Introduction to Non - Linear Optics and Its Applications
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Lecture - 55
Stimulated Raman Scattering

So welcome back student to the next class of Non-Linear Optics and Its Application.

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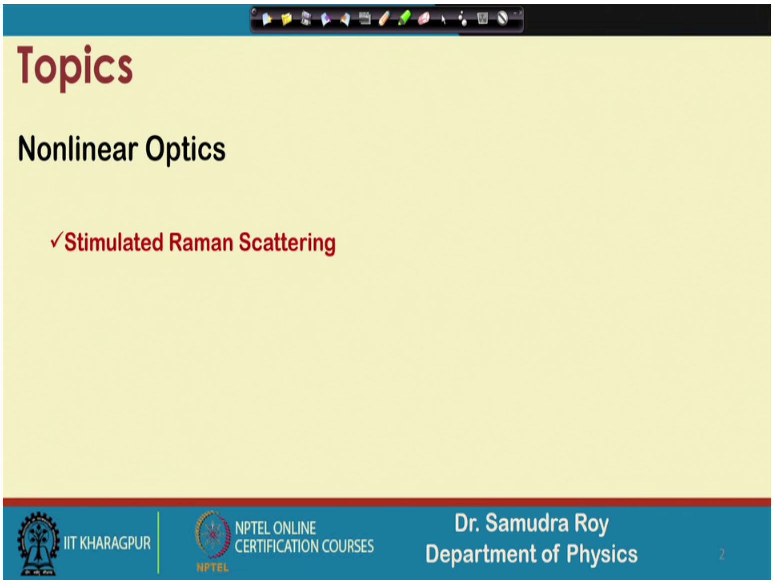

So, today we will going to today it is lecture number 55 and we will going to study the stimulated Raman scattering in last class we have already started stimulated Raman scattering.

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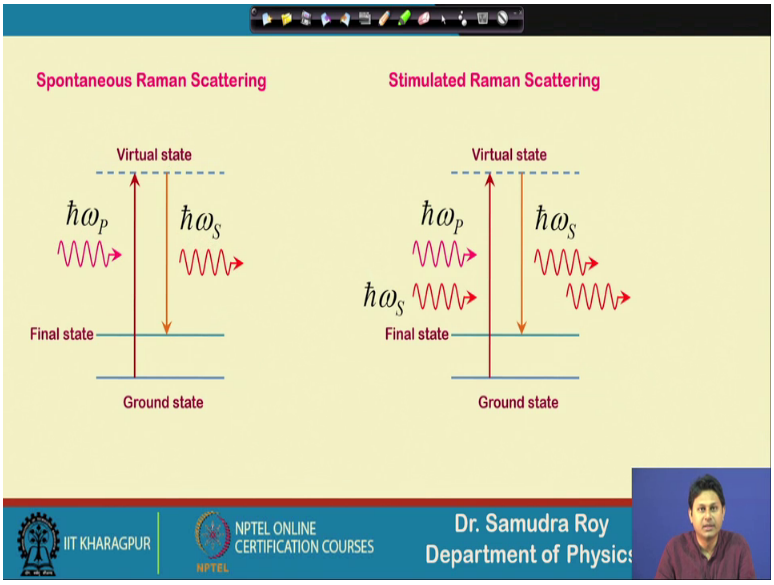

So, let us go back to the topic once again in spontaneous process, what happened that the molecule is excited by some external photon having energy h cross omega p. And then go back to some final and then have some final state private some go to some external some higher energy state, and as a result we are getting some kind of photon, which is excited from the medium having a frequency less than that of the launched frequency.

So, this is the stoke process; in stimulated Raman scattering process we want to amplify these waves omega s. In order to do that we launch not only one omega p photon, but also h cross omega s which is the energy of the stoke wave and in the output what happened these omega s is amplified.

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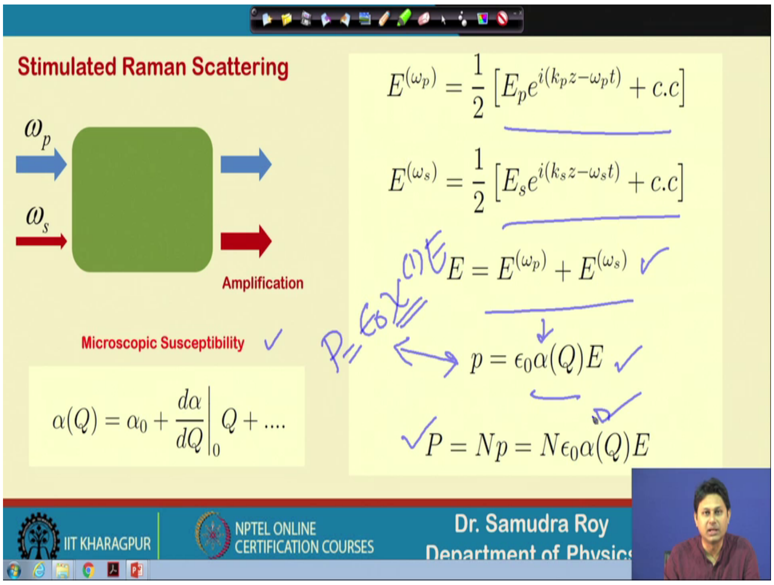

So, if I now try to understand these things in terms of mathematical treatment, then omega p omega s is launched. Omega p and omega s corresponds to the field frequency are the two frequencies corresponds to the field E p and E s which is the pump field and stoke field. In the plane wave form I can write the pump and stoke in this particular form, which we already used. Total field is omega p and omega s E omega p plus E omega s this is a total field.

Microscopic polarizability should be there, it is epsilon 0 microscopic susceptibility multiplied by total electric field the form is exactly same. The way we represent a polarization P is epsilon 0 chi 1 E, if I compared these two form it as it is the same form in state of this I am using a new term alpha which is microscopic susceptibility, because we are dealing with one molecule. But the total polarization can also be figure out by multiplying this some sort of approximation by multiplying in the total number of dipoles or molecules here per unit volume and still these susceptibility microscopic susceptibility term is still there which is a function of Q.

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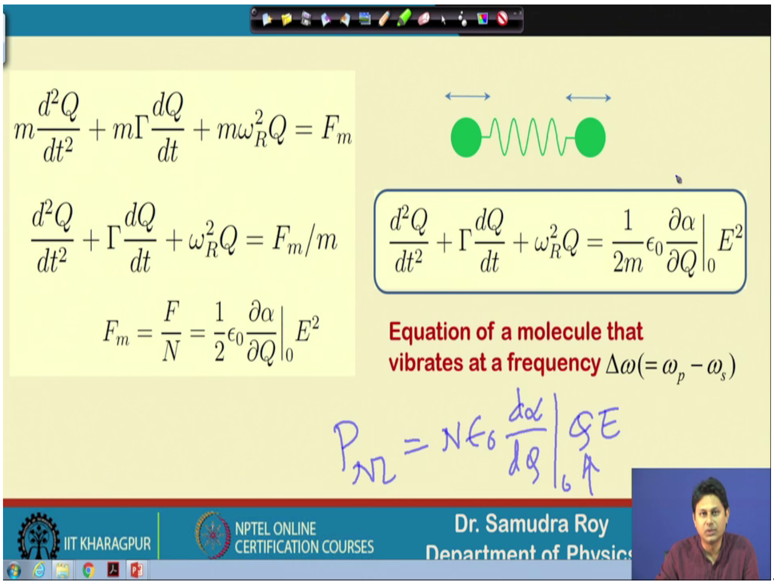

So, in last class we try to find out the differential equation of the molecule that is vibrating because the form of Q is important. And we were try to find out what is the form of Q. And once we know what is the form of Q then only we can find out what is the non-linear polarization; P non-linear is important thing that is our aim. And we know that P non-linear which is, it is N epsilon 0 d alpha d Q evaluated at 0 multiplied by Q and E we have derived that. So, this Q the information of this Q is needed and how do I find this Q? For the molecule that is vibrating because of the electric field.

So, I know I need to solve this differential equation which is a forced damped oscillation equation. So, force term I calculated last day, so today we will going to solve this equation.

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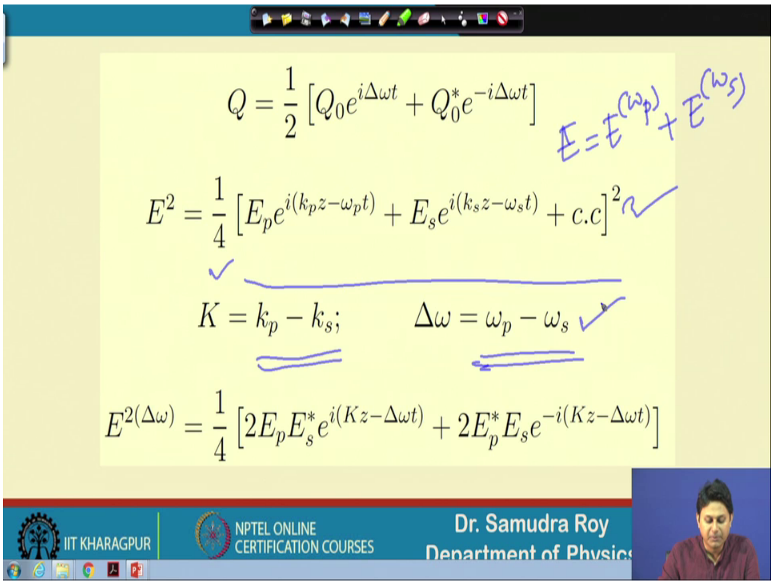

Well in order to solve this equation I write the Q in these two form half Q 0 e to the power i omega t plus Q 0 star e to the power minus of i delta omega t. So, Q and Q 0 are complex amplitude which has a frequency delta omega, because we know that the molecule is vibrating with a frequency omega.

That is why these differential equation the generalized coordinate should have a frequency component delta omega, but the amplitude may not be real it is a complex amplitude. So, in general I can write Q is equal to half of Q 0 e to the power i delta omega t plus complex conjugate of the total term whatever is used. Also the E square information is important because here you can see in the right hand side we have E square term which is a force term.

So, E square is the total field because E is E total field is E of omega p plus E of omega s. When I make a square of that and write the explicit form of E of omega p and E of omega s we will get this term. Since we are making square this half term will be 1 by 4 E p e to the power i kpz minus omega t as usual E s e to the power i ksz minus omega t plus complex conjugate and square of this.

Now, I write K as kp minus ks and omega as omega p minus omega s to reduce sum or to make the expression compact E square please note that E square is having many frequency components when you make the square term, then you will have one frequency omega p with a multiplication of 2. So, 2 omega p 1 frequency will be there 2 omega is another frequency will be there.

So, these 2 omega 2 two omega s is some sort of second harmonic generation kind of stuff, but apart from that there are a few frequency components and one important frequency component that we have is omega p minus omega s. This omega p minus omega s term is important because E square is a force term, if I go back to this equation Q is vibrating with a frequency Q is vibrating with a frequency delta omega.

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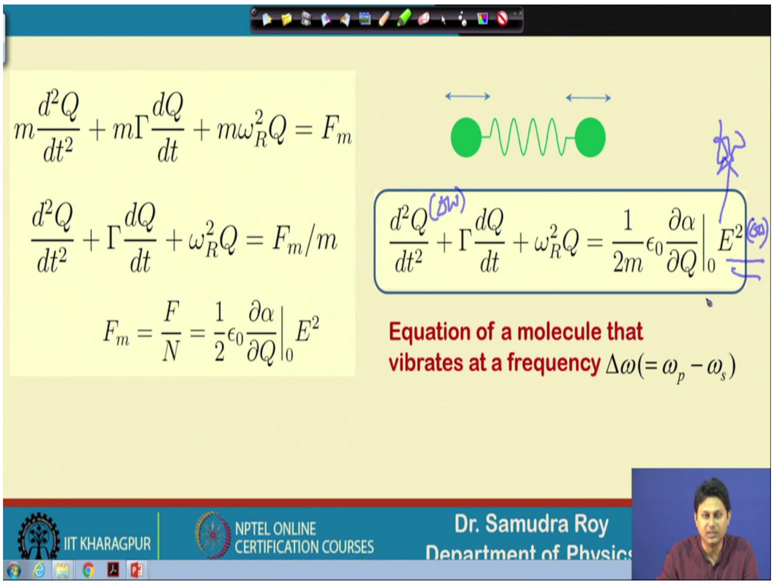

So, in the right hand side I need this is a force term, so that has to be one component delta omega. So, E square having a component delta omega basically force the molecule to vibrate with a frequency delta omega. So, we need to find out what should be the frequency component omega of the term E square. Exactly this the thing we are doing here when you calculate E square, then we derive or take out only those term here, which is vibrating as a frequency delta omega the component of E square with delta omega frequency is this one.

So, how we will get this frequency delta omega so omega p minus omega s it is simple. So, omega p minus omega s means this has to be omega p and omega s is negative so that means, we should have a star of this quantity, which is complex conjugate is here. So, which we will have so that means, E p multiplied by E s star should have a frequency component delta omega like this and also the complex conjugate of this term should be there.

And if I make the complex conjugate of this we will have a delta omega term again here, but with a negative sign you can see a negative sign is sitting here. So, I will have a plus delta omega term this is nothing, but the complex conjugate of whatever the term we have here. So, E p E s star is the amplitude two E p E s star is amplitude that is important also note that the phase term should contain kp minus ks. So, kp minus ks I reduce this kp minus ks term to k so kz minus delta omega t this is the frequency the term corresponds to frequency delta omega.

So, E square may have different frequencies only we are taking those terms, which are the frequency component delta omega because this is the source term for the molecule that will going to vibrate at the frequency delta omega.

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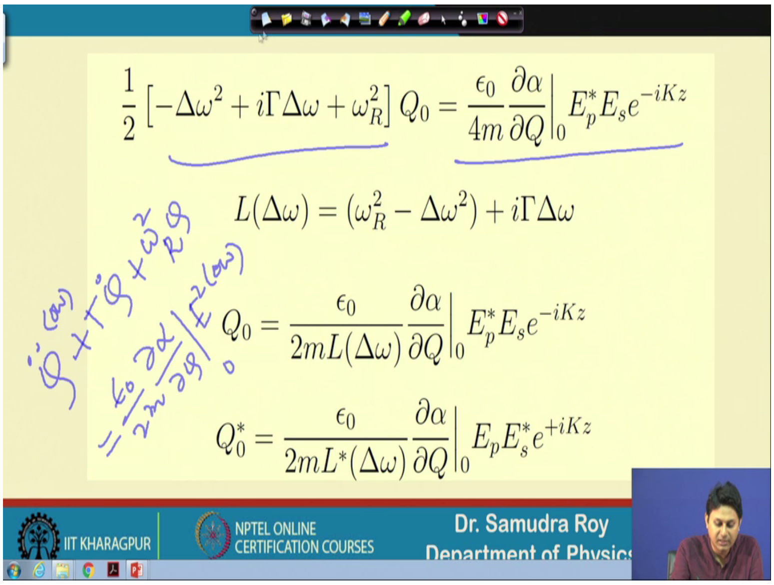

Well after having everything in our hand, we will put that to the equation when you put that to the equation we will get this kind of expression.

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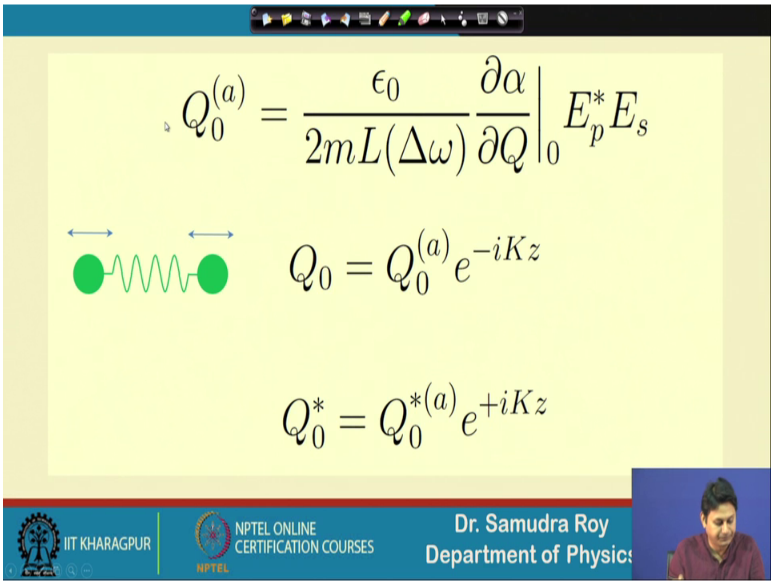

So, we had the equation in our hand which is Q double dot let me write this equation here. Q double dot here double dot means the derivative with respect to time double derivative with respect to time then plus gamma Q dot plus omega R square Q is equal to the total force term this total force term was epsilon 0 divided by 2 m so far I remember del alpha del Q at 0 and E square.

So, E square we know it should be the component of delta omega and here E also the component of delta omega. So, when I put the value of Q and E having the component delta omega and put it here in this differential equation, I will really have this expression in our hand, we readily have these expressions and when I put these expressions then we need to solve because we have here.

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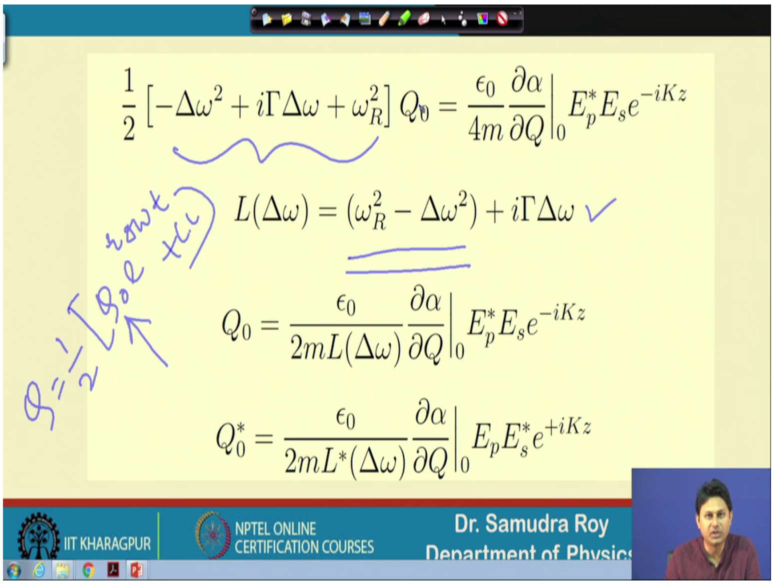

The delta omega thing I write this entire stuff as L function of delta omega to reduce or to make the calculation more compact. So, L of delta omega is omega R minus delta omega square plus i of gamma delta omega.

So, L of delta omega is a complex quantity please mind it and if I put it here then Q 0 is a solution of this equation because we have already a form of solution I already have a form of solution which is Q 0 e to the power of i delta omega t plus complex conjugate. So, this was the solution and in order to have explicit form of the solution we need to find out what is my Q 0. And exactly this is the thing we are doing right now put this solution here and extract the value of Q Q 0.

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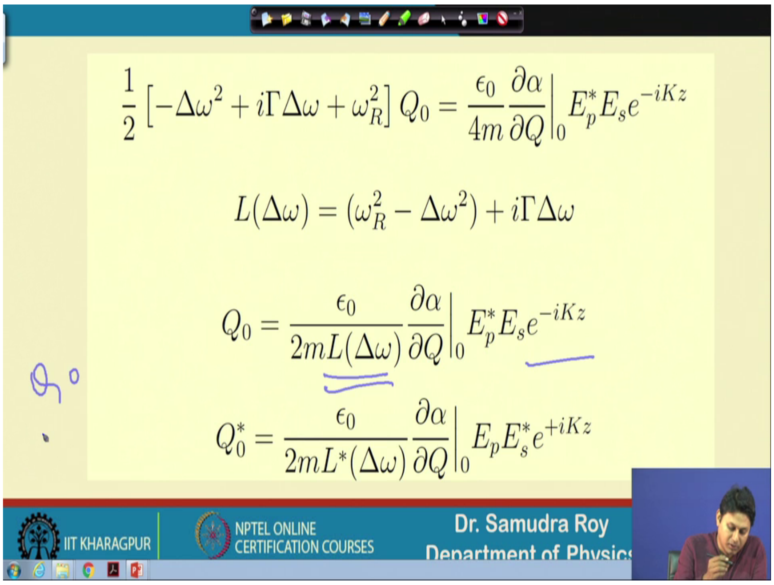

So, when we extract the value of Q 0 it is now simply epsilon 0 2 m L of delta omega del alpha del Q 0 E p star is E s e to the power of minus ik z.

When I will write Q 0 mind it when I write Q one thing I should note here. So, Q is written here the frequency component of plus the frequency component is plus delta omega t here the frequency component is minus delta omega t, but this is plus delta omega t. So, if I want to find out Q 0, then I need to write this frequency component both the side when I write this frequency component both the side these term will appear 2 E p star E s.

If I look very carefully in this equation here we have this term E p star E s because we are using that from e to the power i delta k delta omega t. So, Q 0 I will figure out once we find out Q 0 then it is easy to find out what is Q 0 star because Q 0 star is nothing, but the complex conjugate of the previous term. So, Q 0 is this, so Q 0 star is epsilon 0 2 m L star delta omega del alpha del Q 0.

And complex conjugate of this is E p E s star which is just the star is reversed and e to the power i kz, so Q and Q Q 0 we evaluated that is important. So, once we have the Q and Q 0 term in our hand, then we can write this Q 0 in form of amplitude and phase Q 0 a here is the amplitude. So, here we find the total value so what we do here to make it more compact I write Q 0 bracket a which is the amplitude part of that.

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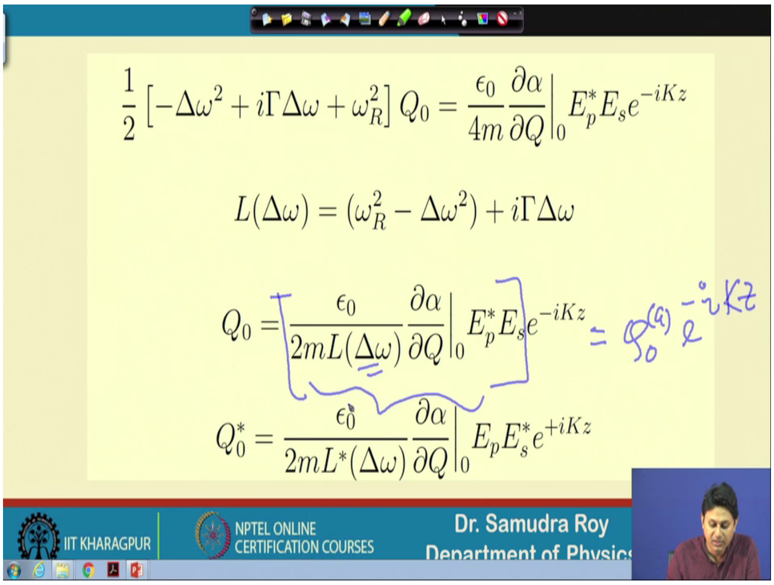

So, the amplitude part contain from here to here with the phase e to the power of say minus i kz. So, the complex amplitude Q 0 a is now in this particular form why this complex? Because E p E s may be complex and here L sitting here which is also complex. So, we can write the total Q 0 as epsilon 0 this, E p star E s and here in amplitude here in amplitude and phase form.

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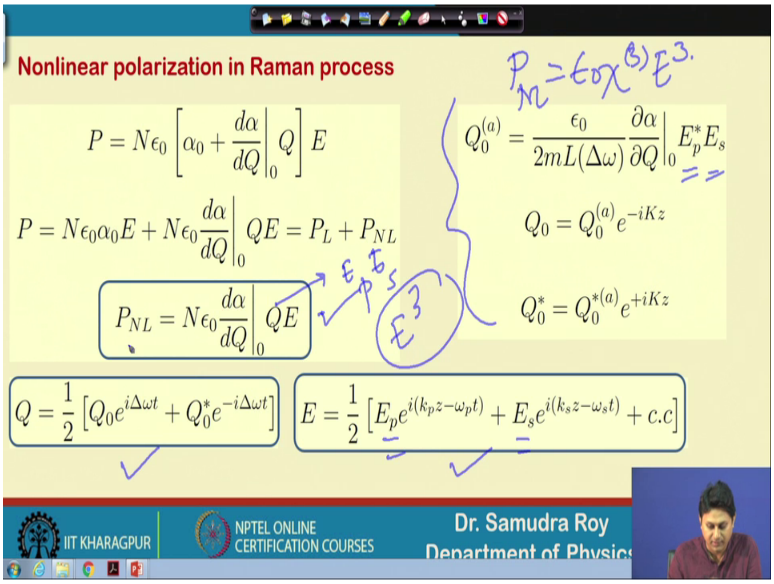

So, once we have Q 0 in our hand the next thing is to find out the non-linear polarization in Raman process that is the ultimate goal because non-linear polarization is something that we need to figure out.

So, let us quickly recap what was on non-linear polarization, so P was N epsilon 0 Q of alpha of Q and E, alpha of Q is expanded as alpha 0 plus d alpha d Q multiplied by Q because is a weak function of generalized coordinate Q. And after that we divide these things into two part P linear and P non-linear we called it P linear and non-linear. The non-linear part contain this Q term and P non-linear is defined in this way, where Q and E are multiplied, next we need to find out what is my Q and E here we write Q this is this was our solution and this was our value E.

So, Q we solve the forced oscillation using the forced damped oscillation model for a molecule we calculate its generalized coordinate Q and when we calculate the generalized coordinate it comes in this particular form. So, when it comes in this particular form you can see that we have E p and E s these two fields sitting here. When this Q has E p and E s term sitting here and then it will going to multiplied with E E is also having one E p and E s.

So, when we multiply this Q to E so overall weightage of E will be E Q this is very important that you need to understand here that why this Raman process is included in chi 3 under chi 3 process. The Raman process is included under chi 3 process because chi 3 in case of chi 3 P non-linear is equal to epsilon 0 chi 3 E cube.

Here also the P non-linear term is of the order of E cube that is why it is some sort of chi 3 process, well Q 0 is in our hand Q is in this particular form E is this particular form. So, the next thing is to calculate P non-linear which is very, very important and now we will going to calculate P non-linear because everything is now in our hand.

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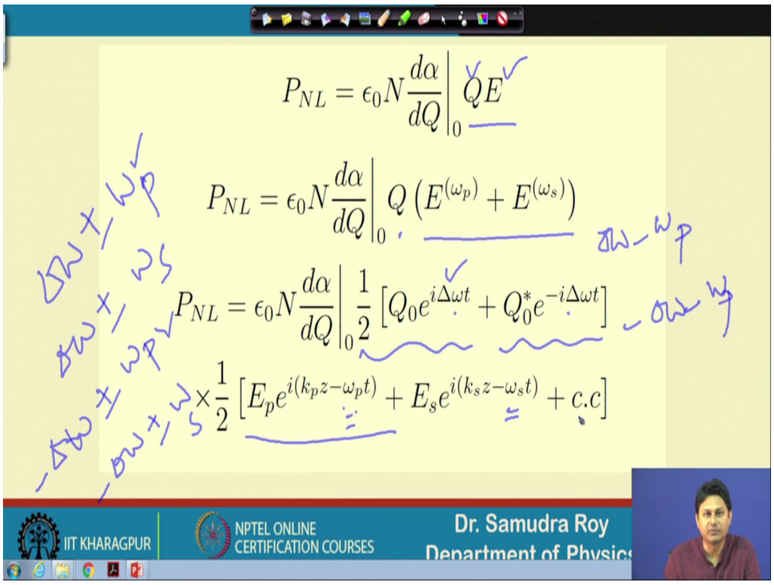

So, P non-linear is P non-linear is the multiplication of Q and these things E with this term. So, Q is sitting here E p is a 2 terms E omega and E omega E omega p and E omega s. So, now I write the extended form of Q which is half Q 0 delta omega t Q 0 star delta omega t with a negative sign with i E p kp z minus omega ts e to the power i kz minus omega t plus complex conjugate.

So, I write the total form of Q I write total form of E and then I am going to multiply that. Before multiplying you should note that how many different frequency component P non-linear should have. Here we have delta omega and when it is multiplied with E p we have omega p again it will be multiplied with omega s.

So, we will have some sort of term delta omega plus minus of omega p. This frequency will be there delta omega plus minus of omega s this frequency will be there, delta omega with a negative sign plus minus of omega p this frequency is also there and minus delta omega plus minus of omega s this frequency is there. So, so many frequency components can be possible here because P non-linear will contain this amount of frequencies which is obvious that when you multiply Q multi with E then this amount of different frequency will come up.

So, when you multiplied omega p with this we will have delta omega minus omega p which is one of these frequencies. When you multiply this quantity with this then we will have minus of delta omega minus of omega p one of the frequency components sitting here. In the similar way we have the complex conjugate term is here. So, we will have all these frequencies in our in your hands.

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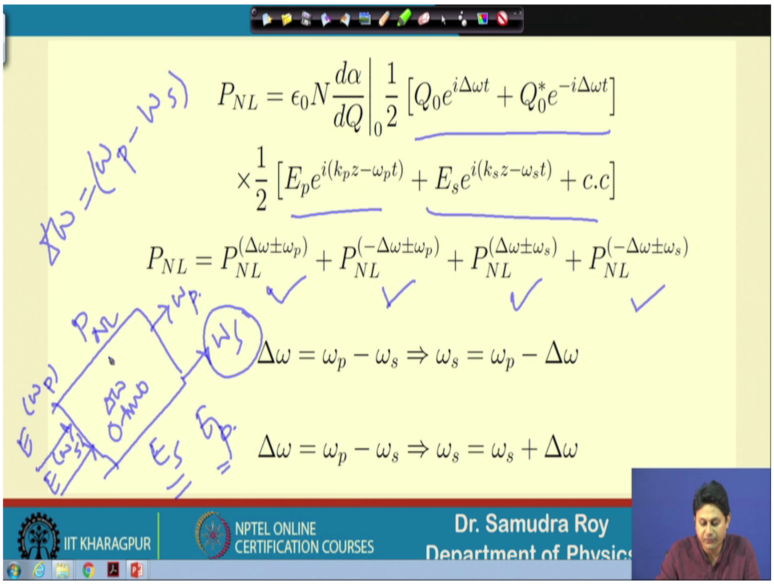

Well we know that P non-linear is now having different frequencies so if I now start writing all these frequency components. So, we will have these amount of frequencies as we mentioned. So, again go back to the equation of P non-linear because P non-linear now we are going to calculate or we are already in the process of calculation and when P non-linear we use the value of P non-linear it is epsilon 0 N del alpha del Q half of Q 0 this quantity and E p this and this.

So, different frequency component as I mentioned in the previous slide should be here and if I write all these components. So, P non-linear should have this frequency component, this frequency component, this frequency component, this frequency component. So, total eight there is two frequency component here we have also two frequency component here also we have two and two total eight different frequency component it should have.

But which frequency component is the most important one that is something we should be careful about, because P non-linear when we try to find out what is the evolution of the stimulated cases the Raman stimulated process what happened that I am launching electric field with frequency component omega p. I am launching another electric field with frequency component omega s molecules are here due to the interaction it is now vibrating with a frequency omega.

So, delta omega is equal to omega p minus omega s now this wave will come out and when this coming out omega s which we call the stroke wave is amplified. So, evolution of E s an evolution of E p we need to find out when omega s is amplified that means, amplitude of this omega s field which is E s we are going to amplify. So, if E s is going to amplify we need to find out what is the differential equation of E s in order to find out the differential equation.

We need to know what is the source term of that which is P non-linear p non-linear is the source term and we find here that the P non-linear is containing these match of frequencies. So, in these frequencies if you look very carefully there is one frequency which is omega s and this is omega p minus delta omega among these frequencies delta omega is omega p minus omega s. So, omega s is omega p minus delta omega when we have omega p minus delta omega you can see this frequency is sitting somewhere here omega p minus delta omega.

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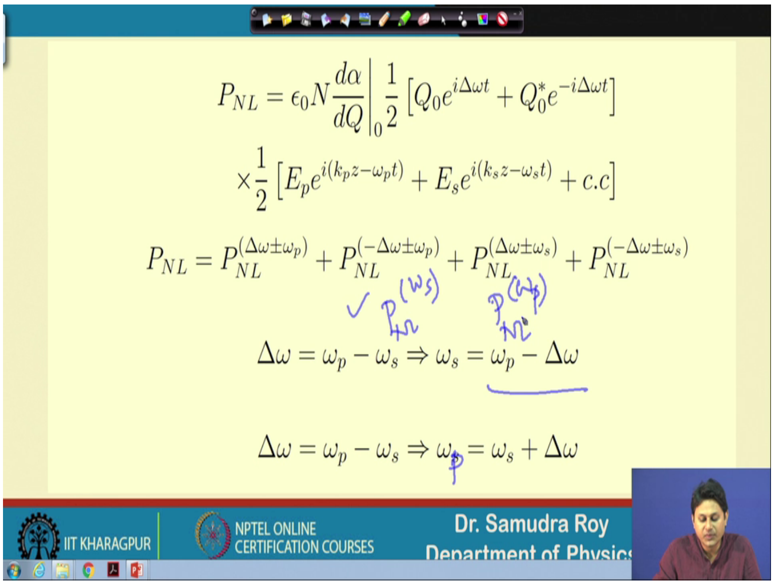

So, PNL omega P minus delta omega is sitting here omega p minus delta omega so I can write that this term will contain one frequency omega s also here it should be omega p there is a printing mistake also omega p frequency is also here somewhere omega p is omega s plus delta omega. So, omega is delta omega is sitting here so one non-linear polarization among these eight frequencies one non-linear polarization is also vibrating with the source frequency which is omega 2. So, we have all the different frequency component is our hand in P non-linear term, so we will only take those frequencies which will going to generate inside the system.

So, in our case stoke frequency we will try to find out what is the P non-linear force to frequencies as well as the pump the frequency for the pump is omega p. So, the non-linear polarization term for omega p is also important that we are going to extract from this expression and in the next class we will construct the differential equation for the stoke wave and the pump wave and try to find out how this stoke wave going to amplify in this process.

So, with this note let me conclude this class here, so thank you for your attention and see you in the next class where we will going to discuss a very important process called Raman amplification and try to understand how the signal is amplified under Raman amplification

So, thank you and see you in the next class.