**Introduction to Non-Linear Optics and its Applications**
**Prof. Samudra Roy**
**Department of Physics**
**Indian Institute of Technology, Kharagpur**

**Lecture – 53**
**Optimal Phase Conjugation**

So, welcome back students to the next class of Non-Linear Optics and its Application.

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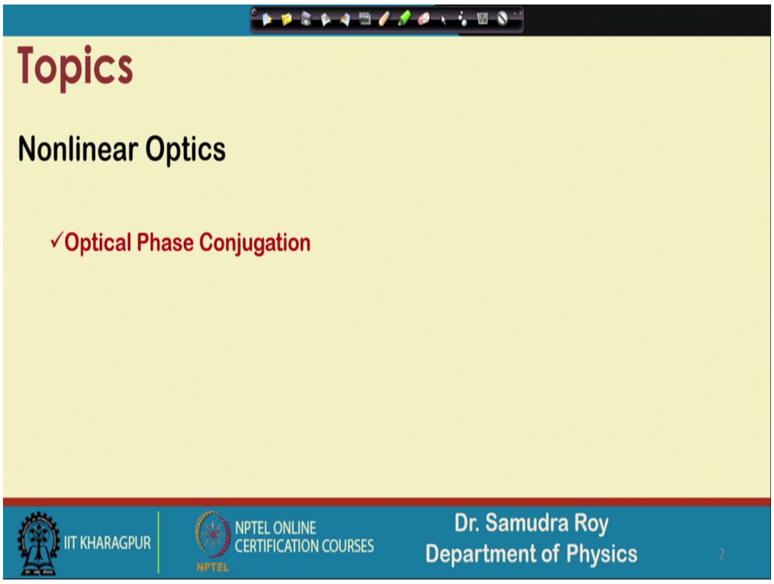


So, today we will going to study one very important thing called optical phase conjugation, for the last few classes we have been studying the 4 wave mixing and the corresponding applications. So, optical phase conjugation is some sort of 4 wave mixing which has some application that we will going to discuss in this class.

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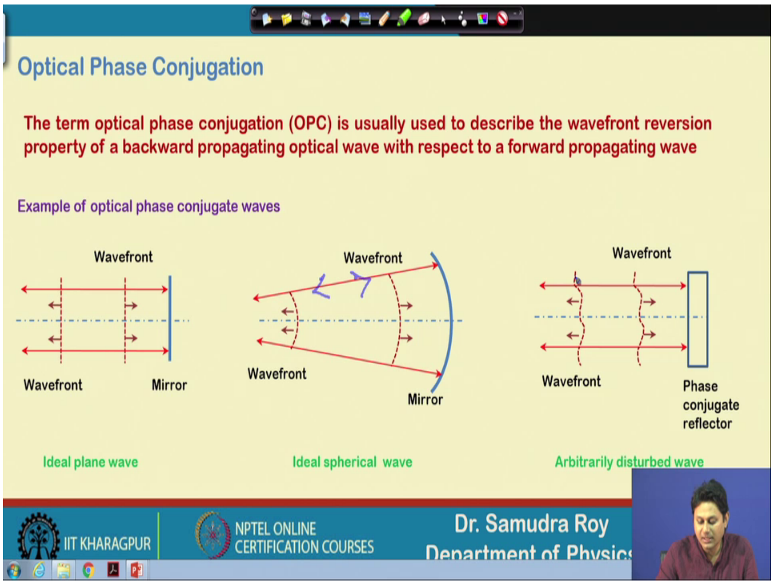


So, what is 4 wave mix, what is phase conjugation under 4 wave mixing? So, before going to that topic let us try to understand: what is basically the phase conjugation. So, the term optical phase conjugation as written here in this slide is usually used to describe the wave front reversion property of a backward propagating optical wave with respect to the forward propagating wave.

So, a wave is propagating in the forward direction and if the wave the same which is propagating in the backward direction then this is nothing, but the conjugate wave of that and this phase conjugate is related to that property. So, there are 3 different examples are shown in this particular slides if you see; in one case we have a mirror and the wave front which is parallel to the mirror is propagating towards the mirror and reflected back. If there is no distortion in the mirror the same wave front will be reflected from this mirror and this is an example of an ideal plane wave and this ideal plane wave which is reflected back from the mirror is nothing, but the phase conjugated wave.

In the similar way; that means, from this example you can readily understand that the light that is propagating in one direction will be reflected by the mirror and propagating in the exactly in the same direction. So, there is a directional property. So, it is not that light is reflecting and then after reflection it is moving in other directions also, that normally happen when we see something then that is the case that light is reflecting all the direction that is why I can see that particular object.

But here, the thing is that the light is reflecting from the mirror and then propagating exactly in the same path and moving in the opposite direction, another example of the ideal phase conjugation system is the ideal spherical wave. So, here instead of having a mirror which is a straight mirror we have a spherical mirror and this spherical mirror have some sort of curvature and if I launch an wave front with the same curvature it will go and then reflected back from this particular mirror.

And we eventually have a reflection, but important thing is the light is propagating in this direction and after reflection this again coming to this particular direction. So, direction is not going to change; so, light is moving in the same part. These 2 examples are related to mirrors the mirrors, I am talking about is a normal mirror and now the third example is something which we will going to discuss in detail and this is called phase conjugate reflector. This is a spatial mechanism on that and we will describe this mechanism in today's class, but the important thing here is that any shape of wavefront you can see that this is a distorted wavefront. And if this distorted wavefront is reflected from this phase conjugative reflector what happened we will return back the same wave wavefront even though it is a reflected even though this is reflected, but the distortion is there.

So; that means, during the reflection there is a possibility that it will not going to move in the same direction, but here we find that it will going to move in the same direction as this is a phase conjugate reflector. So, how by using the 4 wave mixing we can generate this kind of reflector we will going to study today and also we will find that this kind of reflect and not only reflect the light under some sort of phase conjugation light, it will going to reflect also amplification properties are there. So, reflected light can be amplified also. So, this can be only happen if the system behaves in a non-linear fashion. So, non-linear optics is directly related to this kind of phenomena. So, let us discuss what really happened and what is the physics behind that ok.

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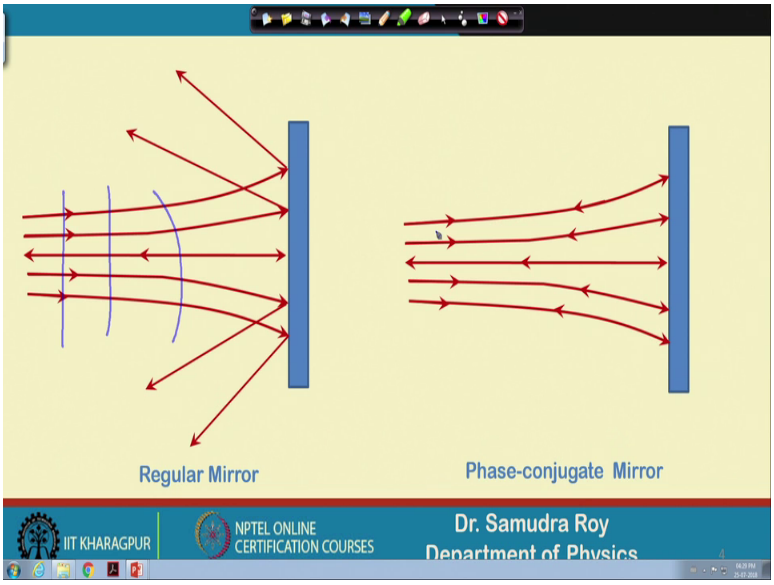


So, this is again the same picture and maybe this picture will be more useful for the students because here you can see that for the regular mirror if the light is coming from one part and then reflected back. So, this reflection may not be in the same direction. So, you can see that this is a light where the wavefront is not a; wavefront is not really a plane wave, but it is some sort of distorted like this after moving some distance and what happened that because of this distortion. So, the light will be; light will be reflected from this mirror and this reflected light can be move in whatever the direction it likes.

But for the same light if it falls on a phase conjugated mirror then what happened that the light will go here and then from here it will again reflected back in the same direction. So, this is a very unique kind of mirror and if some image is distorted then during, by using this phase conjugated mirror we can we can make this image more perfect. So, distortion can be removed by this phase conjugated mirror because whatever the shape is coming to this mirror and after the reflection we will have a similar kind of phase. So, further distortion is not possible here. So, if since the light is propagating the same path so, that is an added advantage and because of this we can find a distortion free system by suitably use this things.

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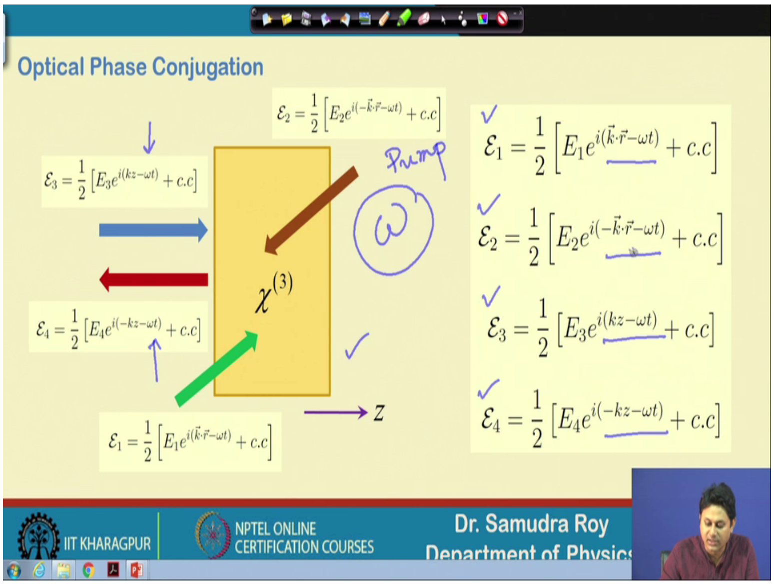


So, now let us try to understand in detail exactly what is going on. So, this is as I mentioned to the start of this class that is it nothing, but the 4 wave mixing problem that we have been discussing for last few classes. So, here we have a 4 wave mixing, but all the waves are having the same frequency that is the first thing that we need to note, that here this is not a non degenerate kind of 4 wave mixing this is a heavily degenerate 4 wave mixing because all the wave length are same for all wave the wave length is same or the frequency is same and we call this frequency simply omega.

But the one important thing here there are 4 wave associated with this system. So, chi 3 is a non-linear system we can see that there are 4 waves associated for this phenomena and this 4 wave are shown here with 4 different colors. In all cases the frequencies are same only thing is that the directions for 4 cases are different, in first case E 3 and E 4 the frequency is omega. So, if I write these 2 plane wave it will be simply omega t omega t both the cases, but since the direction is different in one case the forward, the wave is moving in the forward direction and other case the wave is reflected back.

So, in one case if I write the plane wave in the form e to the power i k z minus omega t in the next case I should write e to the power minus k z minus omega t. So, this is the same wave, but it is moving in opposite direction, here also E 1 and E 3 we use as a pump, why we will going to use as a pump we will describe, but you should note that during this course we study many amplification process and this is in some sort of amplification process I am talking about.

So, that is why the pump term is there. So, whenever we have some amplification or amplification some wave. So, the associated pump should be there. So, pump basically gives the energy to the wave or the signal and signal get amplified, there also we will going to get the similar kind of phenomena. So, these things are basically behave as a pump, but important thing is that this is not in the direction of z, but this is some arbitrary direction and moving in the opposite direction.

So, in one case if I write it is k dot r minus omega t since, this is the arbitrary direction I cannot write it is k z minus omega t because it is not in the z direction at all you can see that. So, it is k dot r minus omega t in other cases it is minus k dot r minus omega t, as I mentioned the frequency is same for all the cases only the directions are different. So, here in the right hand side all the 4 waves are written E 1, E 2 E 3 and E 4; E 3 and E 4 are moving in the z direction. So, that is why this is their plane of structure E 1 and E 2 which is which behave as a pump is moving some arbitrary direction say r. So, that is why the plane of structure is k dot r minus omega t and here also minus k dot r minus omega t because it is in opposite direction. So, 4 waves are now in our hand.

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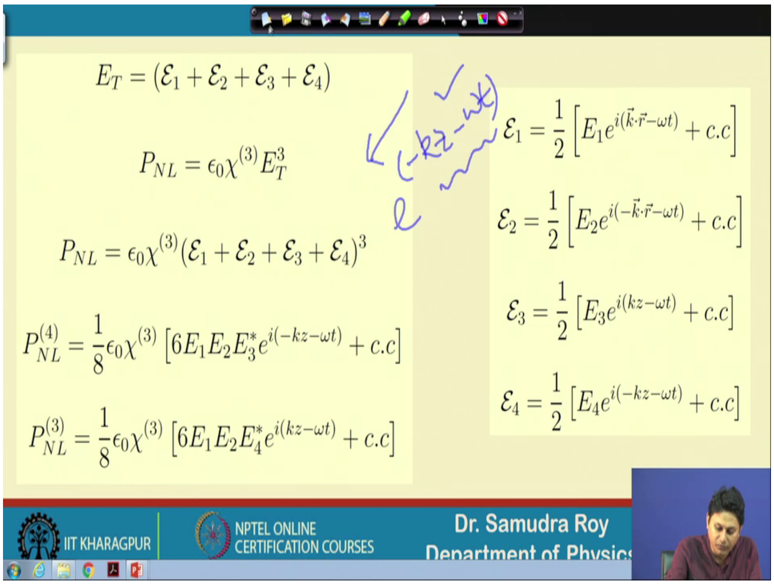


So, next thing that we will do we will use the simple process that we have been using instead in the 4 wave mixing. So, non-linear polarization we need to find out, what is the non-linear polarization for that? So, non-linear polarization is nothing, but epsilon 0 chi 3 the total field cube because this is the third order process. So, in third order process we know that the total field has to be cubed. So; that means, the non-linear polarization inside this system is E 1, E 2, E 3, E 4 and cube of that were E 1, E 2, E 3, E 4 is written in the right hand side. Now we need to find out the polarization of; the polarization that will give rise to the wave E 4 and E 3, why E 4 and E 3? Because let us go back to the previous slide then maybe we can understand these things better.

So, E 1 and E 2 behave like a pump. So, we are not going to study the evolution of E 1 and E 2 because normally pump is very high amplitude; with very high amplitude and were not bother about the evolution of the pump normally, here we will do the same thing E 1 and E 2 will be pump. So, it will it will give rise to some sort of energy to it will supply some sort of energy to the wave E 3 and E 4. So, evolution of the E 3 and E 4 is important since the evolution of the E 3 and E 4 is important.

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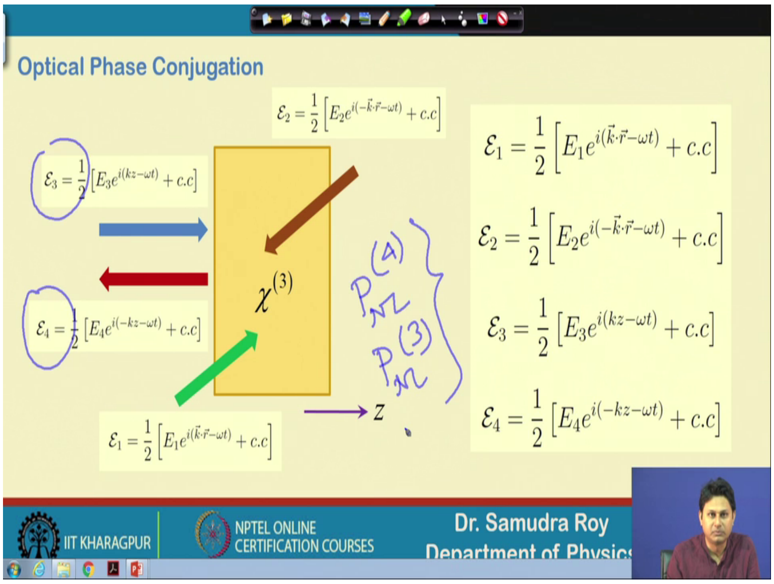


This E 3 and E 4 is important, we need to find out what is the source term of E 3 and E 4 mind it E 4 is not here. So, because of the mixing of this 3 wave E 3, E 2 and E 1; E 4 we will going to generate this is a 4 wave mixing. So, mixing up 3 frequencies can generate other frequency and that is the thing we need to find out the how E 4 we will going to evolve inside this system.

So, in order to find out how E 4 will going to evolve one thing that we need to derive first is the non-linear polarization corresponds to the E 4 and also E 3 if I want to find out what is the evolution of E 3 because E 3 is also important this is some sort of signal. So, P non-linear of 3 we need to find out. So, these 2 non-linear polarization term basically the source term of E 3 and E 4. So, that is why our first aim is to find out the non-linear polarization for E 3 and E 4 and here we find that E 1 E 2 E 3 E 4 is there see if I say I want to find out the non-linear polarization of wave 4. So, eventually what I am trying to find out that a phase that a phase of the form e to the power of minus k z minus omega t this is for 4, because we know that the 4, E 4 wave is moving in the opposite direction.

So, the phase of these things is in this form so; obviously, the p non-linear term should have this kind of phase and that will be the source term of E 4. So, the next question is how to form E 4 by using E 1, E 2, E 3 where E 1, E 2, E 3 is given here and how I will get this stuff this phase which is very easy because this kind of calculation we have been doing for last few classes. So, here if I see; so, E 1, E 2, E 3, E 4 cube are there. So, I need to find out how E 4 will going to generate using E 1, E 2, E 3.

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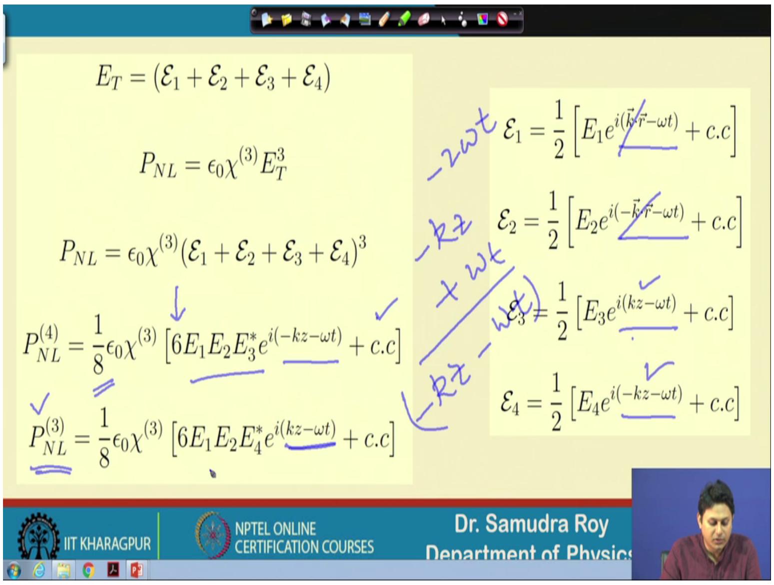


E 1 has a phase this, E 2 has a phase this and E 3 has a phase this with complex conjugate. So, very easy to find out what should be the form of E 4 and if I know the form of E 4 is e to the power of i minus kz minus omega t; that means, if I multiply these 2 stuff E 1 and E 2, I will this minus k r this kr will cancel out we will have a phase with the term 2 omega t. And if I make a complex conjugate of these things then I will have minus kz and then plus omega t, if I add all these 2 all this term then I will eventually have minus kz minus omega t which is exactly this stuff.

So; that means, I need to multiply E 1, E 2 and then I need to multiply the E 3 star with that. So, exactly the same thing is done here because it is a plus b plus c plus d whole cube. So, a cube plus b cube plus c cube plus d cube this term will be there also the mixing term is also there. So, in this mixing term if 3 terms are not same we know the degeneracy factor 6 should be multiplied with that. So, that is that is why this 6 term is here all the fields have half term with them. So, that is why if I multiply 3 fields. So, 1 by 8 will be here and the phase will be sitting simply like this with a complex conjugate term.

So, this is a very simple way to find out the P non-linear 4 I believe you can easily find out if you were given a different kind of phase you can readily find what should be the value. In the similar way exactly in the similar way you can find the non-linear polarization term corresponding to the field E 3, in case of E 3 everything should be same and just by inspecting the nature of the field you can readily find out that just replace E 3 to E 4 star and you will readily get this term. This is important term this is the phase term and we are looking for this phase term that this phase term has to be equal to the phase term of E 3 then only I can say that this is a non-linear polarization term corresponding to E 3.

So, once we find the corresponding phase term and when I combine E 1, E 2, E 3 to get E 4 or E 1, E 2, E 4 to get E 3 then we are ready because P non-linear term is in our hand, mind it our aim here is to find out the revolution of E 3 and E 4 and in order to find evolution of E 3 and E 4 we need to find out the non-linear Maxwell’s equation and this non-linear Maxwell’s equation p non-linear is a source term. So, very first thing is to find out the source term we are doing that throughout this course. So, I believe now you are familiar with this procedure.

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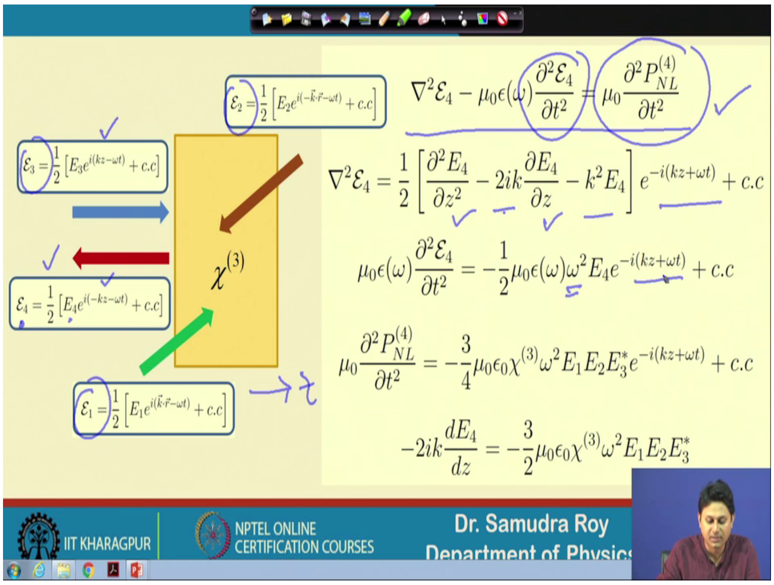


So, this is a very old kind of equation this is this equation I do not know how many times I am using, but again this is our non-linear Maxwell's equation and this non-linear Maxwell's equation for E 4 field. So, for E 4 field we have a source term and this source term should contain p non-linear 4. So, when p non-linear 4 time is there it is easy because P non-linear 4 I already derived in the previous slide. So, this is our structure this is the same structure that we have an incident wave and 2 other waves moving opposite direction and this is our z this is the z direction.

and these 2 behave as a pump and the mixing of E 1, E 3, E 2 and E 3 the mixing of E 1, E 2 and E 3 give rise to another term E 4 and that term should have a phase exactly the phase exactly in the way which is exactly in the opposite direction. So, if the phase of E 3 is kz minus omega t which is propagate in the forward direction if I say this is a forward direction then we will going to find out, what should be the form of the wave that will be moving out and it will be exactly in the opposite direction of E 3.

So, the mixing of E 3 E 2 and E 1 give rise to a wave E 4 and E 4 we will moving in the opposite direction of E 3 that is the same that is the simple thing that we are doing right now. So, in order to find out the evolution of the E 4 we need to study the non-linear Maxwell's equation or non-linear wave equation whatever. So, this is a source term and E 4 we know that E 4 already it is written here what should be the form. So, now, if I try to find out this grad square value of the E 4 this is the same old equation that we are using, the second order derivative of the E 4, E 4 here is the amplitude curly mind it curly is the total field here E 4 is the total electric field of total electric field that is moving in the opposite direction e is the amplitude and the phase term is written here.

So, E 4 will be derived twice with respect to z E 4 will be derive once and then the kz is there. So, kz will be there. So, 2 I kz and then k square E 4 this is the derivation or the grad square value of E 4 once the E 4 value is in our hand; obviously, the phase term will be there and this way and next I will calculate the time derivative of this term E 4 is half Carl E 4 is equal to half E 4 e to the power i minus kz minus omega t. So, when I make a time derivative. So, omega square term will come out with a negative sign. So, here we can see omega square is coming out with a negative sign here half will be as usual and we will have the phase.

And finally, the source term so, p non-linear we have already derived in the previous slide. So, let us go back to the previous slide and check what was the value, it was it was 1 by 8 epsilon 0 chi 3 6 E 1 E 2 E 3 star minus kz minus omega t. So, if I make a derivative of this term with respect to t and if I make the derivative twice then what happened the epsilon this omega will come outside and there will be a negative sign that is all.

So, we will do the same thing, here in this slide and if we do we will find it should be 3 by 4 and this quantity the 6 it to it was if you remember it was 6 and there was a 8 term. So, this 6 and 8 will boil down to 3 by 4 mu 0 epsilon 0 is already there mu 0 is here epsilon 0 is already there chi 3 omega square will come out and this is the amplitude term this is the important term its amplitude term we know the it will come it will be a term E 1 E 1 E 2 E 3 star and this is the corresponding phase.

So, when I put all this equation together we know that these 2 term will cancel out and sorry this is this term will cancel out because this term will cancel out because slowly varying approximation this term and this term will cancel out because, if I replace this k in terms of mu and epsilon then it will be exactly the same this term. So, this term and this term will again cancel out and finally, we will have 2 i k with a negative sign d E 4 dz is equal to 3 by 4 with a negative sign mu 0 epsilon 0 this. So, this is our equation for E 4. So, we can further modify this equation or further simplify this equation.

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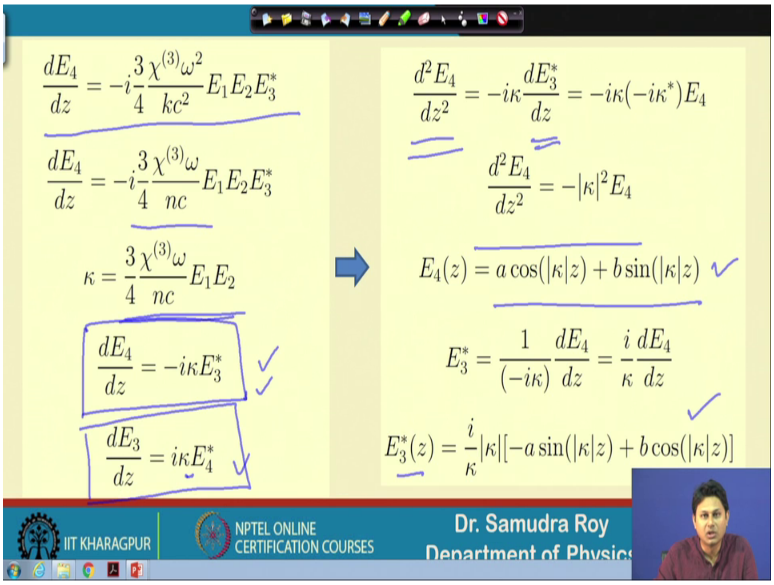


And if we simplify this equation, it will be simply like this; it will be simply like this d 4 dz is equal to minus of i 3 by 4 chi 3 omega square k c square E 1, E 2, E 3 star. So, k again I can make it a more compact form these or the usual form the chi 3 omega divided by n c 3 by 4 is the term with the negative I and E 1, E 2, E 3.

Where, if I put this quantity if E 1 and E 2 are constant if I consider this is a constant as I mentioned E 1 and E 2 is amplitude of this pump. So, if the amplitude of the pump if the pump is strong and if I consider that there is no depletion of the pump then the amplitude of the pump E 1 and E 2 become constant. So, in that term we will remain constant and if I write this constant as kappa as a coupling term then finally, we will have a very important equation in our hand which is evolution equation of E 4 and which is dE 4 dz is equal to minus of I kappa E 3 star.

Exactly in the same way if I derive E 3 it will be simply this. So, E 3 E 4 the at the differential equation of E 3 E 4 we derive, now what we will do we have 2 coupled equation and we will do our usual procedure that to solve this. So, if I make a derivative of E 4. So, in the right hand side this term will be we have a derivative of this term. So, E 3 is e z and E 3 is z star is this. So, I just replace here. So, I will get this equation.

So, this is a very straightforward differential equation homogeneous differential equation whose solution is sinusoidal. So, I write the solution here. So, the solution of E 4 is in our hand, once we have the solution of E 4 then we can readily find what is the solution of E 3 because E 4 and E 3 are related to this differential equation. So, this differential equation suggest that E 3 will have this particular form, I am moving little bit fast here because I believe these basic calculation you can do by yourself and can understand easily. That this is a solution of this kind of differential equation this is solution of this where the equation is given.

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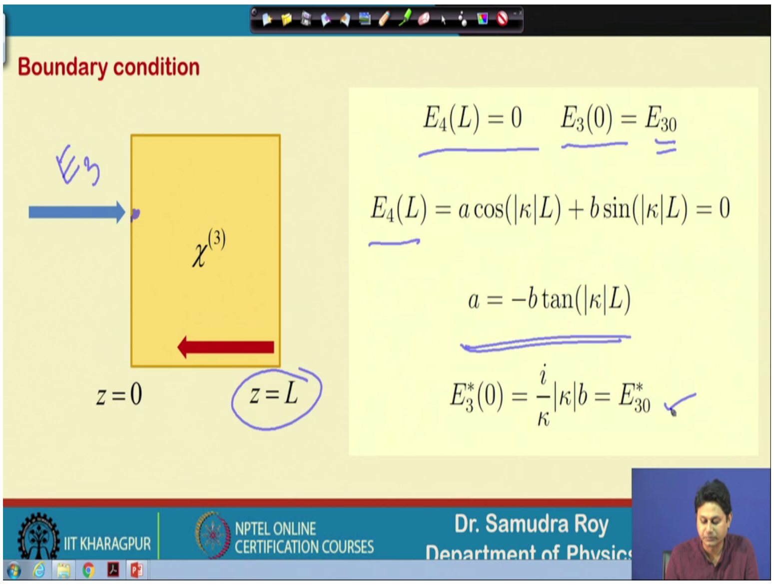


Well once we have these 2 solution in our hand the next thing is to find out the boundary condition. So, boundary condition is important here as I mentioned E 3 is coming from this side this is our E 3, but E 4 is generated inside and then moving out. So, initially E 4 when come and reflect here so; that means, at z equal to L eventually we will we should not have any kind of E 4 so as it is reflected.

So, if I put this boundary condition and also if I write that E 3 at z equal to 0 is some value E 3 0 because here we have some E 3 then we will we will have E 4 L is 0 because L, E 4 L is 0 if I put z equal to l then we have a relationship and this relationship suggests that a equal to minus of b tan of mod of kappa L. And E 3 is 0 is also this quantity, well once we have the value of a and b the relation of a and b.

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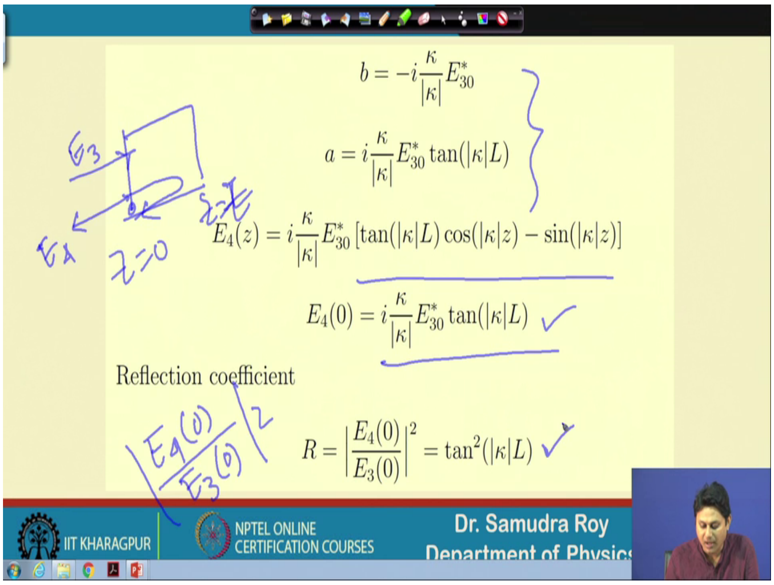


So, this relationship of a and b in our hand which we derived in the last slide. So, E 4 I can write in total, I just replace a and b and write the total equation of E 4 also we know what is E 4 0; that means, if this is the system I have what is here at z equal to 0 point and z equal to l point we know. So, now, at z equal to z point I know because z equal to l this quantity 0 so; that means, something is reflected like this here. So, now, if I try to find out what is the reflection coefficient because if 3 is here and it is reflected E 4 is reflected back.

So, the reflection coefficient is simply E 4 at 0 point z equal to 0 point divided by E 3 this incident wave at 0 point and mod of square. So, from this equation which we have already derived it is easy to find that it is something like tan square kappa L.

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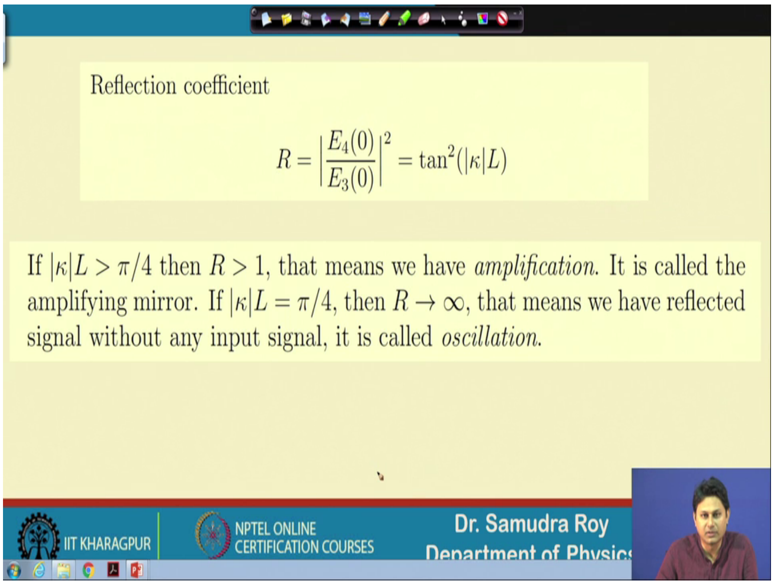


Now, we know that tan square value can have a range. So, this reflectivity suggests it is tan square kappa L. So, if kappa L is greater than 4 pi by 4 then the reflectivity is greater than one; that means, we have some sort of amplification as I mentioned if it is 1, then whatever we have in the input we will have the same thing as a output.

Now, if it is greater than one then; that means, we have amplification. So, that is an added thing and this amplification is possible because of the fact that we are pumping, if you remember the system we have some sort of pumping and this pumping basically give rise to the fact that some sort of amplification may possible now if it is pi by 4 then r tends to infinity. So, we have some sort of wave without any kind of input is something like that. So, it is nothing, but the oscillation.

So, several possibilities or several thing can be happen here and it is a very interesting process, but the important thing is that you can have a reflected wave if just opposite phase. So, this is really a phase conjugated kind of wave. So, using this 4 wave mixing we can have the conjugate wave or this conjugate wave is very useful in different or in order to remove the distortion in this kind of things the physics today we will we have understood. And there are several condition in one condition it is reflected in one condition it is oscillated and in one condition it is even amplified. So, amplification is also possible this is added advantage because of this non-linearity.

So, with this note let me conclude here. In the next class we have we will going to start a completely new topic which is Raman Scattering. So, see you in the next class.

Thank you for your attention.