Introduction to Non-Linear Optics and Its Applications
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Lecture – 51
Parametric Amplification Under FWM

So welcome student to the new class of introduction of Non-Linear Optics and Its Application. So, today we have lecture number 51. So, in the previous lecture, we discussed about four wave mixing and generation of cross phase cross talk frequencies, how the crosstalk frequencies can be generated with making some phase matching condition which is delta k is equal to 0.

So, today we will learn more about the four wave mixing because this is a fascinating process and how this process is used to amplify a signal that we will going to learn today. So, today our topic will be Parametric Amplification under Four Wave Mixing.

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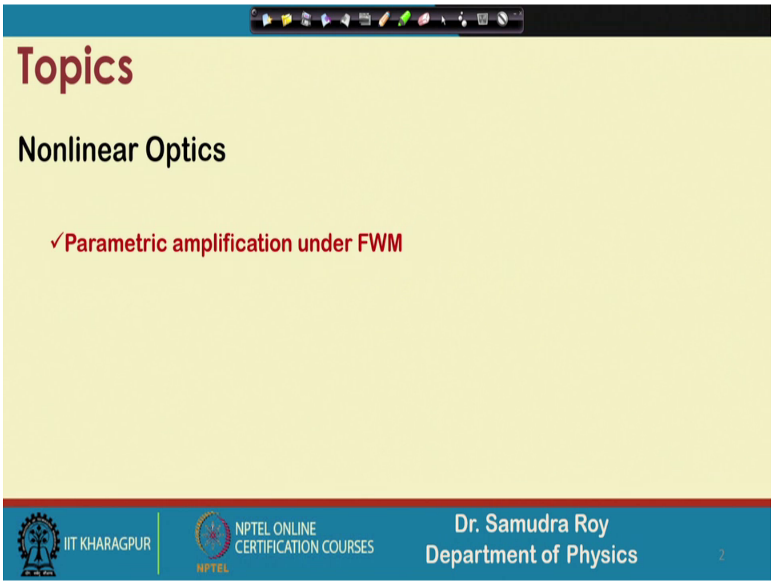

So, parametric amplification means we launched the parametric amplification process we have already learned in the second order effect where we launch a light and another signal is there and we call this light is a pump light. And because of these non-linear interaction what happened? The there is the increment of this signal also another wave is generating which is called the idler wave and this was entirely a K 2 effect; that means, the second order effect.

So, only two waves are mixed there to generate the third one or amplify one of them. Today we will going to learn a similar kind of process, but instead of having two wave; we have interaction instead of having the second order interaction, we have the third order interaction. So, the interaction will be now not restricted with the two wave we will have three wave; however, this three wave not necessarily be distinct to each other. If two waves are launched, then there will be mixing of these two wave in such a way that one wave can come twice and we will generate a four wave mixing kind of stuff.

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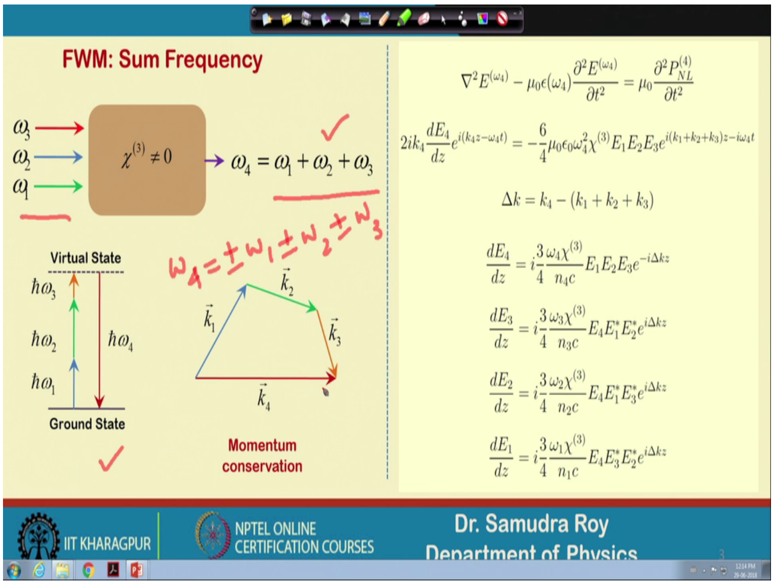

So, let us try to find out what we try to say here. In Four Wave Mixing, there are different kind of categories because the generation of the frequencies are large. So, this is a typical example of some frequency generation. So, what is happening here, I am launching 3 fields with frequency omega 1, omega 2 and omega 3 as shown in here. K 2 is not equal to 0; that means, we should have a non-linear interaction between these frequencies, this non-linear interaction will be of the order 3.

So, E cube term is will be associated with that. So, when cube term is associated with that so we will have the combination of frequency. So, this is a very specific combination the sum, simply the sum of all frequencies whatever the frequencies we launch. We should have a 4 frequency omega 4 which is sum of omega 1, omega 2 and omega 3. You should always remember that omega 4 in general can be generated in this many waves plus minus of omega 1, plus minus of omega 2, plus minus of omega 3. So, these are the numbers there are huge numbers I think this is a 8 different combination. You can consider and omega 4 can be generated with 8 different configuration, if we consider omega 1, omega 2, omega 3 are distinct.

So, among this eight configuration or frequencies, we choose a particular frequencies which is just omega 1 plus omega 2 plus omega 3 which is this one. If I want to generate a frequency omega 4 which is sum of all the frequencies under this Four Wave Mixing process, then the corresponding energy diagram will be simple like this.

So, omega 1, omega 2, omega 3 these are the 3 frequencies that will merge each other and generate a omega 4 frequencies the energy conservation is h cross omega 1 omega 2 omega 3 with a plus sign. So, h cross omega 1 plus h cross omega 2 plus h cross omega is simply cross to h cross omega 4, which is this equation; this is the energy conservation.

But at the same time if I consider the momentum conservation, then I can write that k 1, k 2 and k 3 are the propagation constant or the wave vectors of the photon with frequency omega 1, omega 2 and omega 3. And they will sum up and generate another photon with a frequency omega 4 whose wave vector is moving in k 4 direction.

So, this momentum conservation figure suggests that the phase matching may not be collinear. So this is a non collinear kind of phase matching that is possible or normally this is the case. As far as the equations are concerned, so we already solved this equation in the previous class.

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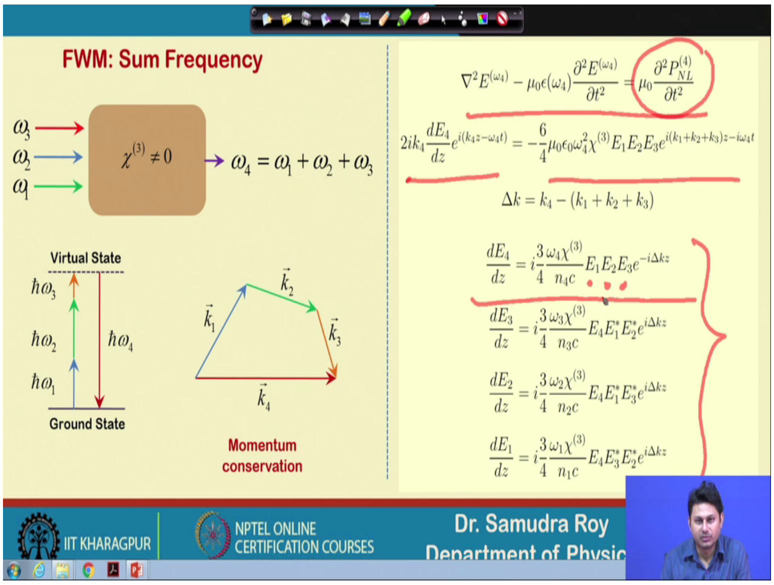

This is the starting equation; that means the non-linear Maxwell's equation. So, this non-linear Maxwell equation, we have a source term the procedure is exactly identical that we have been using. From that we now put E and when I put E omega and P NL omega, I will have evolution equations of this. So, P non-linear omega will contain omega 4 frequencies. So, that is why E 1, E 2, E 3 will be here.

And eventually we will be having differential equation for 4 fields E 1, E 2, E 3 and E 4. E 4 is more important and in E 4, I find that it is associated with E1, E 2, E 3. So these are the source terms. So, these are the coupled equation. So, if I want to solve this coupled equation we need to check that how this equation can be solved. It will be not that easy, but in principle we can solve this coupled equation numerically and find out how this field will be evolving.

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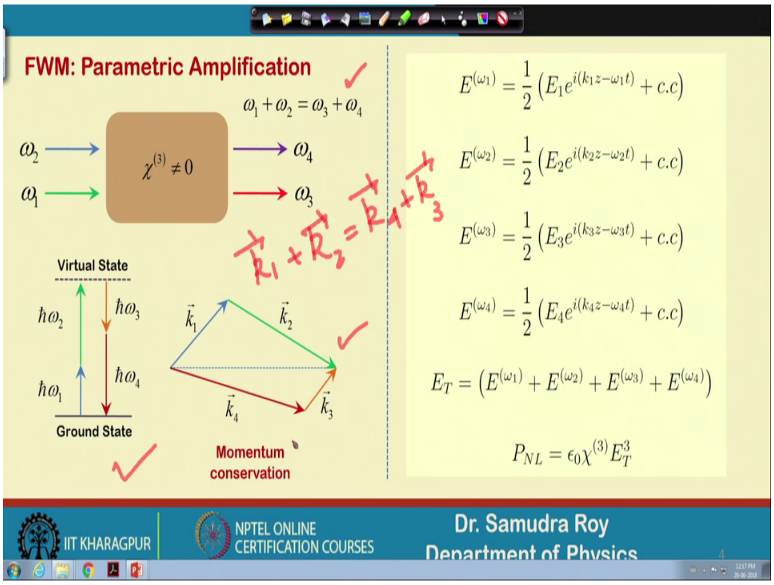

Now, we will go to the next thing which is Parametric Amplification. So, some frequency generation is one kind of Four Wave Mixing, but parametric amplification is quite interesting, where omega 1 and omega 2 are launched and we are getting omega 4 and omega 3 at the output. So we are launching two frequencies omega 1 and omega 2 and we are getting omega 3 and omega 4.

So, here the energy relation is or the energy diagram is something like this omega 1 and omega 2 are merge to generate omega 3 and omega 4. So, this is the energy conservation equation. Similarly in the momentum conservation equation is also like something. So momentum conservation equation suggests k 1 plus k 2 has to be equal to k 4 plus k 3. So, this is the figure of momentum conservation.

Now, I want to find out in parametric amplification how this omega 4 and omega 3 will going to evolve and the concept is if I now launch something here, small signal omega 3 this omega 3 will be amplified because omega 3 will be generated inside this four wave this under Four Wave Mixing this parametric process. Also omega 4 can be evolved. So, I can amplify certain signal having frequency omega 3 and omega 4.

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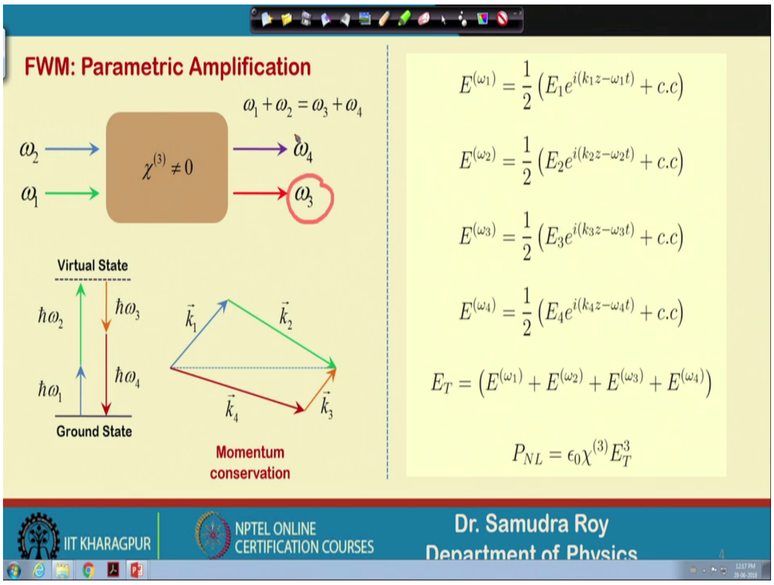

So, let us try to find out how the field equations one can generate which is again the similar process. The starting point is to define our electric field of E 1, E 2, E 3and E 4 where E 1, E 2, E 3, E 4 are the corresponding frequencies the field of the corresponding frequencies omega 1, omega 2, omega 3 and omega 4.

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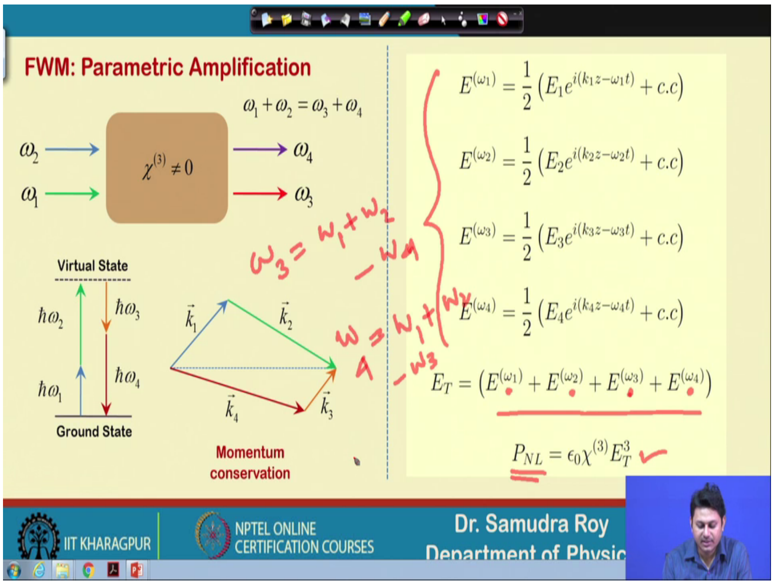

So, these 4 fields are now combined together to generate the total electric field ET. So, total electric field ET is now P Non-Linear related to P Non-Linear and this P Non-Linear is epsilon 0 chi 3 and total electric field cube. So, when I make the total electric field cube, so omega 1, omega 2, omega 3 and omega 4 will combine because I am making a Q of that and as a result we will going to generate different kind of frequencies. But our aim here is to find out the frequencies omega 3 and omega 4.

So, if I do, then quite easily I can say; omega 3 is omega 1 pulse omega 2 minus omega 4 and omega 4 is omega 1 plus omega 2 minus of omega 3. The structure is again some sort of this crosstalk frequencies, that omega 1 plus omega 2 minus omega 3 or omega i plus omega j minus omega k where i, j, k are not same.

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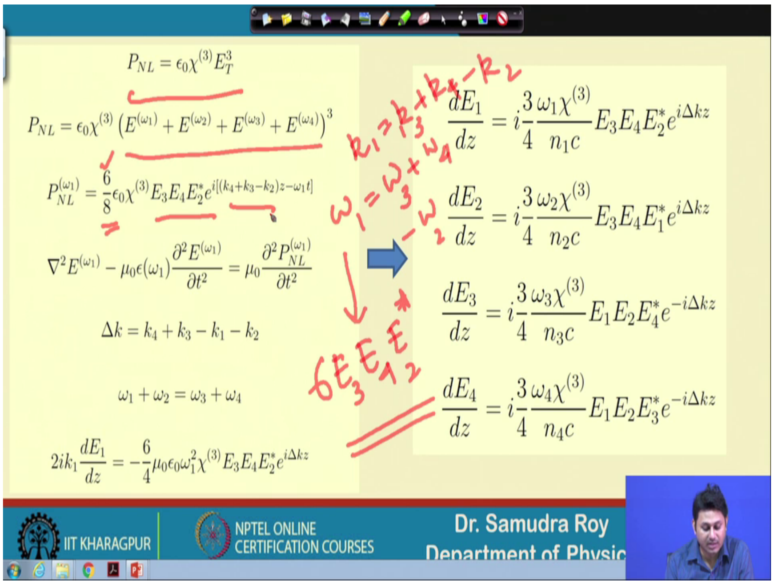

So, next let us see how these things. So, P Non-Linear will be epsilon 0 chi 3 ET whole cube. ET is the total electric field, as I mentioned, it is e omega 1, omega 2, omega 3, omega 4 whole cube and now I try to find out only the corresponding frequency omega 1. So, omega 1; if I want to find out how this omega 1 can be there, so, omega 1 is omega 3 plus omega 4 minus omega 2.

So; that means, in P non-linear term, this frequency component omega 3, omega 4 are plus; that means, I should have electric field E 3, E 4 multiplied and omega 2 minus; that means, I will have E 2 star. So, this should be the structure of the electric field in P non-linear term and here this is the thing.

Since these 3 fields are distinct, so, we will have a 6 term as a degeneracy factor. So, this 6 is sitting here we are making cube of this whole field. So, according to our definition it was a half of that. So, 1 by 8 term will come, because I am making a cube of that thing exponential term should be there also. So, exponential term since I am talking about omega 1, it is omega 3, omega 4 minus omega 2. So, k 1 should be k 3 plus k 4 minus k 2.

So, here it is written which is a k component; the phase term and omega 1 will be 3 because I am generating the frequency omega 1 here. So, my P non-linear term is ready. Once my P non-linear terms is ready and total electric field the expression of the total electric field is ready, I can use again my non-linear Maxwell's equation.

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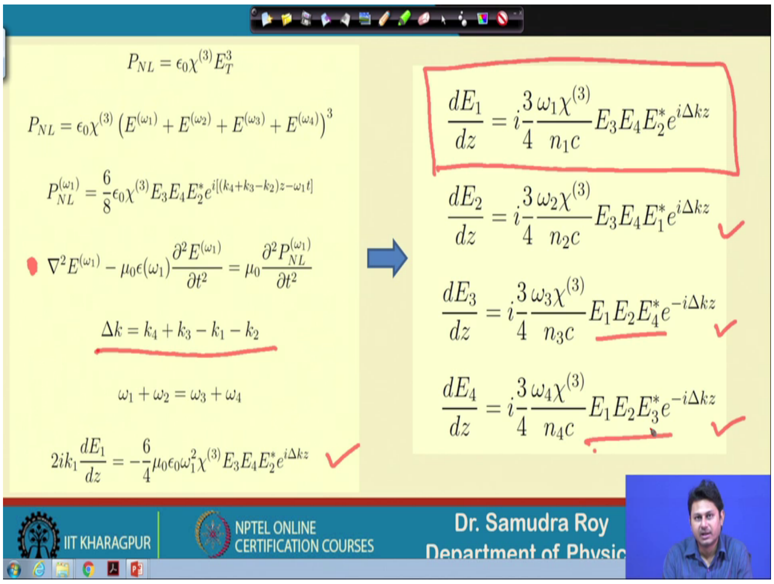

When I use the non-linear Maxwell's equation, I readily find out what is the field equation and the field equation is simply 2ikE i z minus 6 by 4 mu 0 epsilon 0 omega 1 square chi 3 the entire term where delta k is the corresponding difference of the corresponding k 1 plus k 3 minus k 1 minus k 2. This is the difference between the propagation constants.

So, if I simplify, it should be like this. I believe now, so many times we are deriving the electric fields of different frequencies. So, you are now quite familiar with the procedure. So, you can apply the same procedure for other 3 fields and you can generate the evolution of all the fields. So, these are the evolution equation or the governing equation of all the fields under four wave mixing. Only thing is that here it is E 3, E 4, E 2 star; for 2 it should be E 3, E 4, E 1 star. You just remember that all the cases, you need to find out the correct frequencies. So, here the frequencies is frequency component is omega 2. So, how you get the omega 2, we will get omega 2 as omega 3 plus omega 4 minus omega 1. So, that is why it is a star. In the similar logic, you will get omega 3 the field of omega 3 and the field associated with omega 4.

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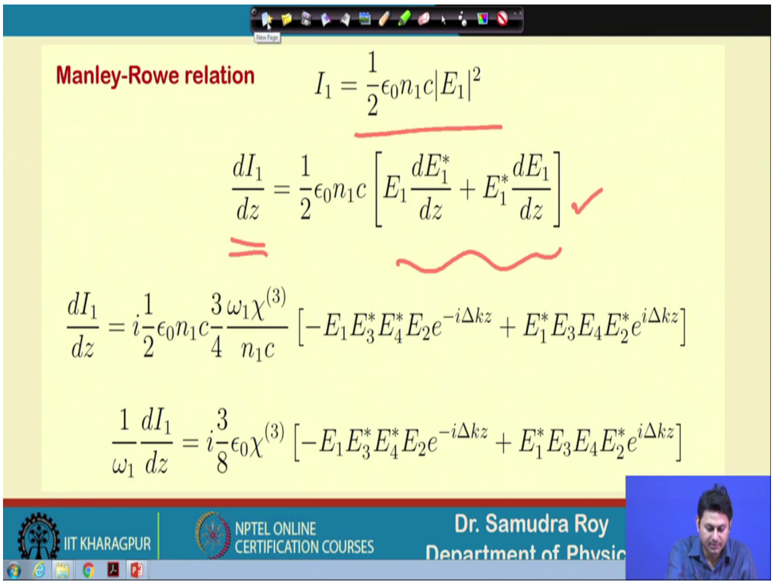

Once you have this four equation in our hand, then the next thing that to from this four equation, we can derive the Manley Rowe relation. So, this Manley Rowe relation is nothing, but some sort of energy conservation. So, all the four equations are coupled to each other, if you look carefully. They are coupled to each other; that means the energy E 1 E 2 E 3 E 4, they are evolving. They are evolving in such a way that there is a interrelationship between these things.

So, this is the source term; in the source term, they are coupled. So, some sort of energy exchange is always there in between and that is the case because if you look that omega 1 and omega 2 is generating omega 3 and omega 4. That means, obviously, some kind of energy distribution is there energy exchange is there. Now the question is the total energy for this case will remain conserved or what kind of things we have.

So, in order to do these things we need to calculate the total electric intensity. So this total electric intensity is something like half epsilon 0 n 1 c E 1 mod of E 1 square which is the total intensity of electric field E 1. So, now, d I 1 d z is equal to half of these things whatever we have as a constant and the correspond derivative. So, the derivative with respect to z gives you this quantity.

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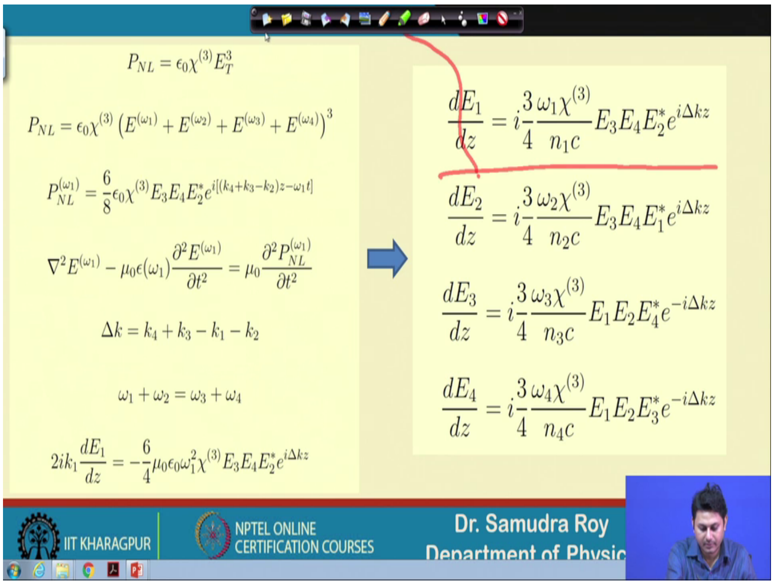

Now d E1 dz is known. From the previous slide if you look, so d E1 dz is here; it is here. So, this I will use in the next slide and if I multiply this d E 1 dz with E 1. So, I will have E 1 E 3 E4 with a star E 2 and the corresponding because this is the star of that. So we have a complex conjugate, once we have a complex conjugate I should have i as minus i; so that is why this minus term is there and i take common outside. For the next term, I will have a similar kind of things, but it is again a complex conjugate of the first term. So, I should have E 1 star E4 E4 E2 star e to the power i delta kz.

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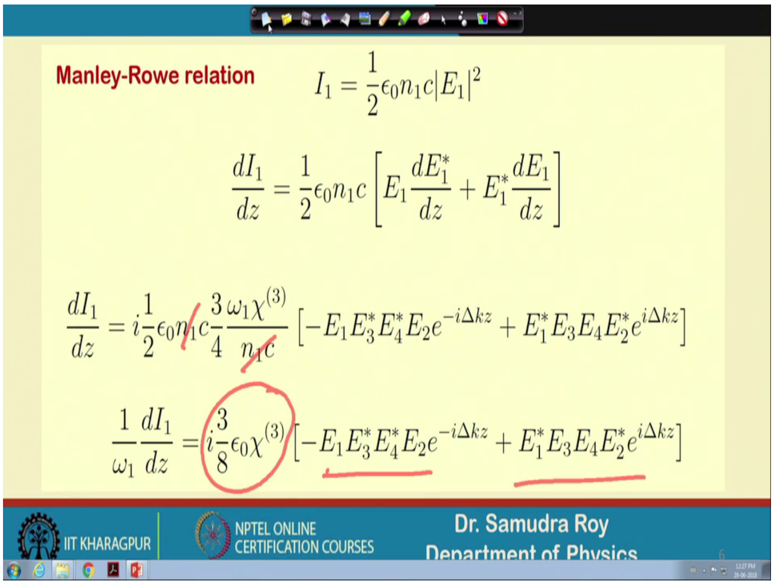

From here also we can see few things that n 1 c and n 1 c will cancel out and eventually we have half epsilon 0 which is a constant chi 3 is a constant and omega 1 is again a constant. But I want to write this omega 1 here in the left hand side like 1 by omega 1 d I 1 dz is this term. Now here you can see this term is really a constant; even it is not depending on omega 1, omega 2, omega 3 or n 1 n 2 n 3. This should be same for all the terms. Inside the bracket we have something here.

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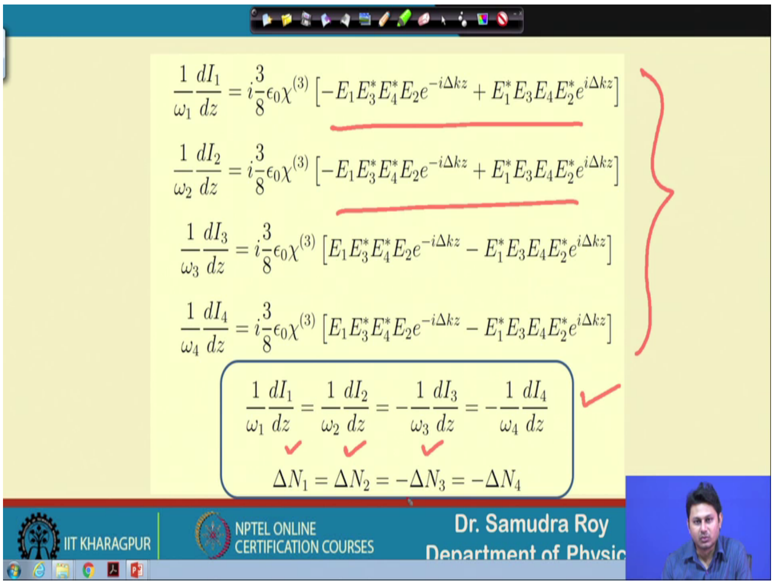

Now, we will do the same thing for E2, E3, and E4. If we do these things for E3, E4, E2 and E4, you will going to get a similar kind of expression here for I 2 so; that means, these two are identical expression. But once we have I 3 and I 4, you will again have a similar expression, but with the negative sign.

So, this term will become positive and this term will become negative again dI dz id omega 4 we will have the same thing, but only thing is this term will be positive and this term will be negative. So, from this four expression, we can write a general thing, and which is 1 by omega 1 dI dz is equal to 1 by omega 2 d 2 dz is equal to 1 by omega 3 dI dz with a negative term and 1 by omega 4 dI 4 dz.

So, this is basically this relation is the Manley Rowe relation. This relation we have already figure out in our second order effects, when we generate some frequency generation or difference frequency generation or other parametric generation. This is always valid. In terms of photon number I can say, the delta N 1 is the photon number the change of photon for omega 1 frequency, then the change of number of photon for omega 2 frequency it is same, because omega 1 and omega 2 are combining to generate omega 3 and omega 4. So, that is why the change of omega 1 photon is equal to the negative change so; that means, if I write a minus sign here. So, omega 1 and omega 2 photon are collapsing to generate the equal number of omega 3 and omega 4 photon.

So, this basically the physical significance of the Manley Rowe relation; and we will readily get this expression starting from our old Maxwell's equation or a non-linear Maxwell's equation. Just evolve the field equation and from this field equation, we can find this expression.

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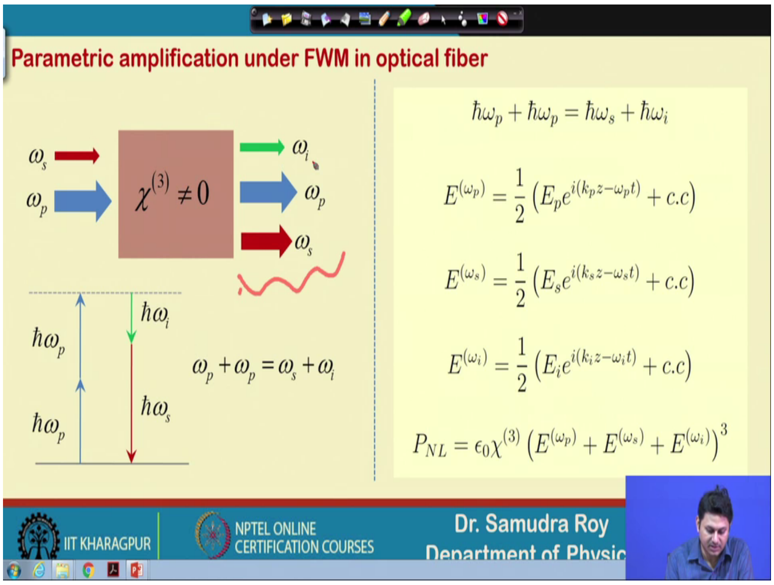

Well, next we will go further we will try to study the parametric amplification under Four Wave Mixing in optical fibers. So, what in optical fibers we do? So, so far we have a knowledge that how four wave mixing can generating different frequencies. It is not generating different frequencies, it is also generating some frequencies and which can be amplified so this can be use as a amplifier. So, using this concept, now we try to apply this in fiber optics where we launch this is the schematic diagram of what is going on here. So, we launch a very strong pump here omega p and very weak signal and in the output we are going to generate omega i which is the idler and omega p which is there because I am launching a very strong signal; a very strong pump.

And here the signal is amplified. So, we have an amplification of this signal that is important. So, I launch a very small amount of signal in the output with that we launch a very strong pump. The strong pump will be there, omega i will generate as idler frequencies and omega s will generate and it will rather amplify because I have some amount of omega s at the input z equal to 0 point.

Now, if I write the energy equation in the previous case which was a degenerate kind of amplification. So, non degenerate rather non degenerate amplification, the previous slide if I go. So, we were generating the frequencies omega 1 omega 2 and an generating omega 3 omega 4. Now if omega 1 omega 2 was same which is the case, we called it this omega 1 omega 2 same thing as omega pump. So, this energy diagram we slightly change omega p and omega p to omega p photon will combine and as a result it will going to generate omega i and omega s two different frequencies.

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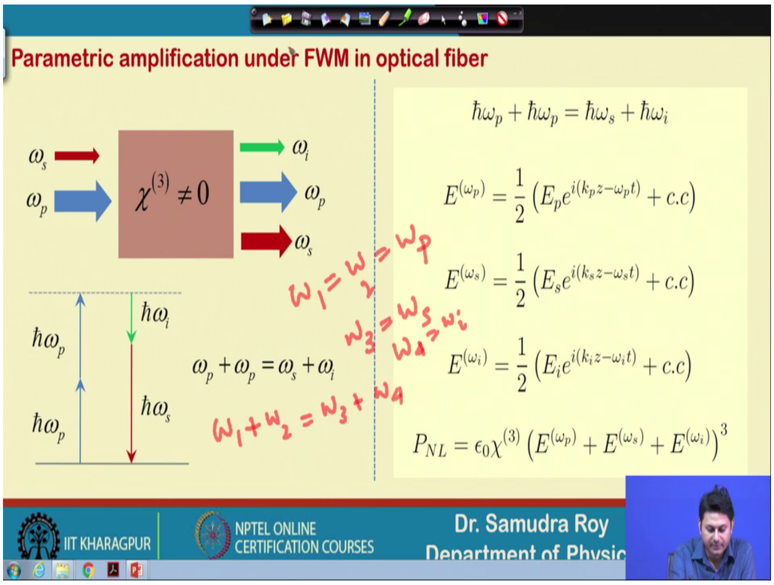

So, in the previous notation it was something like omega 1 plus omega 2 is equal to omega 3 plus omega 4. Now I just modify this I say, change the name omega 1 is omega 2 which is omega p omega 3 is suppose omega s and omega 4 is idler frequency omega i. If I replace these things, so, I will have an expression like this is energy expression; So, energy conservation expression.

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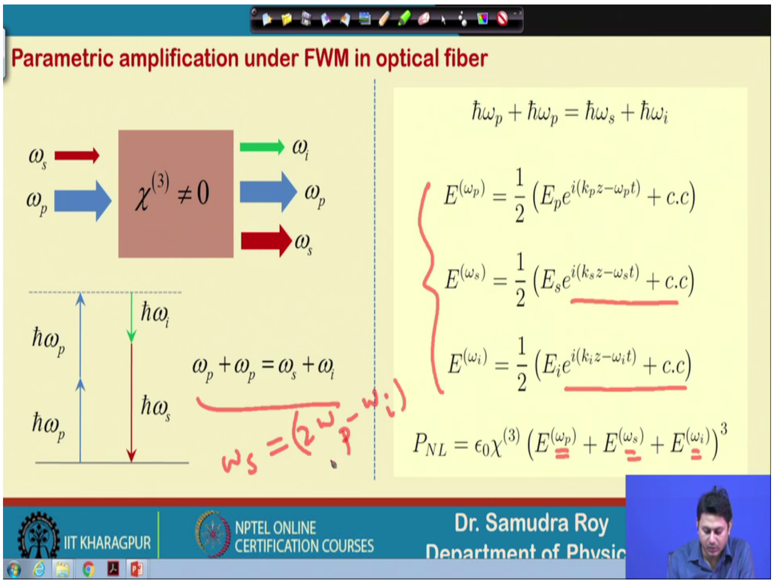

So, now we have 3 fields; omega i omega s and omega p. Omega p is half E p e to the power i kpz minus omega pt plus complex conjugate omega s is half E i e to the power i k is z m inus omega z and similarly for omega i. Again p non-linear should be the combination of these 3 field because inside we have these 3 fields cube of that epsilon 0 chi 3 and E cube where E is the total fields I make a cube of that so; that means, there will be combination of omega p omega s and omega i will be there.

But which frequency I will consider I will consider omega s. So, if I consider omega s, then omega s should be 2 omega p minus omega. This is the component, this is a frequency component we will concern about.

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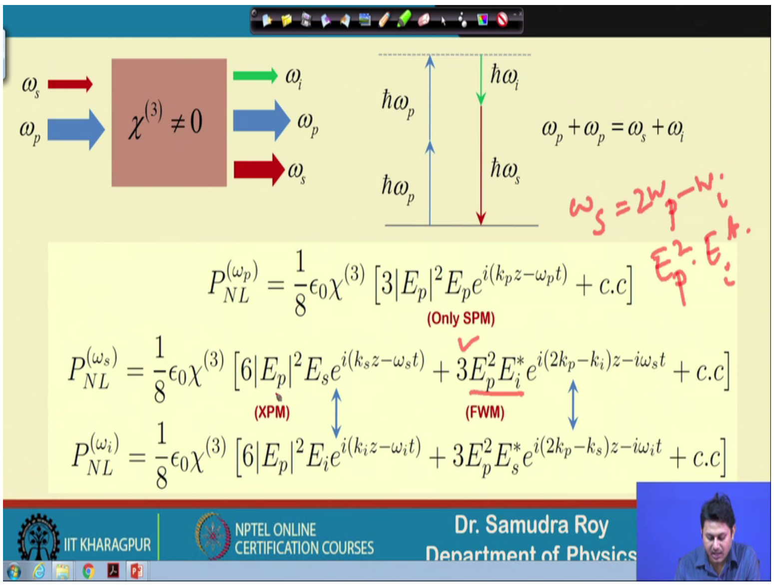

So, this is the structure energy diagrams are there. The next thing is to find out what is my P non-linear. So, p non-linear will combination of all these frequencies in optical fiber what happened I need to find out my P non-linear this is important and we should be careful enough to find out how these p non-linear terms are there.

So, p non-linear omega p; that means, how the electric field of E p will experience the thing; if I write, I just write this way. It is 3 E p square E p e to the power k p z minus omega. So, there will be many terms. So, omega p term can be generating in other way also, but I only take the self phase modulation term because pump is very strong. So since pump is very strong, we can consider pump as a constant. So, since it is a constant, I will not going to take any other source term as p non-linear and I just consider only the self phase modulation term.

Once I express the self phase modulation term in this way, for other two cases which is a small signal omega s and generated idler, I will have 1 by 8 epsilon 0 chi 3 as usual. And inside that we have two term: one is the cross phase modulation and another is the Four Wave Mixing. This Four Wave Mixing term we have discussed earlier several time. So, I will not going to discuss this again. So, you know that if I want to generate omega s, it will be 2 of omega P minus omega i. So, the my corresponding electric field will be E p square with generate 2 omega multiplied by E i star exactly, If you look here so, E p star E p square E i star is there. The corresponding electric the corresponding exponential term with the field propagation constant is also written here. And the degeneracy factor three is coming because two distinct there are not three distinct field rather two fields are distinct. But E p and E p are same because I am multiplying twice. So, that is why the degeneracy factor should be 3. But another term should also be considered here which also give rise to omega s frequency and that is the E p E p star. So, we will have a combination.

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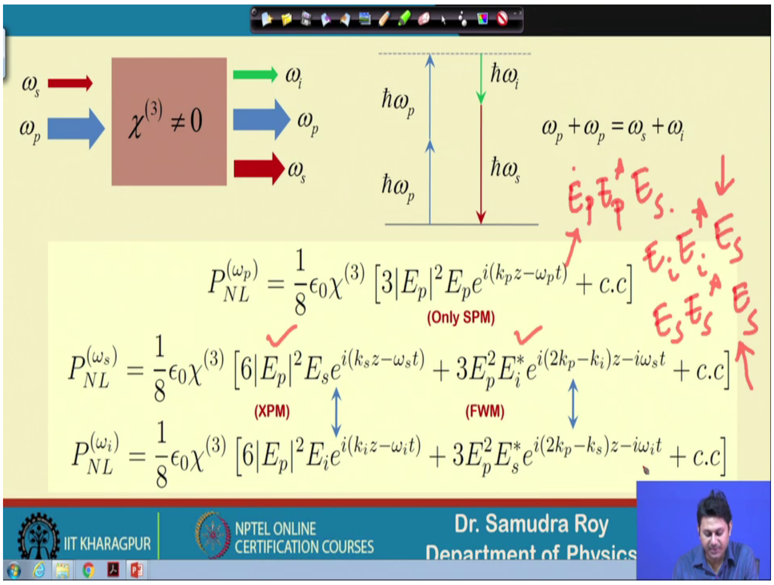

E p E p star and E s; E p E p star basically, this is omega p. This is minus omega p and this is omega s. So, I will have this term also. And another term the self phase modulation and cross phase modulation for other two is also possible, if I write very carefully so; that means, E i E i star E s also give rise to a frequency omega s and another term E s E s star E s also give rise to a same frequency. So, this is a cross phase modulation with E I, this is a self phase modulation with E s, this is a cross phase modulation with E p.

So, E p will be more important because pump is very high. So, that is why only I took this term here and the corresponding four wave mixing k system which is this. So, these are the two terms you should be very careful. There will be many terms, but which term I am taking here, I am taking only the cross phase modulation term with respect to E p because E p a strong field. So, this effect will be much stronger than the cross phase modulation of E i which is a weak field or the self phase modulation of E s which is also very small. We can neglect that and the corresponding Four Wave Mixing where E p is associated with that because E p is strong.

So, all the strong field related to E p will be considered in p non-linear which is a source term. In the similar way p non-linear omega i this is also a source term and we can also generate or write this expression for p non-linear omega. So, here I should stop my class because so far we are deriving the equation for deriving the equation for the signal amplification. So, for signal amplification, we derived the source term. There will be three different source term one is for omega p another for omega s and another for omega l. Eventually we will going to use the source term for omega s, but how these source terms are generated is important to note and this that is why I in detail I try to make you understand; how this different field combination should be there in the source term. So, with this note let me conclude here and see you in the next class.

Thank you for your attention.