Introduction to Non-Linear Optics and its Applications
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Lecture – 52
Parametric Amplification Under FWM (Contd.)

So welcome student to the next class of Introduction to Non-Linear Optics and its Application. So, today we have lecture number 52.

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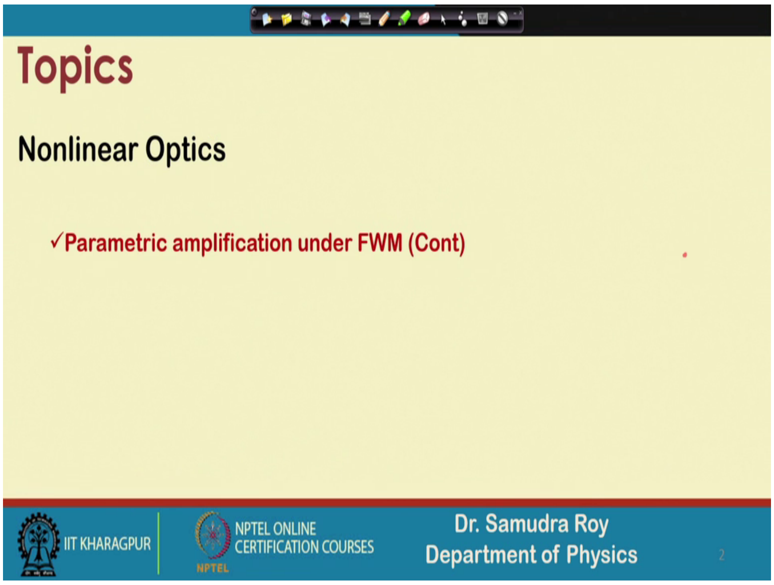

So, we will be continuing with the old topic that we have started in the last class that is the Parametric Amplification process under Four Wave Mixing.

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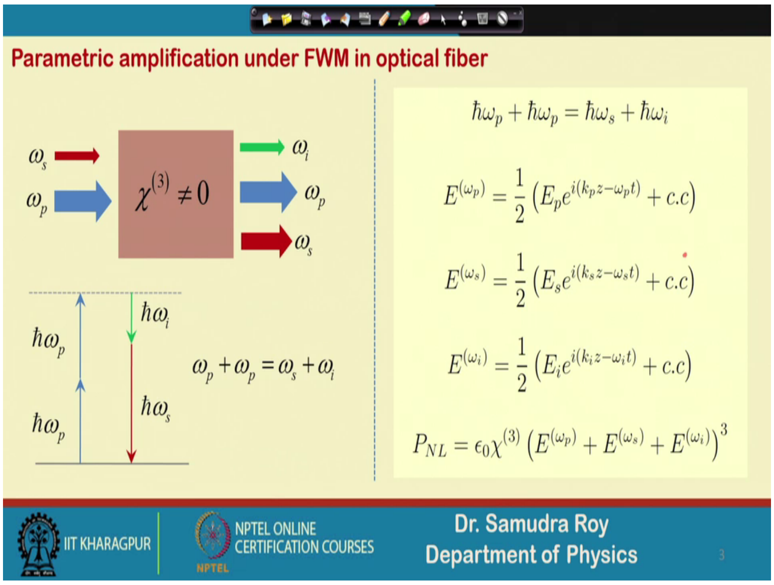

So, in parametric amplification process what happened that, in optical fiber we launched a pump omega p and if this optical fiber having a third order non-linearity which is quite strong, then what happened we can use this as an amplification of a signal omega s.

How this omega s will be generated can be understood by this energy diagram; that two pump photon will going to absorb and will generate one idler photon and one signal photon. So; that means, omega p plus omega p will be equal to omega s plus omega i. Since, three waves are associated and we are generating omega s out of that. So, it is essentially a four wave mixing process.

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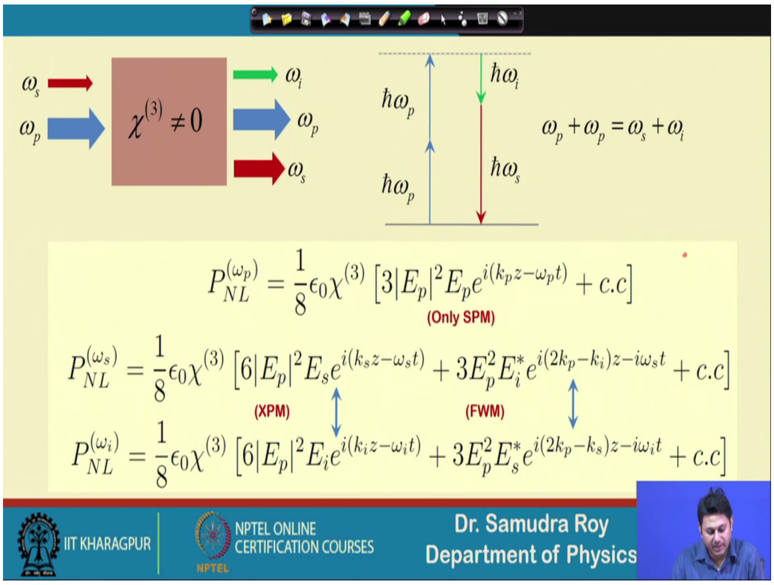

And, in this four wave mixing process the important thing to find out the evolution of the electric field containing frequency omega s; that means, the signal, the corresponding source term. So, P NL omega p is a non-linear polarization term, that will going to excite the omega p field or the pump; and in order to consider this pump field, we only take the self phase modulation term because E p is quite strong. So, since it is quite strong I will only consider its own interaction with itself that is the cross phase self-phase modulation. The cross phase modulation term with other effects omega i and omega s is neglected because they are quite weak.

In the similar way if I want to find out the source damper signal and idler, we only take the cross phase modulation with the pump and also the four wave mixing term related to the pump. Here the four wave mixing term related to the pump is the second one; both the cases we have taken these two terms. So, why these terms are taken is important and I believe you can understand that why these terms are taken because there are many terms in omega s frequency.

Omega s frequency can be generated in many way with the self phase modulation also we can generate by combining three omega s omega s omega s star E s E s star and E s also will generate omega s. Also E i E i star E s will also generate omega s, but we will not going to consider these cross phase modulation and self phase modulation term because they are weak only consider this cross phase modulation with pump and four wave mixing.

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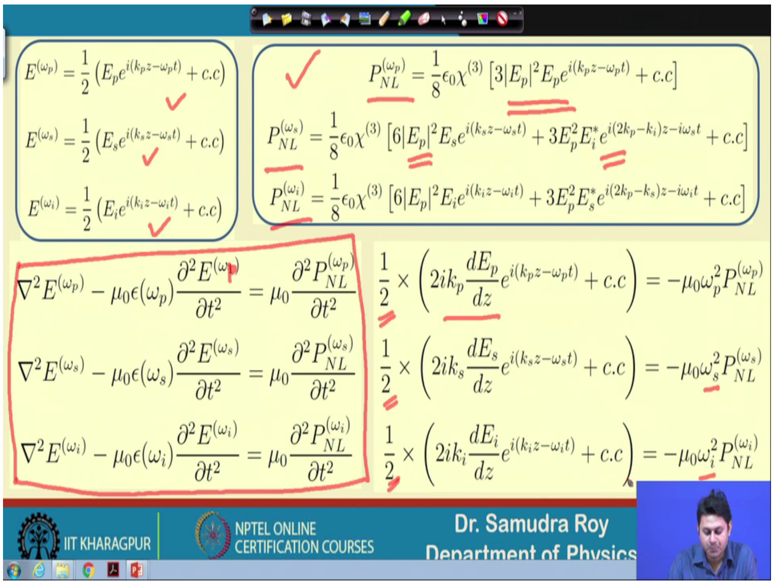

Well if I now make a total picture of whatever we are getting, then this will be our a summary. So, it looks quite heavy that many equations are there, but you if you look carefully you will find these are merely the identical equations written. Only thing that is the frequencies are changed. So, let me explain one by one, do not get confused with so, many terms it is quite straightforward. The first I have written the definition of our electric fields that E p is defined by a half E p e to the power i kpz minus omega t plus complex conjugate. In the similar way I write E omega and E i this is the definition of the electric field that we have been using for several times. So, E p so, E omega p E omega s and E omega i is known.

Next thing that one should know in order to calculate the Maxwell's equations is the corresponding source term, source term also I calculate in last slide and these source terms are for P NL omega s P NL omega p and P NL omega i is this. In the first case with P NL omega we are having only the self phase modulation term in the next case we have a cross phase modulation term with E p and the four wave mixing term and for idler we will have the same thing.

So, now I have the electric field a corresponding source term; in this portion I just wrote all three Maxwell's equation that you can use. First case how the pump will going to evolve is shown in the second case how the signal is going to evolve is shown in the third case thought the idolize is going to evolve is shown. Here there is a slight mistake it should be omega p it is written omega 1, it should be omega p anyway. In the next slide in the next part of this slide in this portion we write the compact form of this Maxwell's equation.

We know that this term will there are many terms that will cancel out and if I take the slowly varying envelope approximation we will have in this term. And meticulous write half both the cases so, that you can have the idea that every term will multiplied by half because according to our notation this half is already there. And also the P non-linear terms in the right hand side is the derivative with respect to P non-linear. So, it will be P non-linear omega with mu 0 omega p square because, when I make a derivative of this quantity only thing that will come out is omega p if I make a derivative with respect to t.

In this case it is omega s square it is omega i square with a negative sign all the cases. So, these are the compact forms of whatever the equation we have. So, now we just replace here whatever the source term we are having and generate the corresponding field, field equation rather.

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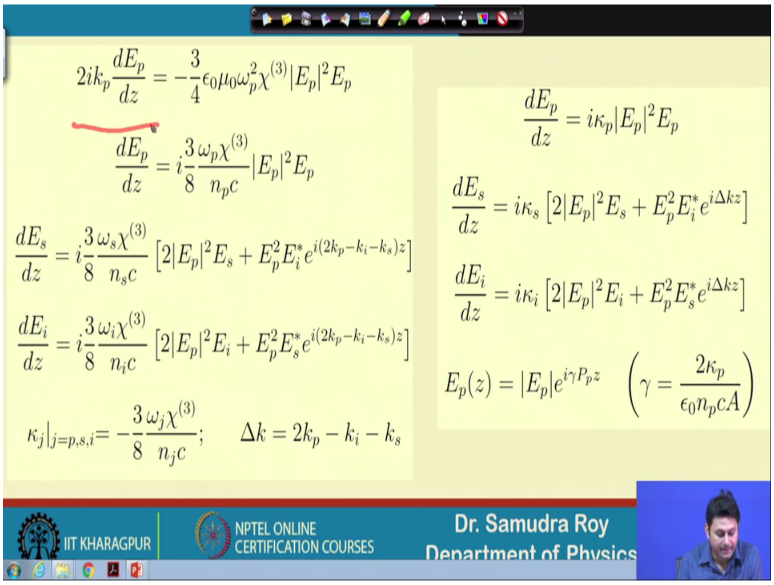

So, let us try to find out what should be the field equation for the pump because pump has a source term, which is self phase modulation kind. So, self phase modulation is the solution of the self phase modulation we know how to do that.

But let me do once again let us. So, it will be 2 i kappa p 2 i kp d E p dz. So, if I go to the previous slide basically we are using these equation we are using this equation right now for omega p evolution of the omega p.

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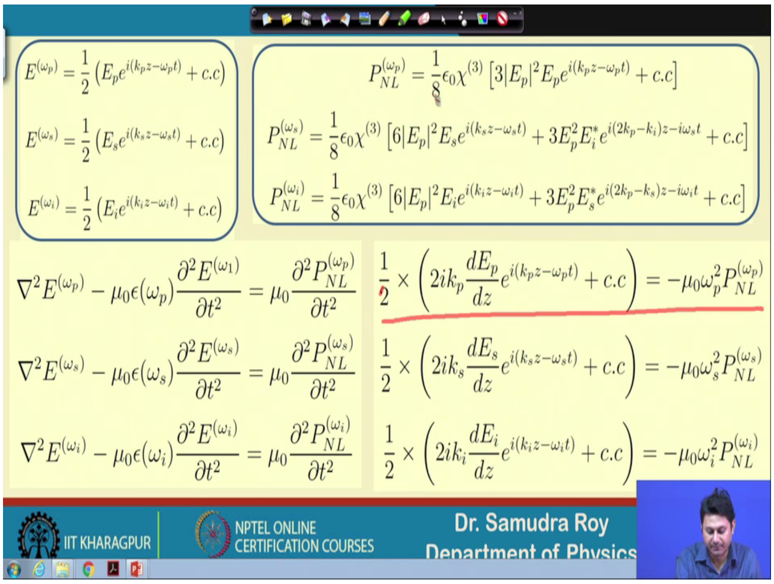

So, this half term will cancel out with P non-linear omega p which is this. So, we have one by four term here. So, eventually you have three by four term here. So, if you look carefully we are getting 3 by 4 term here with a negative sign.

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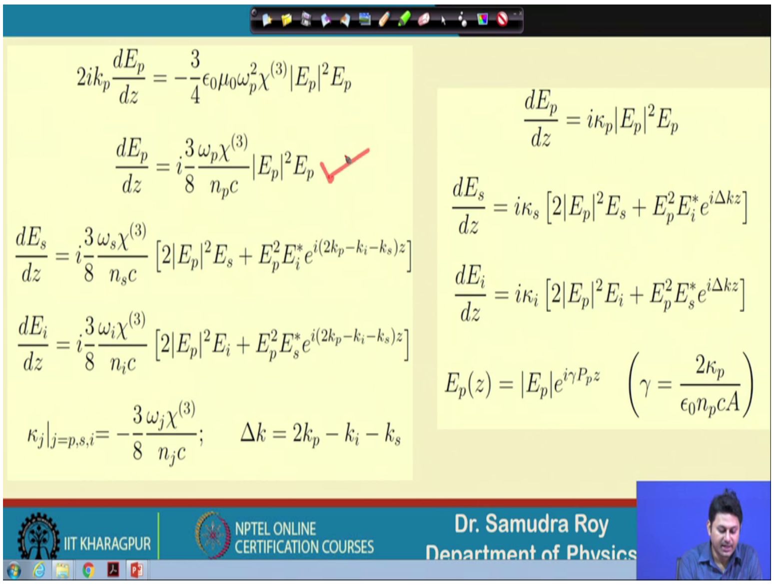

Mu 0 epsilon 0 omega p square chi 3 as usual and apart from that we have E p square E p, E p mod square E p and this is a phase matching is the phase matching is automatic. So, your exponential term will not be here, with both the term from both the side it will cancel out.

So, we will this leads to an expression like this. This is the equation of pump what should be the equation of signal and idler, you just use the same logic or the same procedure and if you do, you will find that I will going to use this is some sort of exercise I want the student to do that I will going to use these two equations where P non-linear of omega s is given P non-linear of omega i is given.

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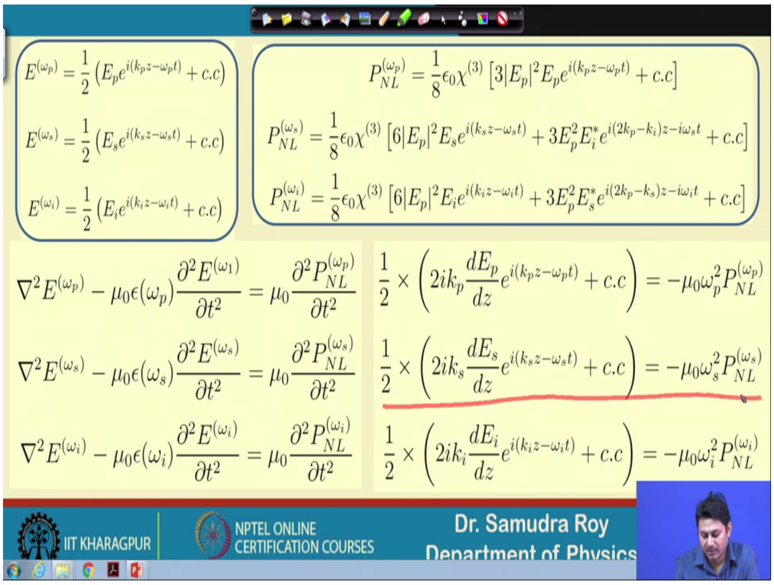

So, I just put this value make the derivative basically the derivatives not required at all because of the derivative we have already have omega eight square and omega i square term and when I put this term together we will simply have this quantity. So, first term is corresponds to the cross phase modulation term.

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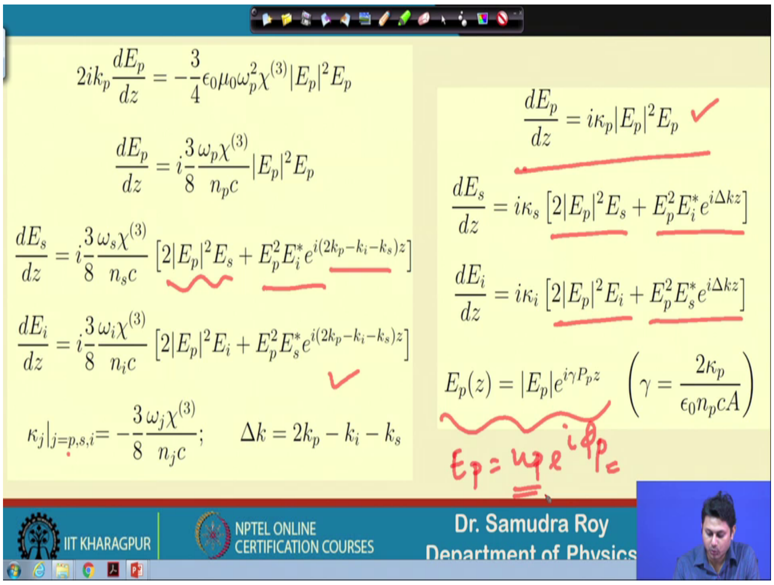

As usual and second term is a corresponding phase corresponding four wave mixing term, the exponential term will modify because in the right hand side we have e to the power i k z it will come this side. So, that it will we will have 2 k p minus k i minus k s.

So, this is the our phase term in the similar way we will have an expression for idler. Now if I this coefficient if I write kappa which is 3 8 omega j chi three divided by nj C and now this j is running can be getting the value ps and I and in that case it will be kp ks and ki for these two these three coefficients, then in the right hand side we have a more compact form and this compact form is dE p dz is i kappa p E p mod square E p which is a straightforward self phase modulation kind of equation.

In the next case we have a self phase cross phase modulation equation with a four wave mixing term and the last case also I have a cross phase modulation with E p and the corresponding four wave mixing term. So, the next thing is to solve this equation. So, now, we are going to solve E s in order to solve E s we need to know what is E p? Because E p is sitting here and we have a solution of E p we can have a solution of E p here. If we do the solution the solution will be something like this and this is nothing, but the solution of self phase modulation.

So, you can do quite easily just put E p as up e to the power I phi p and if I put this here you will have a equation of up which is a amplitude and equation of phi p which is a phase, and divide it into two part of real and imaginary and on the right hand side I can also have real and imaginary, if I combine these two real and imaginary we have two equation one for the differential equation of amplitude and another for phase we will find the amplitude will not going to change, but there is a change is phase and if you put this together then we will have a solution as shown in here.

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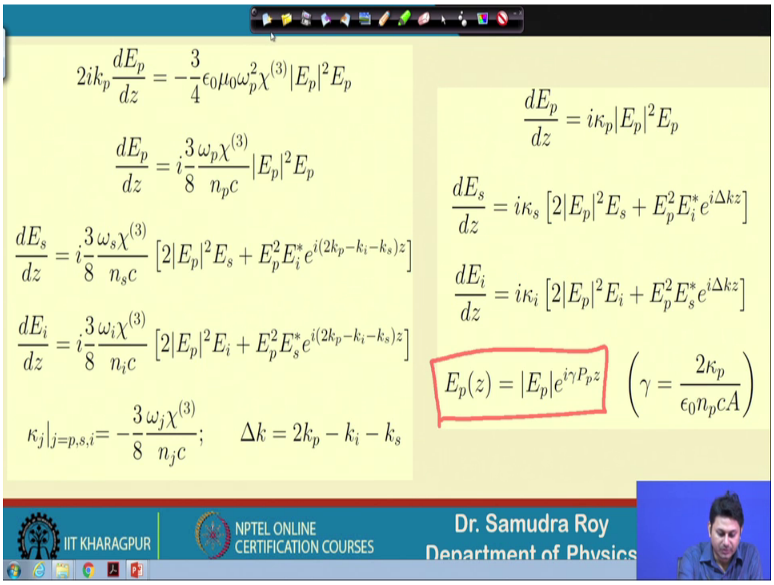

So, this is this will be our solution for Ep. So, now, once we have the solution of our E p and you can see it is not going to evolve because it is. So, strong that it can be considered as a constant, but since it is self phase modulation effect is there. So, we should not consider this as a constant its phase is changing may be the amplitude is constant there is no evolution in a amplitude, but phase is changing.

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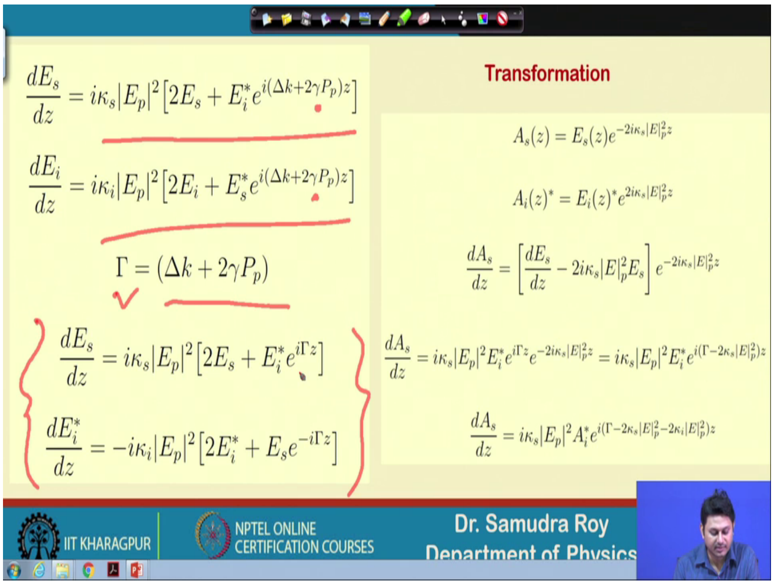

So, we need to consider this phase term here in this third order effect which was not there in second order effect, because there is no self phase modulation kind of terms are there. The next thing is we just put here I have Ep. So, what I will do? I will put this E p here and E p here to rewrite the expression of E s and E i once again if I put these things together then my E s and E i the differential equation of E s and E i will slightly modified and it will be like this. Please note that now this additional phase will be inserted here. So, E to the power i k z term was always there it is a natural phase term, but on top of that we will have another phase and which is coming because of the fact that my pump is now change pump is now having a phase term. So, this phase term will be added here.

Next in order to reduce all these terms I write delta k plus two gamma P p is big gamma. So, the equation for differential equation for E s signal and idler is something like this is the compact form of differential equation, but still we need to solve this two coupled equation that is the something which is important here. So, so far we are dealing with the differential equation of E s and E i, but here we have a differential equation of E s and E i, but not only one term there are two term associated with that one is corresponding self phase modulation term and another is the four wave mixing term.

So, self phase modulation and four wave mixing terms are together which should be because this is the demand this is the due to the physical process this will be there. So, in order to solve this coupled equation, we will going to have some sort of transformation.

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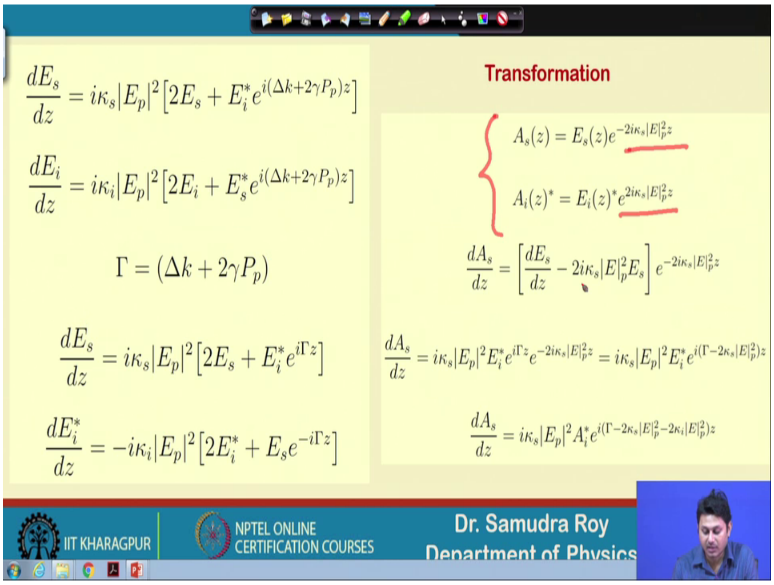

So, transformation is that I write two new variables as z and A i z star why I take A i z star because induced to couples equation you can see that one case it is E s, but in another case it is E i star. So, I take A s as E s e to the power of 2 i ks E p mod square 2 z this is the transformation. I took also for A i z I took a another transformation which is E i z star e to the power 2 i kappa E p squared z.

Why I took this transformation and why this is the why we took this is this particular form you should know quite I mean right now. That if I now make the derivative with this transformation if I now the derivative of A s with respect to z.

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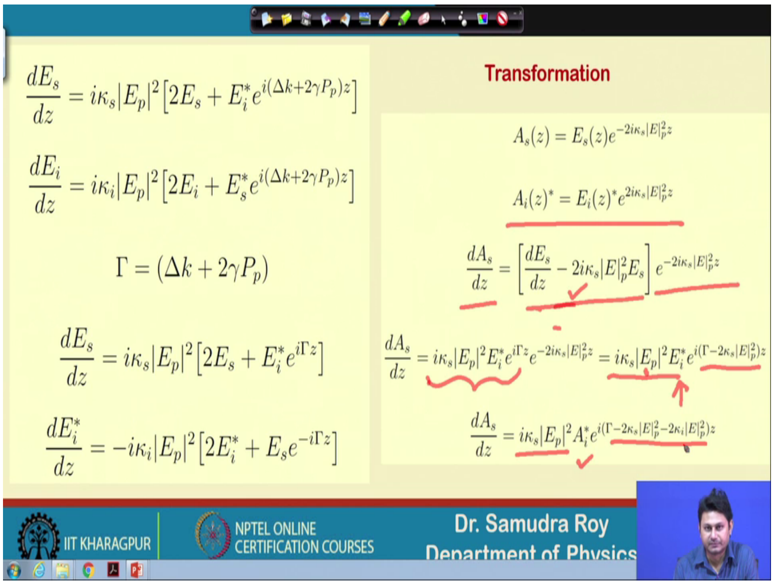

In the right hand side it will be E s dz minus 2 i ks ep mod square E s and E to the power of 2 i kappa s E p E p star z. Now this quantity if I look carefully from here ds d E s dz minus of 2 i kappa s E p star E s if I put it here. So, I can this quantity I can replace as i ksep mod square E i star e to the power i gamma z.

So, this is the term I replaced here. So, this term are now di dz is now simply becomes i kappa s E p square E i and with exponential term like this. This now I write E p I know: what is the solution of E p. So, E p is mod of E p square is here. So, next thing is I just replace this E i from here. So, that the entire equation become in terms of As and A i in the right hand side now I have E i and the left hand side I have as E p is a constant. So, we should not bother about Ep. So, I will make E p as usual.

So, i kappa s E p if I now change this E i star, from here this E i star will be Aiz star which is this, multiplied by E to the power of minus two i kappa s E p mod square z. So, if I write this term it will be exponentially it it this term will combine and give exponential gamma minus 2 kappa s E p square minus 2 kappa i E p mod square z. So, this is the final expression we have in terms of A s.

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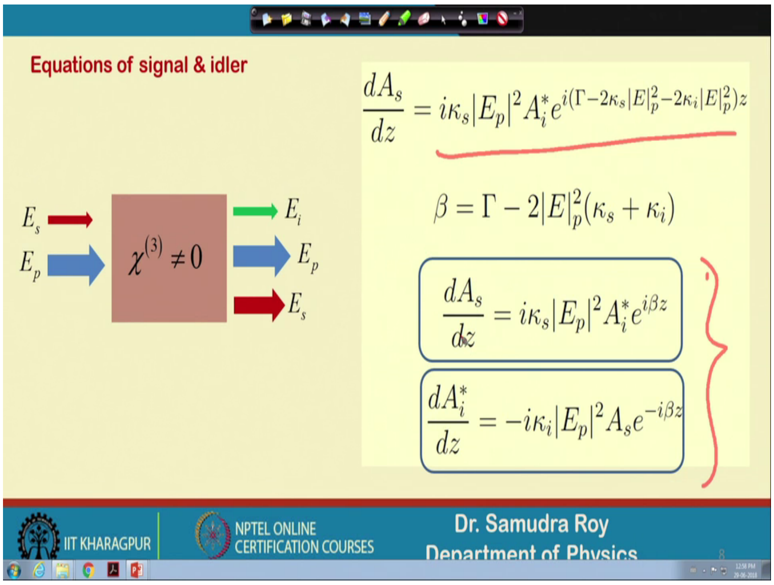

So, in terms of A s now, I have this now equation of signal and idler we want to derive. So, this is the expression of A s which corresponds to the signal. So, beta is now in a another constant I want to introduce so, that I can reduce this part. So, if I reduce this all things are constant because Epi consider constant it is a strong field. So, Ep is considered. So, when I put this. So, we have a rather compact form for as if you calculate the compact form of A s in the similar way you can calculate the compact form of A i also. So, A s and A i are the fields corresponds to frequency omega s and omega i.

So, now I write the differential equation for As and A i with this particular form. So, this now allow us to solve. Now we are in a position to solve this differential equation and how we solve this? This is the old equation old kind of equation we have solved this in the previous classes. So, we make a derivative both the side of A s. So, here we have a derivative of A i. So, this A i derivative I will replace from here; so, that the equation become in terms of As. Another term is also appearing from here which is e to the power i beta z with i beta and A i. So, again I will replace this A i from this equation and I will get a more straightforward expressions.

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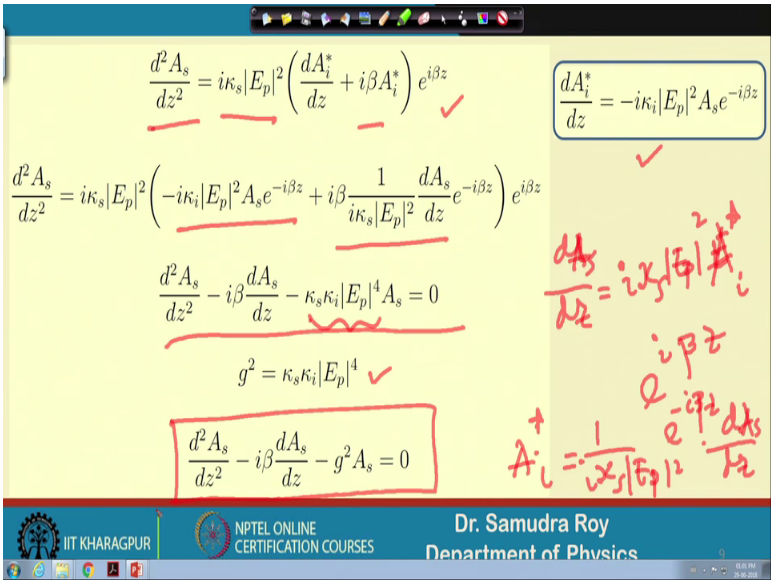

So, as I mentioned. So, what I what we do that I will we will make the derivative with respect to A s. So, when you make a derivative with respect to A s there will be two terms. So, one is this which is a constant. So, di dz will be here and plus i beta A i star and e to the power I beta z will be there. So, this is the derivative with respect to z for both the side as.

Next what we will do? I know that A i z is this. So, I will replace this here I will replace this term here once I replace this term here. So, this become a function of As in the left hand side I will have As. So, our goal here is to eliminate A i because I then only I can solve the differential equation. So, I just decouple the equation. Now we have A i star. So, A i star I just replace A i star because I have dAs dz is equal to i kappa s mod of Ep square and then A i star and then E to the power of i beta z that was the expression we have for as.

Now, what I will do I will just replace this A i star as simply 1 divided by i kappa s Ep square, e to the power of minus i beta z multiplied by dAs dz I will replace this term here if I replace this term, this is the term after replacing. So, now, you can simplify these things and when I simplify we find this is the differential equation we have in our hand. So, if it is differential equation is there now I write kappa s kappa i Ep whole to the power 4 as g square. So, I will have a differential equation of A s which is a signal amplitude in this.

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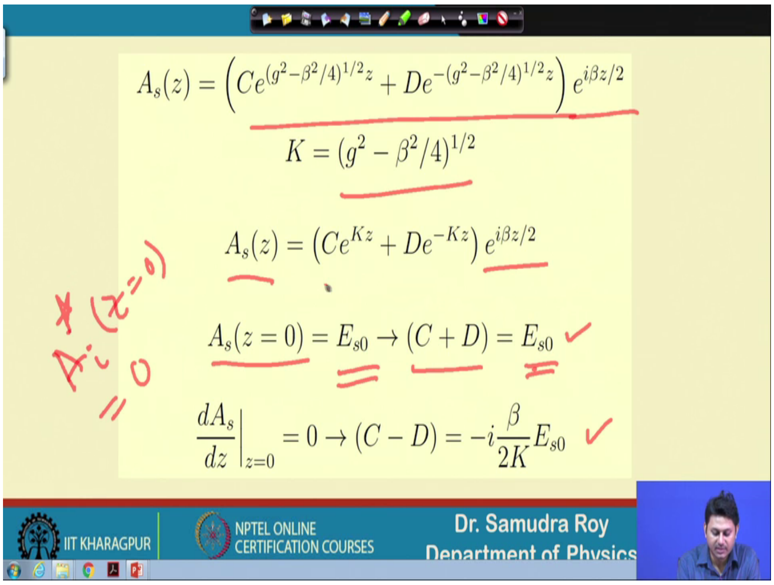

Next we will going to solve this equation which is not a big deal. So, if you solve this equation. So, I can solve this differential equation this is a homogeneous second order differential equations, I believe all of you who are taking this course are quite unable to solve this equation. So, I am not going to show how you solve this, but if you solve this differential equation you will going to get an expression something like this. So, I ask the student to please solve this equation by your own and one you once you do that, you will get a straightforward solution in this particular form.

So, this solution if I write these things as K; so, again I replace the different variable, to make it more compact to another variable. So, I just write g square minus beta 2 divided by 4 to the power half is K. So, my A s is now C e to the power kz plus D e to the power minus kz e to the power beta z by 2. So, this is the way the signal will going to evolve. So, the boundary condition is that when A s at z equal to 0 is 0 we should have a if you remember the figure, we should have a small amplitude of a signal. So, this on small amplitude we say E s 0. So, E s 0 is C plus D which is.

So, if A s 0 is e E s 0 then C plus D will be 0 if I put the boundary condition, then z equal to 0 becomes c z equal to 0 become d. So, C plus D this all this is one will be E s 0. So, this is the first boundary condition. The second boundary condition is the derivative of this quantity is 0 because A i at z equal to 0 is 0. So, that is why the derivative which is containing A i or rather A i star is 0. So, I will have C minus D this. So, I have C plus D this quantity C minus D this quantity. So, with these two boundary conditions I can eliminate E and D.

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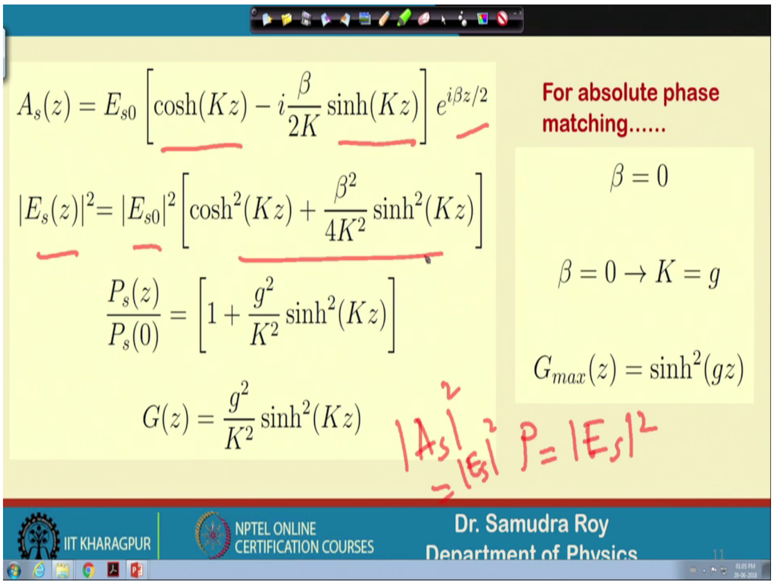

If I eliminate E and D then finally, then finally, we have the expression of A s. And this expression of A s basically if I put this d e and d together c and d whatever the value of c and d we have then from here I can eliminate d by just adding and I have a value of C and I can also eliminate C by just making a minus sign.

So, I will put this here and after putting that if I manipulate slightly, then these two equations the A s become this. So, this is the solution of the signal under four wave mixing this is the solution of signal under four wave mixing. So, we have a cosh hyperbolic term K z plus i beta k sin hyper hyperbolic K z term, and then e to the power i beta z by 2. Next I want to find out what is the corresponding power. So, I know that power is proportional to the fields square, and a if we go back to the transformation you can see that A and s is related in such a way that mod of A s squared is equal to E s square.

So, amplitudes are same only the phase term is associated with this transformation. So, once we make the mod term. So, this phase term will cancel out. So, I can write readily right it is E s it is E s 0 and if I take an mod term I will have this quantity.

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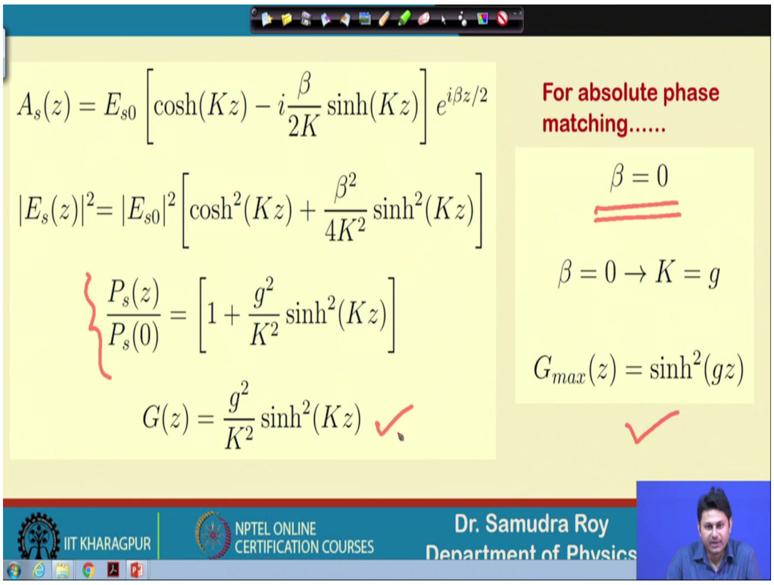

So, if I make a ratio then this ratio E s z divided by E s 0 square will give me the ratio or the amount of gain this is the efficiency that if I have P s 0 as a input and something we have at z point, then if I make a ratio then we find that this ratio is greater than 1 that means, we have some sort of amplification.

So, this ratio gives 1 plus g square k square sine of K z. So, this is basically the gain term that is gives you this is a nonzero positive term; that means, we are getting some kind of gain out of that if this term is not there the ratio is one. That means, there is no gain whatever the value we have at z equal to 0.4 ps the same value we have for z, and this gain value will get a maxima when there is a absolute phase matching that is beta is equal to 0 for beta is equal to 0 we have a maxima of gain which is sin hyperbolic square Gz.

So, this is the standard expression for gain for four wave mixing in optical fiber. So, we just derive these things and understand that how the parametric amplification can be generated in optical fiber and there are certain phase matching conditions. So, beta is equal to 0 means we should consider a phase matching condition, but if you look carefully you will find that in this phase matching condition, the phase associated with the field phase associated with the pump field is also there.

So, the pumps the field the pump basically acquires some kind of non-linear phase that should be compensated by the by the propagation lead vector of signal and idler. So, this non-linear phase matching is important to generate the efficient amplification. In fiber we can readily match these phase by just changing the dispersion profile and we have a four wave mixing gain. So, today we learn these things. So, in the next class we will go further and try a different application of four wave mixing and that is the phase conjugation. So, with this note let me conclude here thank you for your attention and see you in the next class, where we explore more on four wave mixing and learn a new concept on new problem related to phase conjugation.

Thank you and see you in the next class.