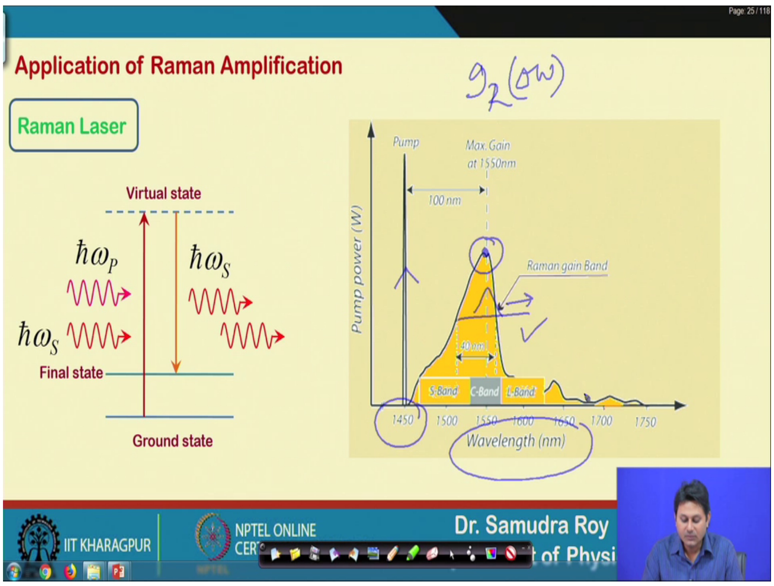

So, finally, we will say something about the application of this Raman amplification and this Raman amplification basically applied in Raman laser it is some sort of lazing action you can see that you launch omega f and you are getting the amplified omega s. So, one can use these as a amplifier and in fibers extremes extensively this Raman amplifier is using the fiber which is made of the silica if I plot the Raman gain. So, the Raman gain means the, it is you mean I mentioned already the gR which is the Raman gain coefficient gR, which this is a function of delta omega, because this is a function of Raman susceptibility.

Raman susceptibility is a function of delta omega. So, it has to be function of delta omega. So, if I plot this gr as a function of delta omega we will have this kind of curve, where the gain is maxima around this and in the here I plot the wavelength. So, this wavelength suggested in if I pump here at this point at say 1450, the highest efficiency will be here. So, I will going to generate some kind of stoke light very efficiently at this point which is 100 nanometer apart. So, it is this frequency is smaller. So, the wavelength is higher.

So, based on this curve we can generate these stokes wave. This is a very well known curve gain curve for silica kind of fiber. So, there are materials where also we have Raman amplification and in other cases this gain curve may change, it depends on in telling the material you are using. So, with this note let me conclude the class here. So, today we learn the complete process of Raman amplification and in the next class we will start a brand new topic, which is the pulse propagation in non-linear systems which will be the last topic and.

Thank you for your attention and see you in the next class.