Introduction to Non-Linear Optics and Its Applications
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Lecture – 56
Raman Amplification

So, welcome student to the next class of Introduction to Non-Linear Optics and Its Application.

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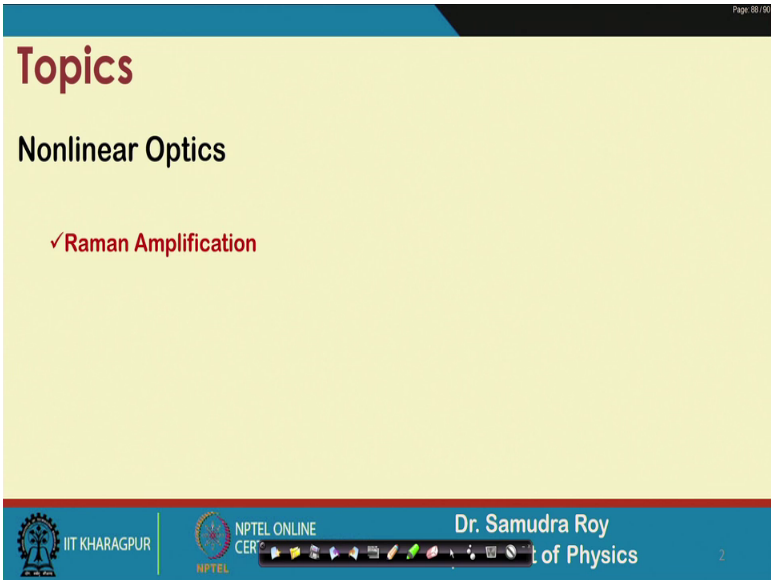

Today we have lecture number 56, and we will going to continue the Raman process. So, today we will going to learn the detail about the Raman amplification. So, the calculation part 2 we have already done in the last class.

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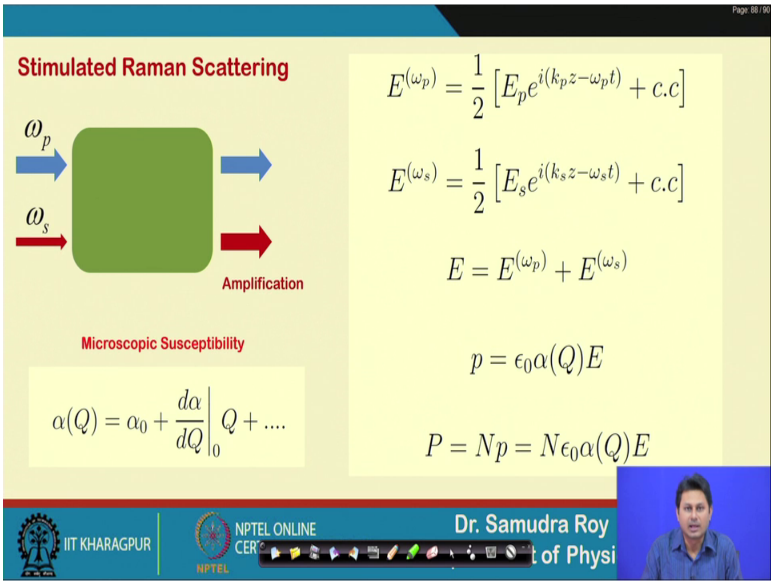

So, we will going to continue with the same calculation and gradually try to understand that, how the stokes wave will going to evolve. Now that was the goal of this particular topic that to understand physically that how this stoke wave will going to generate. And not only generate it basically amplify through this pump.

So, this was the basic figure that we have used in the last class that, how stimulated Raman scattering process is happening. When a pump omega p is launched along with a stoke wave omega s then the stoke wave omega s is going to amplify. So, E p and E s are the amplitudes of the pump wave and the stoke waves both are assumed to be plain waves, having the propagation constant K p and k s and the corresponding frequency omega p and omega s.

The total field E is E omega p plus E omega s that is the summation of these two fields and the total polarization is N into microscopic polarization small p which is n epsilon 0 and then microscopic susceptibility alpha we should be a function of Q which is shown in the bracket multiplied by the total electric field E this was the basic structure we have being we are using for last few classes.

So, since the microscopic susceptibility is a weak function of Q, we can expand this as a Taylor series and around a around 0 point or some reference point, and we can expand it to only first order we are not going to take the higher order terms here assuming in the displacement is relatively small.

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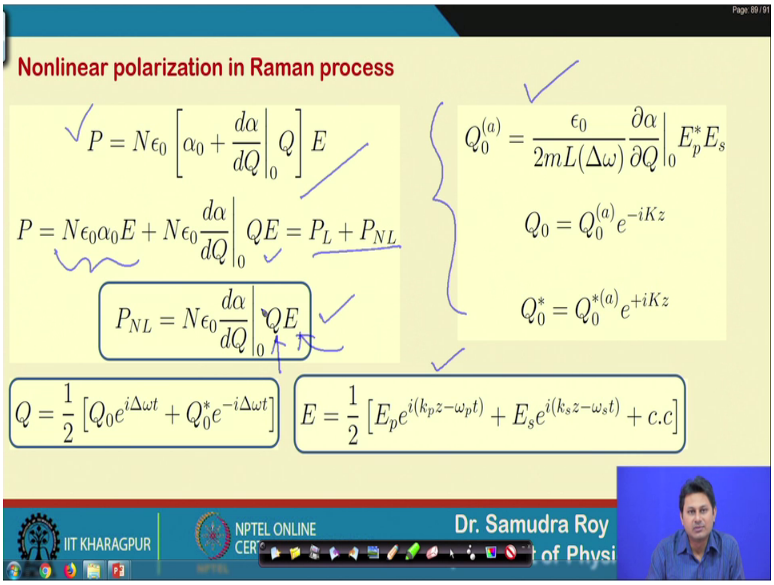

Well the non-linear polarization of the Raman process we need to calculate, because that is the term which is important in order to understand how the stoke wave will be evolving because in this non-linear polarization term we have the corresponding source effect. So, if the source is nothing, but the p non-linear.

Well these calculation is also done in the last class, but in order to make a recap I just like to briefly tell you how these equations are there. So, first this polarization term, the polarization term is this one, it can be divided into two part one is the linear part and another was the non-linear part these things as written here and we are interested in the non-linear part because this non-linear part should be in the place of the source term of non-linear Maxwell's equation.

In order to calculate the P non-linear to important quantity we need to calculate one was this Q that is the displacement, and another was this E the field. The distribution of the electric field is already shown, this is summation of the pump and signal field and Q is also calculated with the assumption that the system in this case the system means the molecule will going to vibrate under the external field. And this vibration will be a force dam kind of vibration and we put the model and solve this differential equation and we eventually have this equation set of equation, which give us what should be the value of Q. Q here is some sort of displacement. So, this displacement multiplied by the electric field basically gives the essential quantity to calculate P non-linear.

So, all these things are now in our hand and next what we will do? We will just find what is our P non-linear.

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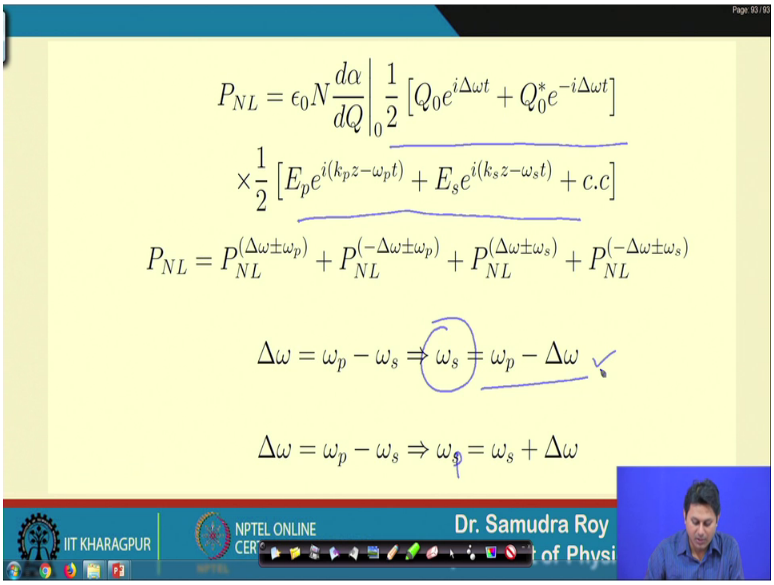

And our P non-linear if I multiply these two things here it is shown that if I multiply this Q part and the E part and then try to understand how many frequencies are there we find there are eight different kind of frequency component that should be in the P non-linear term.

So, we are not interested in all the frequencies, but certain frequencies is important in our case and this frequencies are those which basically govern the strokes and anti-stokes. Since, we are dealing with stoke a wave then the frequency which is containing the source term or the source the stoke frequency omega s will be this. So, this frequency will be the important term in our calculation. So, we need to find out from where we can get these two frequencies, and we find that there is a painting we say it should be p last it is the last day also I mentioned that.

So, from here we can find that there is a frequency omega p minus omega s omega p minus delta omega which is equal to omega s. So, this is the frequency I am looking for and P non-linear term will be vibrating all these frequencies, but this is the frequency which is important in our case well.

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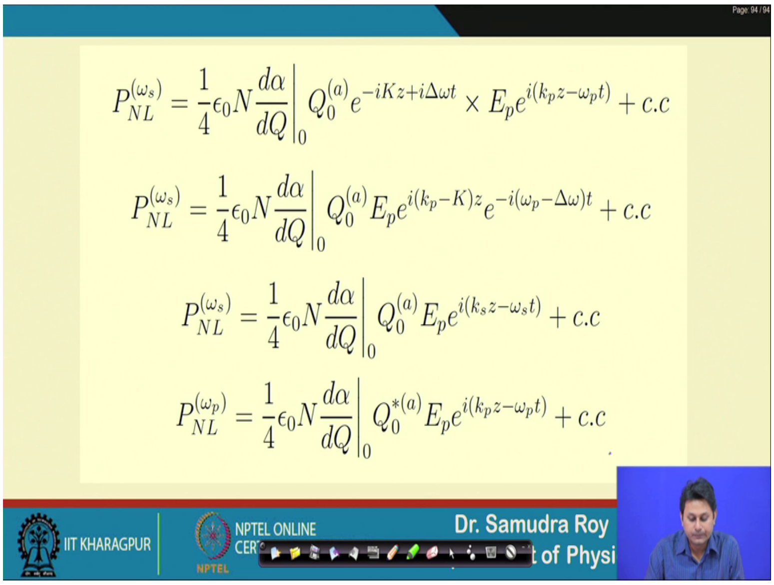

Now, if I extract this frequency and from this equation I need to extract this frequency.

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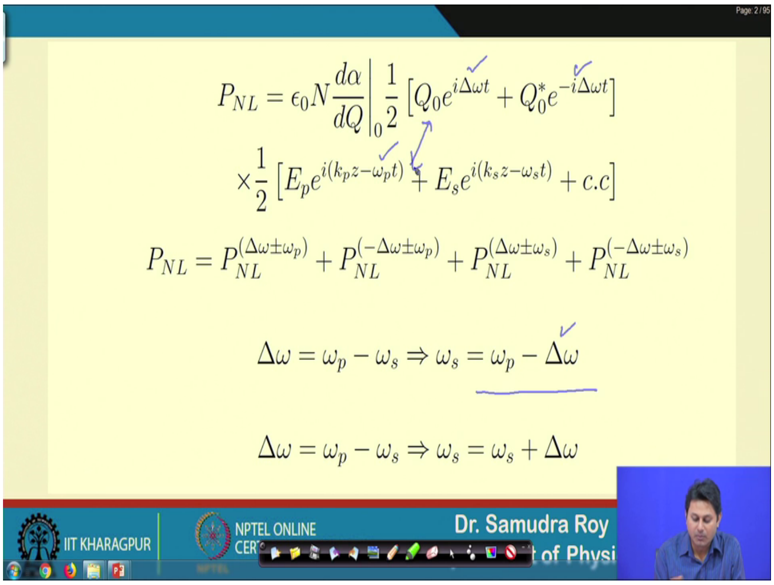

So, how do I extract? So, I need to have this frequency. So, omega p is somewhere here and if it is multiplied with minus of omega p this term in this case here is this term.

So, these two term basically multiplication of these two term basically give rise to if polarization, which is having the frequency component omega s. So, by the time I believe you know how to extract the different frequency component and exactly here we have done the same thing.

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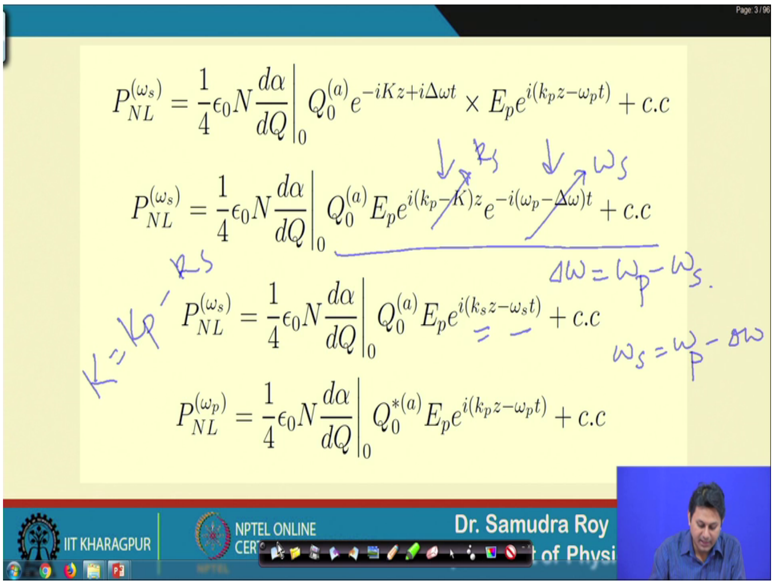

So, if you look carefully in this equation here we have Q 0 e to the power i delta omega t and here E p. So, E p multiplied by Q 0 this term will only stands. So, here we are doing these things E p multiplied by Q a Q 0 this term is there with the phase term where delta omega and omega p is here.

So, if I combine these two things we will have our desired non-linear polarization term which is this one and here we have a phase term and a frequency term and if we look carefully delta omega was omega p minus omega s. So, omega s is essentially omega p minus delta omega. So, exactly these frequency is here. So, it is nothing, but omega s.

So, I can replace this frequency to omega s total K p minus k s I can also replace as k s because you remember that K b k was K p minus k s. So, K p minus k is equal to k s I can replace these are also as k s which is here and form my total polarization term, which is containing f frequency omega s that is important.

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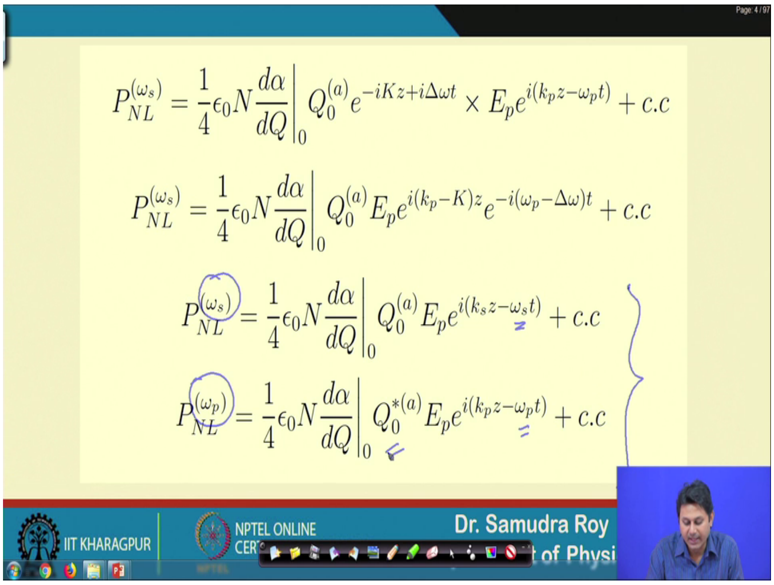

So, that is why here this omega s each zone that I try to mention that that non-linear polarization here is containing the frequency omega s. Exactly in the similar way you can calculate the non-linear polarization of the pump omega p and those two equation looks exactly same, only difference here we have this thing and here we have Q 0 a.

So, these two difference you need to mention just one thing I need to check maybe this p seems to be this p seems to be s here if we multiply these. So, in order to find out the frequency term omega p what we need to do here there is a slight printing mistake that basically creates that some little confusion that I want to.

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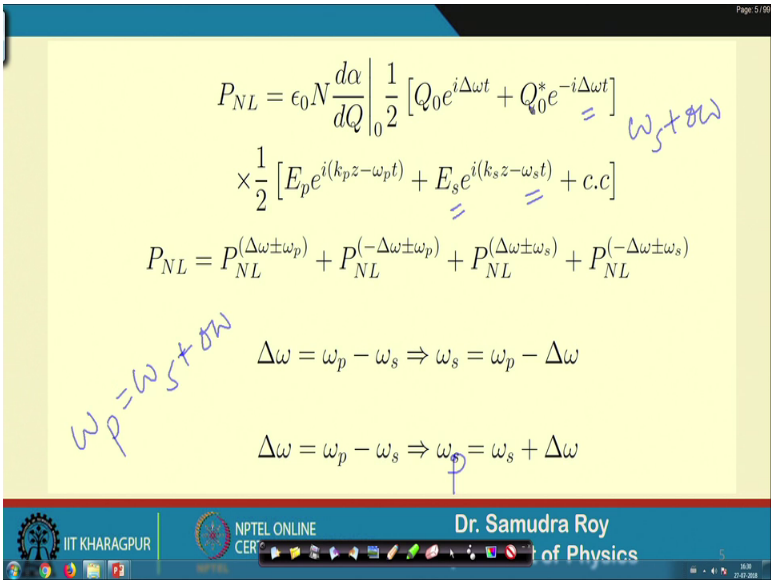

So, omega p is omega s plus delta omega. So, omega s component is here and delta omega component is sitting here. So, this complex conjugate basically gives plus omega s. So, this complex conjugate and here if I multiply these two this then I will have a frequency component. So, I need to have a plus sign. So, E s has to be multiplied with Q 0 stars so, that this frequency component become omega s plus delta omega. So, this omega s this E s this E s and Q 0 should be there in P non-linear term where it is containing a frequency omega p.

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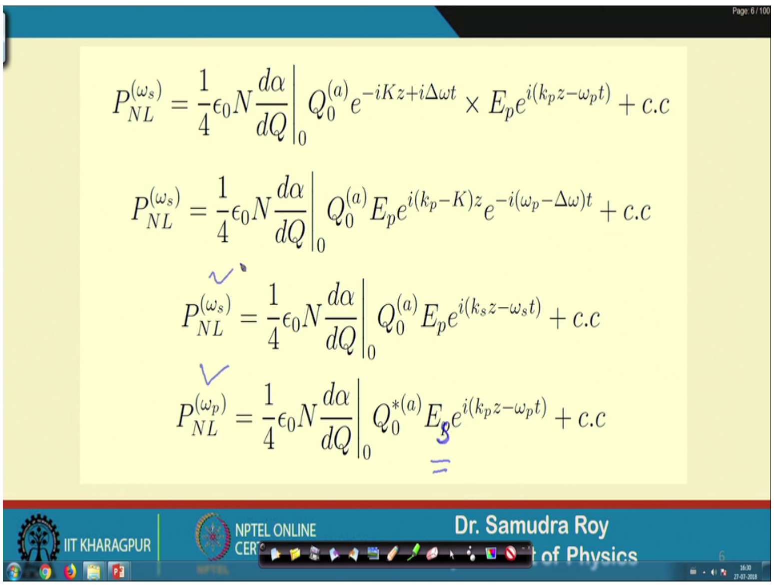

So, here by mistake I have written E p it should be E s please note that it should be E s because, E s and this should multiply if it is E p then; obviously, it should be E s so, that the frequency component can come correctly anywhere.

So, once we have the information of P non-linear with the frequency component of omega s and omega p the next thing that we do that I will readily calculate the evolution equation or the evolution of the stoke field.

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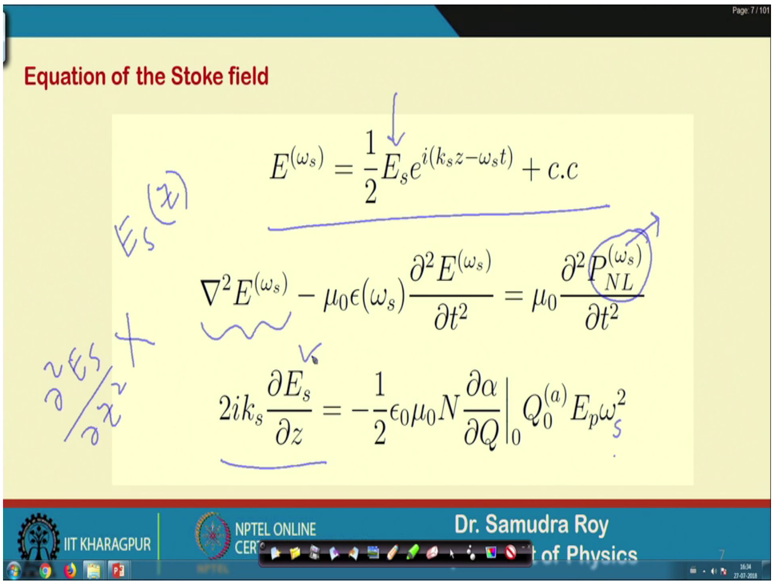

So, in order to find out the evolution equation of the stoke field, what essentially we need to do we need to solve the non-linear Maxwell's equation. So, every time in this throughout this course we find that starting from this non-linear Maxwell's equation, we can derive whatever the field evolution we want. Only thing that we need to extract is the P non-linear term for that particular frequency, for which you were generating or you want to find out the equation of evolution equation of amplitude.

In this particular case we want to find out the evolution of the stokes wave. So, stokes this stokes waves has frequency omega s. So, that is the reason why we need to find out the polarization term and this polarization term should have the frequency component omega s. So, exactly here, first this is our field I need to find out the evolution of this quantity. So, mind it E s is the amplitude of the stoke field, which is evolving inside the system. So, it has to be a function of z. And here in this equation if I solve this equation or simplify this equation I have the evolution equation of stoke filed.

So, this quantity I am not doing, I will not going to do these things in elaborate fashion because several time we use this equation and know how to handle that. So, again we will going to use the slowly varying envelope approximation or slowly varying approximation so, that I can neglect the second order derivative term.

If I neglect this term the first effect will be the first order derivative and this term will be cancelling out with other term that is coming from this derivative and eventually we have this in the left hand side. In the right hand side on the other hand I will have some term and what term I will get will depend on the P non-linear omega s the value of P non-linear omega s, which we have already calculated.

And if I make a derivative of this quantity then since is the frequency is omega s we will have a omega s term here. So, it should be so, here it is reading omega, but it should be omega s square with a negative sign omega s quite with a negative sign. So, this equation basically give us how is will going to evolve. Well this is still a complicated looking equation. So, I need to make some sort of simplification.

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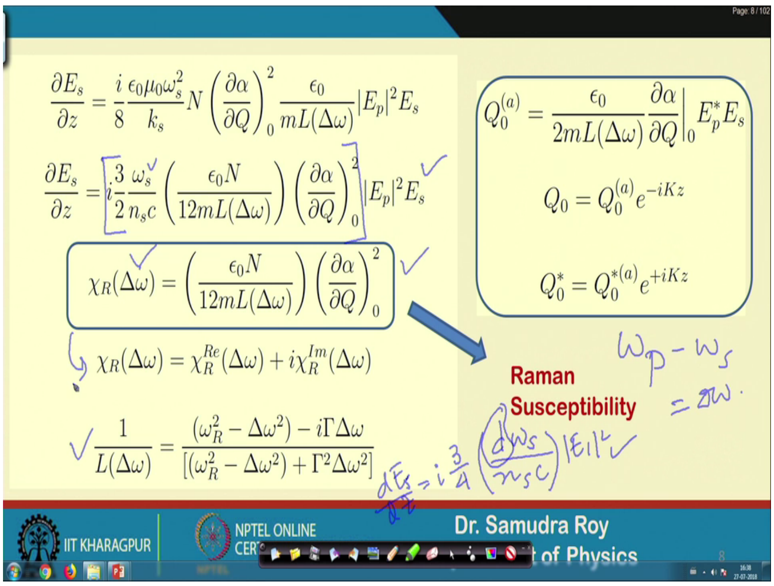

So, if I do so, simplification how I do just replace Q 0 a, because value of the Q 0 a is somewhere here this is the amplitude Q 0 a, when you put this Q 0 a we have E p star E s here. So, I just replace this Q 0 a here. So, that this term become E p star is there. So, E p multiplied by E p star there should be one mode of E p, and 1 e is term will appear.

So, here you can see I have E p mod square multiplied by E s in the right hand side, the Q 0 term is again written. So, then you can understand when you put this Q 0 how this term will going to evolve and other terms will come accordingly. Because there are a few terms here if you put these things it will come automatically. And now what we do that we will define one quantity which is Raman susceptibility why we are doing? Because if you remember the process of second harmonic generation of third harmonic generation, and the form of the equation. The form of the equation looks quite similar to this equation.

So, only thing here is the term that we have here and in this term we have one omega s if you remember. So, there was a term in second harmonic generation process there was a term d omega s divided by ns c for second harmonic, if I write the second harmonic omega 2, it should be d omega 2 divided by into c. So, this was the multiplicative term you always had and then there was some term say E 1 square and some coefficient was there, it was something like 3 by 4 I am doing remember right now what I have won i was there and in the right hand side we had E s e E s or E 1 E z.

So, this kind of form we had when we calculate the second harmonic and third harmonic, the important thing is that in the bracket term. We had a susceptibility term multiplied by the frequency divided by the refractive index in to c. If I recollect this things suitably then in place of d I can have one term which is shown here in box, epsilon 0 multiplied by n divided by 12 m L of delta omega. Where L of delta omega is some function also it is written here in the last line.

So, the term that is in the bracket is basically the Raman susceptibility term. So, this Raman susceptibility is something which is equivalent to the third order susceptibility term or the second order susceptibility term. It is Raman surface susceptibility and you can see that this Raman susceptibility term is now depending on delta omega; delta omega is the frequency dependent term this is the frequency dependent term. So, omega p minus omega s is delta omega. So, it depends on delta omega.

I can write this term in because this is a complex term. So, I can write this term as x plus i y form, real plus I into imaginary form which I did here. If I now see this equation here, this coefficient previously this coefficient was purely real at least we consider this coefficient to be real. But here we find that this coefficient is not real, but complex now this equation for example.

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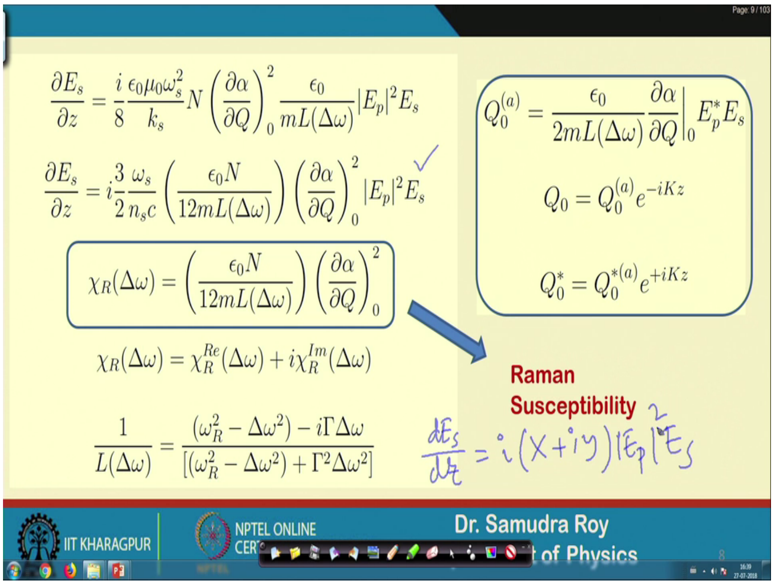

This equation if I write, it looks like del E s dz is equal to i and this is a constant thing. So, i, but complex terms so, I write it as x plus iy and then the rest term which is E p square E s. So, mind it this x plus iy is now has a real part and imaginary part, but one I is there and if I multiply this I with these things. So, again this quantity will change and we have ix this quantity and minus iy this quantity.

So, there will be two different parts of that. So, one basically give rise to some sort of thing which related to amplification and other term will give you something which is related to the phase or something. So, we will see what happen.

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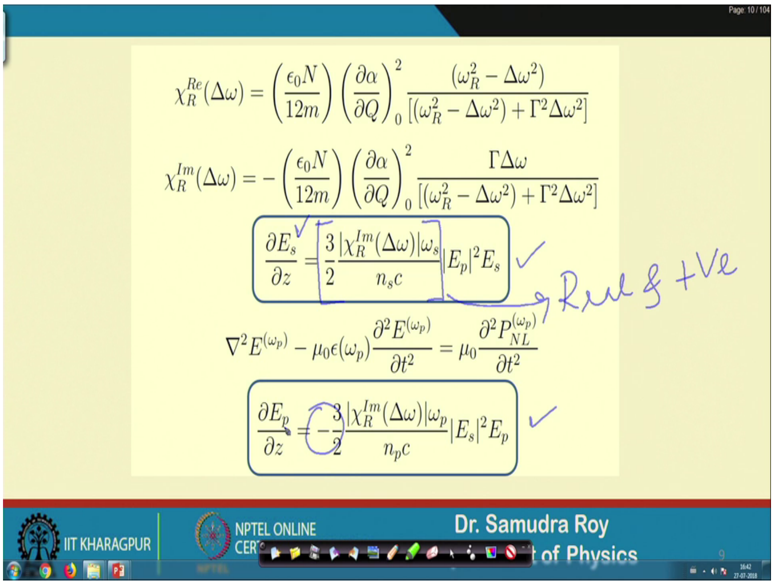

So, real term and imaginary term we calculate separately. So, how I calculate this real and imaginary term it is quite easy, because L omega is a function of omega and this is the complex term here is the only complex term here. So, I write this 1 by L omega in terms of x plus iy. So, just multiply whatever we have with the complex conjugate we know that how to write a term in x plus iy form.

So, here initially it was 1 divided by x plus iy. So, I just multiply these two complex conjugate, here numerator and denominator and have these things. So, now, I can write 1 by L omega 2 real an imaginary and if I separate out this. So, I will have the susceptibility real term this and susceptibility imaginary term this one.

So, the imaginary term basically give rise to some kind of gaining effect because in the right hand side we has to have some term which is independent of any kind of i. Because whenever we have i so, that basically designate that the phase will going to modify not the amplitude. But if we have everything this coefficient is a real coefficient, then this equation basically give us something related to loss or gain.

Here everything is positive and this sign is there is a positive sign here. So, this quantity in the bracket, this quantity is real and positive real and positive that basically gives you the idea that this equation the solution of this equation basically gives us something, where is will be amplified. In the similar way you can find out E p and when you find out E p you will find a similar kind of expression, but the notable thing is sitting here that you will have a negative sign.

This negative sign suggests that E p will go into decay what is E p? E p is the pump and what is E s? E s is our stoke field so; that means, the stokes feel will going to amplify and pump will going to decay. We can write these two equation in more suitable way.

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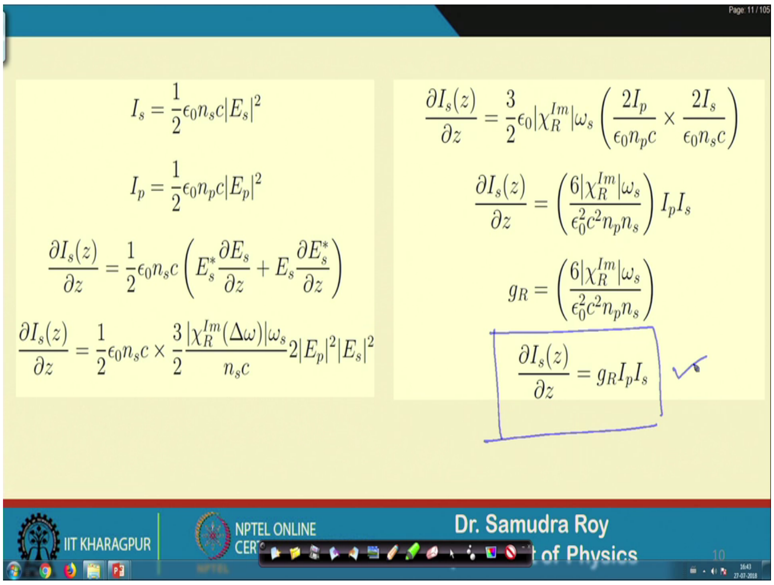

And in terms of intensity and we know that what is the relationship between intensity and the electric field, and if I use this term intensity and electric field then we can find that Is an I p can be represent in this way. So, derivative of Is an I p if I do then I will have this stamp and now d E s and dz dE s dz is known to us and when I put this term and calculate these things which is a very straightforward algebra then I land it up, then I have this expression in our hand. So, this is the intensity expression for signal and how the intensity will going to evolve. Now, we have two equation, one for signal and one for pump.

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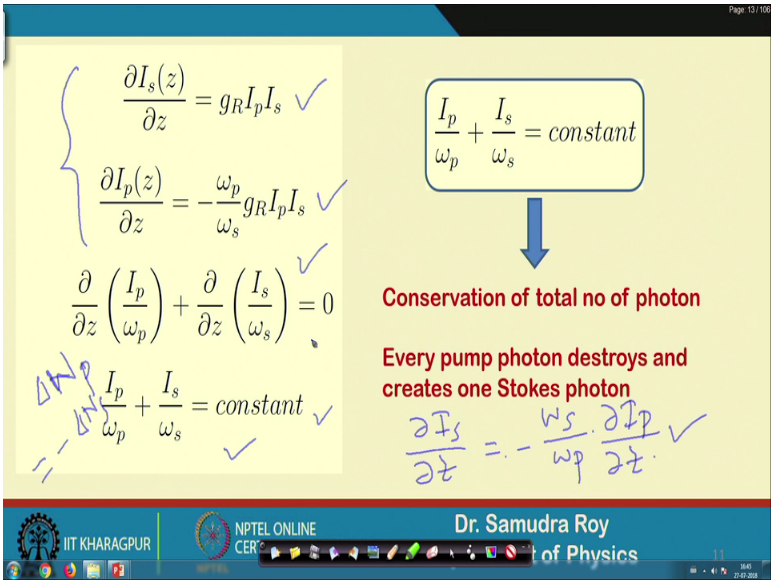

If you calculate the thing in a similar way for pumping we will get the similar kind of equation, I can put this as a homework to you that if you calculate the pump equation this is a signal equation, and this is a pump equation. So, this pump equation will come exactly in the same way.

So, g is a gain coefficient gR which is also defined. So, this till equation basically tell us something and this two equation tell us that if I replace this g R I p I s then d I s will be let me write d I s del z and gR I p Is I will replace from this equation it will be minus of omega s omega p d of I p I z. I can write this equation in this particular form, this is a well known form and some sort of manly row kind of relation that we have been using for long time for second harmonic generation, third harmonic generation for remix in all cases, we find that the photon number remain conserved.

So, if I do this simple calculation we find that I p divided by I p divided by omega p plus I divided by omega is this quantity is constant. So, this is nothing, but the conservation of the total number of photon so; that means, every photon if I calculate here. So, then we will find these things is nothing, but del z I p del omega is nothing, but delta N p which is equal to minus of delta N s; that means, one photon pump photon will destroy to generate one signal photon.

So, that is the simple thing that we find from this equation and this is nothing, but the conservation of total photon; well I would like to stop here because I do not have time today. So, in the next class we will going to solve these two equation and when we solve this equation we will find how the signal will going to amplify. With this node I would like to conclude this class here.

Thank you for your attention and see you in the next class.