Introduction to Non-Linear Optics and its Applications
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Lecture – 50
Four Wave Mixing (Contd.)

So, welcome student to the new class of Introduction to Non-linear optics and its Application.

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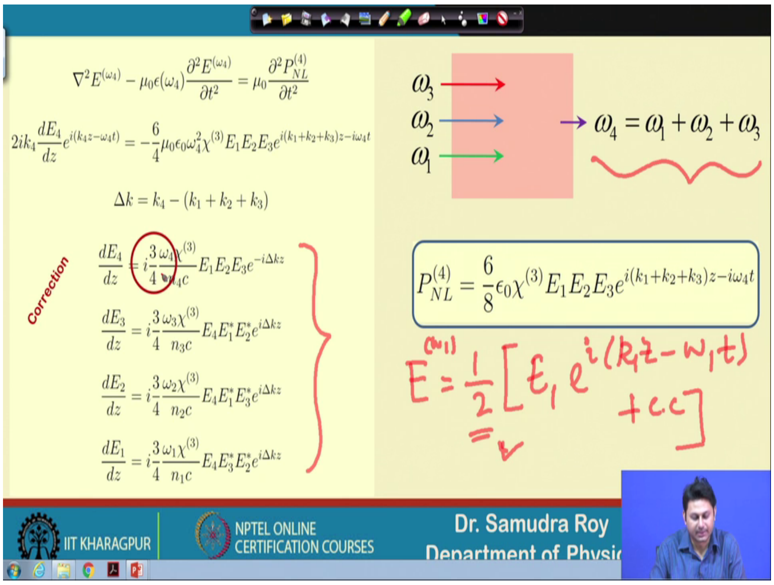

So, in the last class we discussed about the four wave mixing. So, today we will going to give you one correction, because in the last calculation it depends on what value you are taking regarding the electric field. So, before starting the class let me once again remind you that our according to our notation the electric field E, for a particular wave length omega I was represented by half of say E 1 E to the power i k 1 Z minus omega 1 t if the frequency is omega 1 plus complex conjugate of that.

Now, in many books the complex conjugate term is not there and, if I want to take this complex conjugate term, then one half term we need to add with that so, according to our convention this half term should always be there so, with this electric field if I try to find out the implication of four wave mixing, or the role of four wave mixing in some frequency generation, then in the last class we figure out the expressions of individual field E 1 E 2 E 3 and E 4 and how they are coupled to each other under four wave mixing.

So, during this calculation since this half term was there, we need to take this half term into consideration and it should be 3 by 4, but in the previous slides or the in the previous class slide it was written 3 by 8, this four term will come because in the both side we have a half term. So, this half term will cancel out and you will get a four term here. So, this small corrections I just wanted to make before starting the class.

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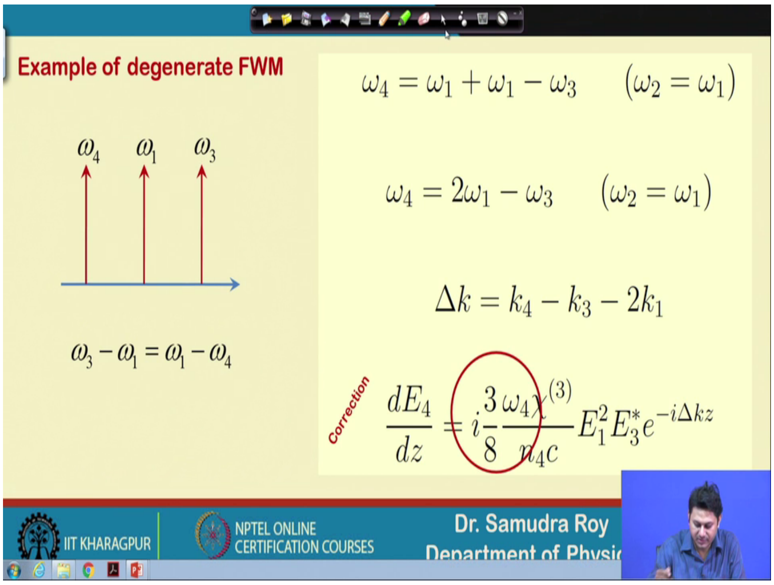

And also for four wave mixing in this degenerate process, if I want to find out the frequency omega 4, which one can generate through omega 1 and omega 3, then what happen that I can excite I can generate the electric field E 4. So, E 4 electric field can be governed by some differential equation and, we also calculated the differential equation and this differential equation there was a 3 by 16 term.

Again this half term we need to consider and it, because of this half term it should be 3 by 16. So, you should be careful enough so, when we calculate the Maxwell's equation non-linear Maxwell's equation what convention you are using, how you define your electric field. Since in this course we are defining electric field with a half term, we need to take account this half term and that is why this term this input term, which is a constant anyway should change slightly, and depending on the convention of your electric field. Well with this note let me now start our today's class. So, today we have lecture number 50 and today we have the elaborate expression of elaborate discussion on four wave mixing.

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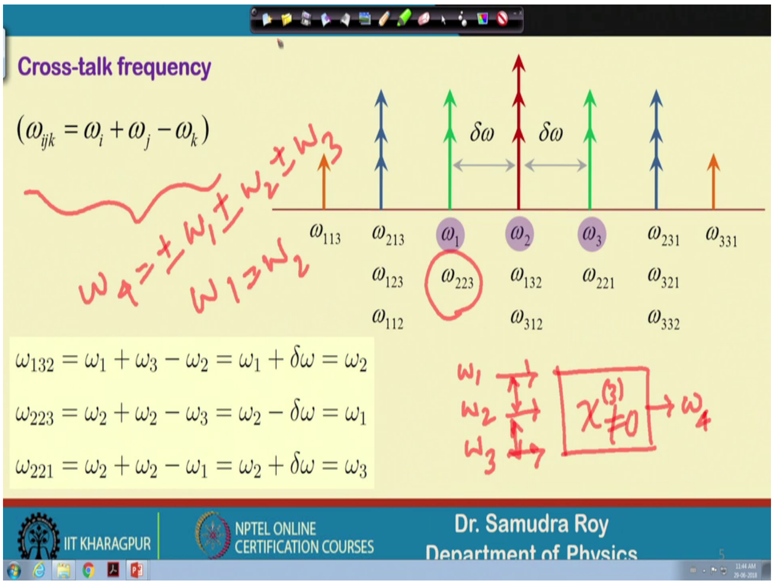

So, what we have let us try to find out. So, we have the cross phase module cross phase frequency effect that we discuss in the last class that under four way mixing there are many possibilities that the frequencies can be generated. So, we find that in four wave mixing what happen this omega 1 omega 2 and omega 3, if these 3 frequencies are there, which is launched together to a non-linear medium, where chi 3 is not equal to 0, then if combination of these 3 frequency can generate another frequency say omega 4.

So, now omega 4 can be the combination of omega 1 omega 2 omega 3 and in general omega 4 can be represented by plus minus omega 1 plus minus omega 2 plus minus omega 3. So, these many terms one can generate, because for all electric field there is a complex conjugate term and for this complex conjugate term we have a minus omega frequency associated with that also minus omega 2 minus omega 3.

So, their combination can generate many frequencies this is one issue second thing is that it is not necessarily that omega 1 omega 2 omega 3 are distinct. So, there is a possibility that we have omega 1 is equal to omega 2, when omega 1 is omega 2 then there is slight change in this distribution and we can see that there is a possibility that we have some frequencies, with the combination of omega 1 omega 2 omega 3 with the degeneracy that omega 1 is equal to omega 2.

Then there maybe some frequency that can generate which coincide with the original frequencies like shown here so, omega 1 can be equal to omega 2 2 3, where how omega 2 2 3 is defined is shown here, this is a specific frequencies where this can be coincide this combination of this frequency can coincide to the original launched frequencies; however, this launched frequencies omega 1 omega 2 omega 3 are separated with a certain frequency difference.

And if this separation is same as shown in here, it is delta omega. Then this combination of omega i j k can give you different kind of frequencies and among these different frequencies, there are few frequency that will coincide to the original frequency this is called the cross talk.

So, in fiber optics communication problem so, this cross talk is the major issue. So, today we will going to learn more about this cross talk, how this cross talk can be eliminated, or if I want to generate a frequency which is with a same separation of the two frequency that is launched, then eventually we are generating a cross talk frequency. So, how we generate the cross talk frequencies we will going to learn this course in this class rather.

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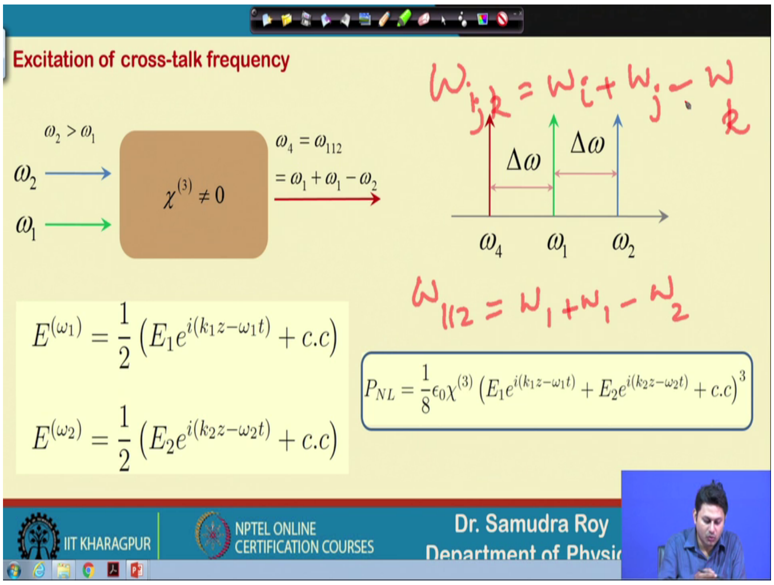

So, the problem is excitation of cross talk frequency. So, we want to excite a frequency which is among these frequencies. So, there are twelve frequencies are there which we can we can say these are the some sort of cross talk frequency with this definition that omega i j k should be equal to omega i plus omega j minus omega k, this is the structure of the frequency. So, I generate want to generate any of these frequency with this combination.

So, if I do so, there is example that suppose I am generating omega 1 1 2. So, what is omega 1 1 2? Omega 1 1 2 we know that omega 1 1 2 according to our definition it should be omega 1 plus omega 1 minus omega 2, because we know omega i j k is equal to omega i plus omega j plus omega k, this is minus omega k this is the definition, this is the definition of the frequency, we are we are interested to find what kind of frequencies are there with these combinations.

So, omega 1 1 2 is such a combination so; that means, if I launch omega 1 and omega 2 together as shown here, where omega 2 is greater than omega 1, then the combination omega 1 1 2 will give a frequency we called it omega 4 and, it will generate exactly in the same difference that we have between omega 1 and omega 2, but in the lower side.

So, I launch the frequency omega 1 here, I launch the frequency omega 2 here, which has a separation delta omega and due to the four wave mixing process omega 1 and omega 2 will combine with themselves and, they will generate another frequency say omega 4 which is generated exactly delta omega apart from omega 1 as shown in this figure. Now, the question is the, if I want to generate omega 4. So, how these things can be excited.

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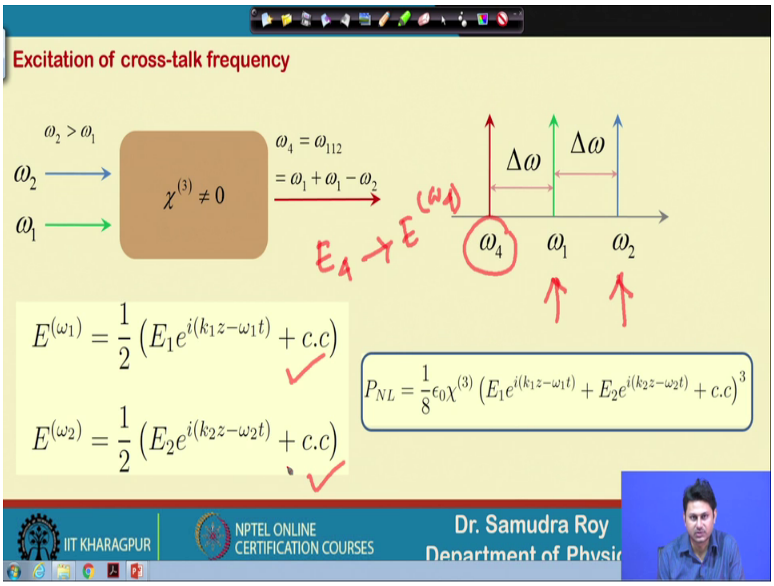

So, what should be the differential equation of this field E 4, where E 4 corresponds to the frequency omega 4. So, omega 4 will generate as a combination of omega 1 and omega 2 so; that means, omega 1 and omega 2 will be essentially the source term the source field that will generate omega 4.

So, I defined first E omega 1 and E omega 2 these are the input fields. So, omega 1 and omega 2 are the two frequencies, for these two frequencies I have two fields and these two fields are defined in this way. So, half E 1 E to the power i k 1 z minus omega 1 this is the field 1 and for field 2, which is if corresponding which corresponds to the frequency omega 2 is i k 2 z minus omega 2 plus complex conjugate.

So, for these two field if I know the next thing is to find out the source term; that means, the non-linear polarization term, that will basically leads to the source of different frequencies.

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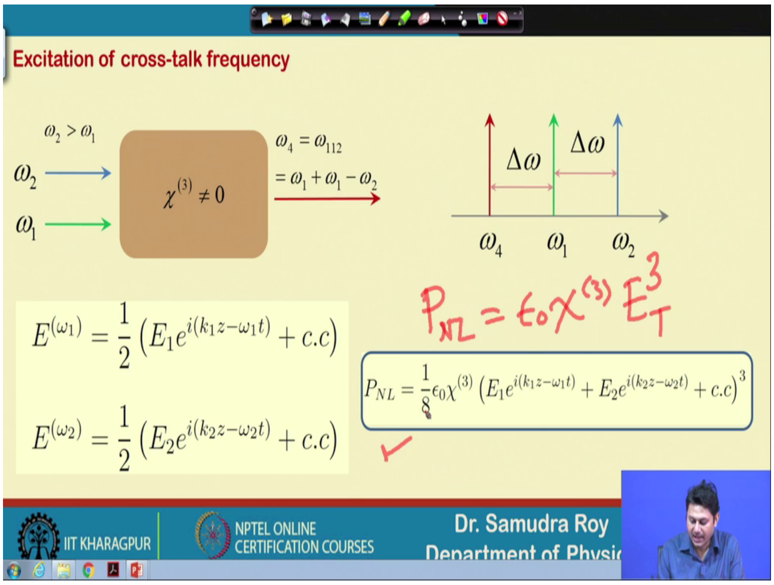

So, it should be simply 1 divided by 8 because I know P non-linear several time we are using this expression so, P non-linear, if epsilon 0 chi 3, then E total cube of that. So, we know E total cube means E omega 1 plus E omega 2 and cube of that because E total means the total electric field that we have inside the system.

So, here we have E 1 and E 2 this is the source term and cube of that in order to generate the omega 4 from P non-linear is something this, half this half term when I make a cube of that we have one by 8 and, it will be one by 8 epsilon 0 chi 3 and cube of the rest of the term associated with electric fields.

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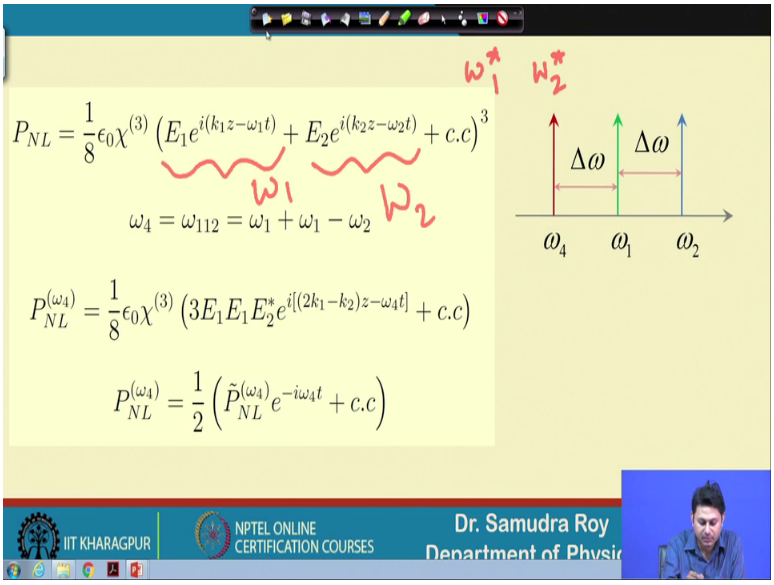

So, now P non-linear is this once we have the P non-linear the next thing that we should note is the how many different frequencies we can generate with this combination. So, there are many frequency component you can generate to very honest.

If I make a cube of this two this there in fact, there are four terms one is associated with frequency omega 1 another is associated with frequency. So, these term is associated with a frequency omega 1, this is associated frequency omega 1, this is associated with a frequency omega 2 and complex conjugate means, we have another two frequency in our hand which is omega 1 star and omega 2 star, omega 1 star and omega 2 star means nothing that minus omega and minus omega 1 and minus omega 2.

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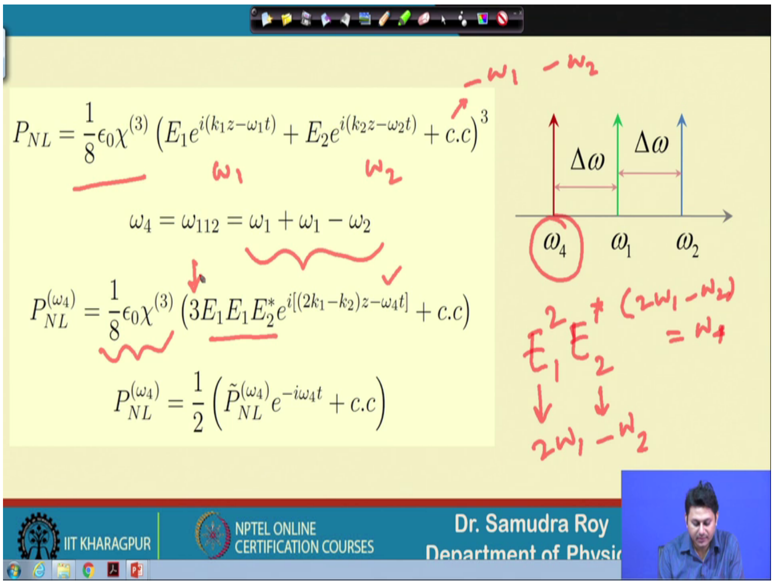

So, I have omega 1 omega 2 minus omega 1 minus omega 2, in this complex conjugate terms and then cube of that. So, when you make a cube of that we know there are different terms that will appear, but our concern here is only one frequency omega 4 which has the combination of omega 1 and omega 2 like this because, I want to generate only this frequency and in order to generate this frequency what should be my corresponding electric field that is my concern right.

Now, so, I just want to extract there are many terms are there so, a plus b plus c plus d whole cube. So, you can understand how many terms one can generate from this expression, but we should not bother about all the terms, we just want to find out which term contain omega 1 omega 1 and minus omega 2, which combination of the electric field.

So, this is easy to find because omega 1 omega 1 will be related to the field E 1. So, omega 1 omega 1 means 2 omega 1 so, 2 omega 1 will be related to the field E 1 square and minus of omega 2 will be related to a field which is E 2 star, because this corresponds to two omega 1 and this corresponds to minus of omega 2, when I multiply these two things that is mean the frequency will be added up and eventually I will get the frequency component of these term is 2 omega 1 minus omega 2 which is nothing, but omega 4.

So, for omega 4 frequency I will have only one term and this is E 1 square multiplied by E 2 star, I just extract this term for the frequency omega 4 and if I write this here, then it should be E 1 E 1 E 2 the term one by 8 epsilon 0 chi 3 will remain here, as it is only thing is this cube from this cube I will take only those term, which are having which will have a frequency component omega 4 like this so; that means, this is this will be our term, but here you should very careful about this degeneracy factor 3, because in the previous class we mentioned about this degeneracy factor. So, how the degeneracy factor is determined in this case?

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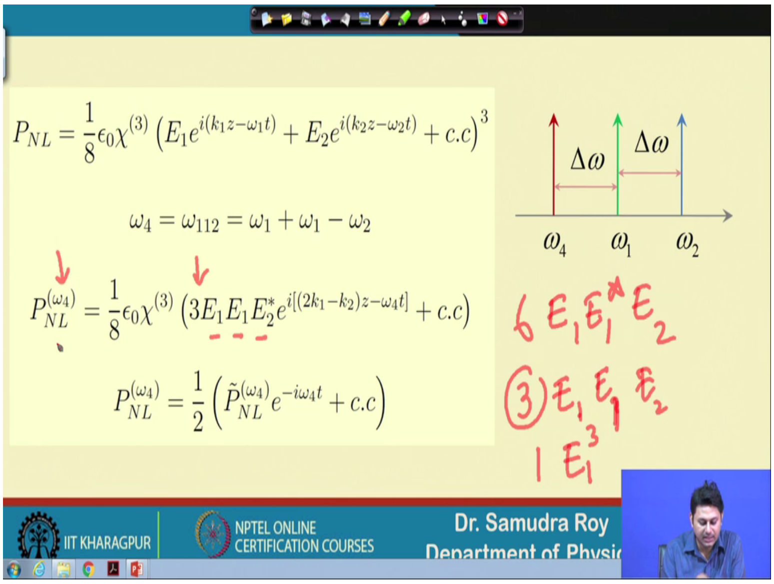

So, we know that if three fields are distinct for example, if it is E 1 E 1 in this case there are four different fields E 1 E 1 2 E 1 E 2 E 1 star and E 2 star there are 4 fields E 1 E 1 star E 2 so, there are three distinct fields, if three distinct fields are there with this combination I will have a degeneracy factor 6.

If they are same any two of same for example, if it is E 1 E 1 E 2 which is our case, then the degeneracy factor reduces to 3. And if all that fields are same like one term we will have is E 1 cube, the corresponding degeneracy factor should be 1. So, this way one can be determine the degeneracy factor.

So, here E 1 E 1 E 2 that means, we have two distinct electric field so, for two distinct electric field by degeneracy factor will be 3 here so, that is why the three term is sitting so; that means, now I have an expression of P non-linear, which containing a frequency omega 4. So, if I now write this P in as a as aplane wave kind of expression than P non-linear omega can be written as half P non-linear tilde omega 4 and the corresponding frequency term.

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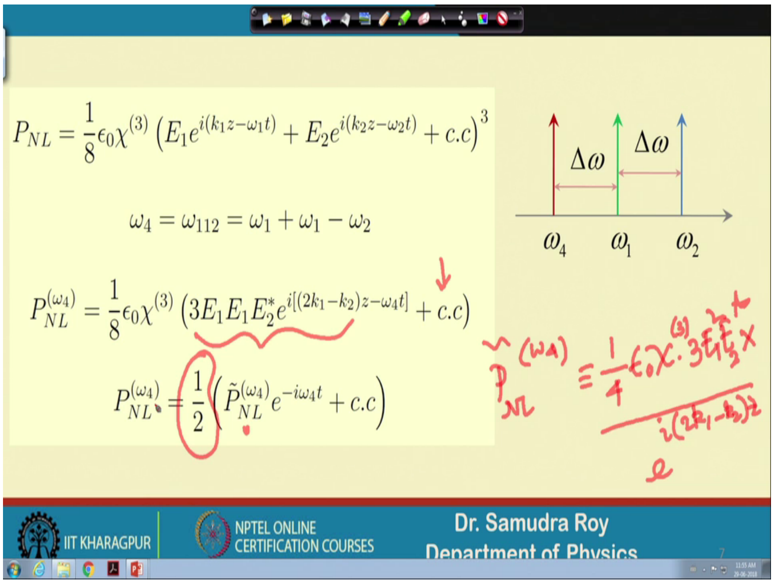

So, I write this entire stuff in this here with so; that means, if I compare these two things so, P non-linear tilde omega 4 is nothing, but 1 by 4 then epsilon 0 chi 3 and, then 3 multiplication of E 1 square E 2 start these are the this is the value of this so, called amplitude also the exponential term will be there we should not miss this exponential term.

So, i 2 k 1 minus k 2 and z so, all z dependent terms is here and the time dependent term I will extract like this and, we have a complex conjugate of that because already complex conjugate term is sitting here, why I write this in this particular form, because I want to extract this half as I mentioned at the starting of this class that this half is important so, that I can cancel this half for both the sides, when we use the non-linear Maxwell’s equation.

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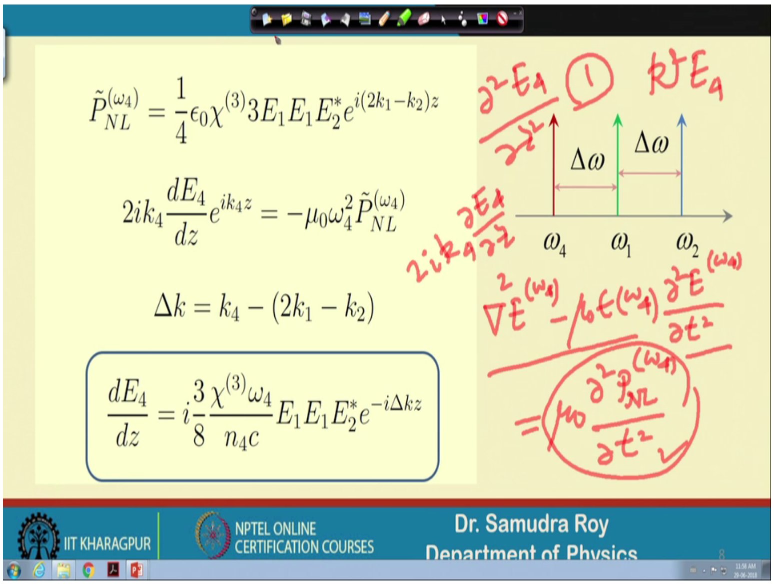

Well after the knowledge of after the knowledge of this P non-linear at the frequency omega 4. The next thing is that to find out the corresponding electric field the expression of the electric field, or the differential equation of the electric field. So, in order to do that, we use basically the non-linear Maxwell's equations, or non-linear wave equation whatever.

So, the equation is this we are using this in several time it should be E of omega 4 minus mu 0 epsilon of omega 4 d 2 E omega 4 d t square is equal to mu 0 del 2 P non-linear of omega 4 divided by d t square. So, this is our source term and this equation we are using almost all the times when we calculate the evolution of the electric field amplitude.

So, now E 4 we know that when we extract this things, or when we solve this part using the slowly varying approximation, we always remove this term that is one step we do, also the next thing is that another equation when we do that one equation related to k square is coming this case k square E 4. So, k square E 4 is cancel out by these term every time. So, when this k square E 4 term there will be a minus sign, I guess is cancel out to the next term then eventually we have the first order derivative term in our hand and, which is 2 i k 4 d E 4 d Z, this is the left hand side and right hand side I have a derivative with respect to t of this term.

Now, my P omega is represented in this way. So, when I make a derivative of the entire term with respect to t this term will simply sit here and E to the power of the frequency term will be there. So, this frequency term will cancel out from both the side because there is an another frequency term 2 E to the power 4 E 4. So, the point is I can have a more compact expression here, which is equivalent to whatever the expression, we have read in here just before that means.

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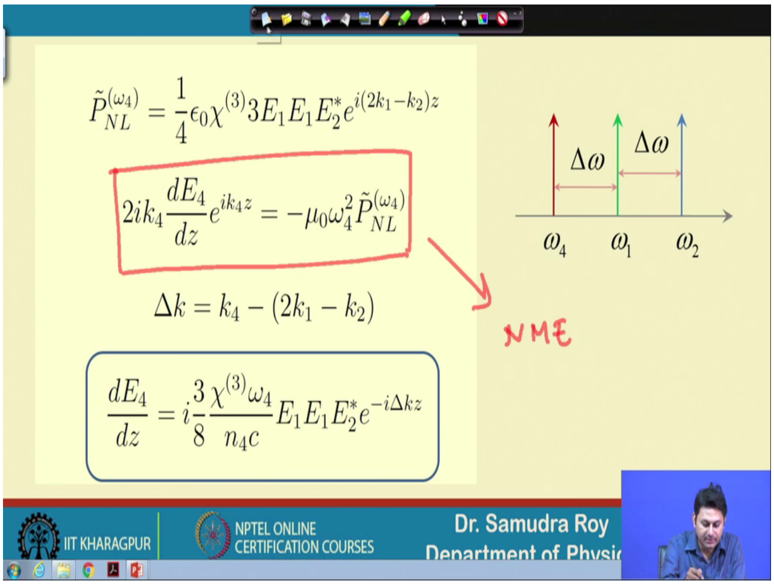

This is equivalent to the non-linear Maxwell's equation, by applying the slowly varying envelope approximation and writing P in this particular form in P tilde form. So, we have a very compact form. So, now what we will do we will just write this things once again and, when I write this things.

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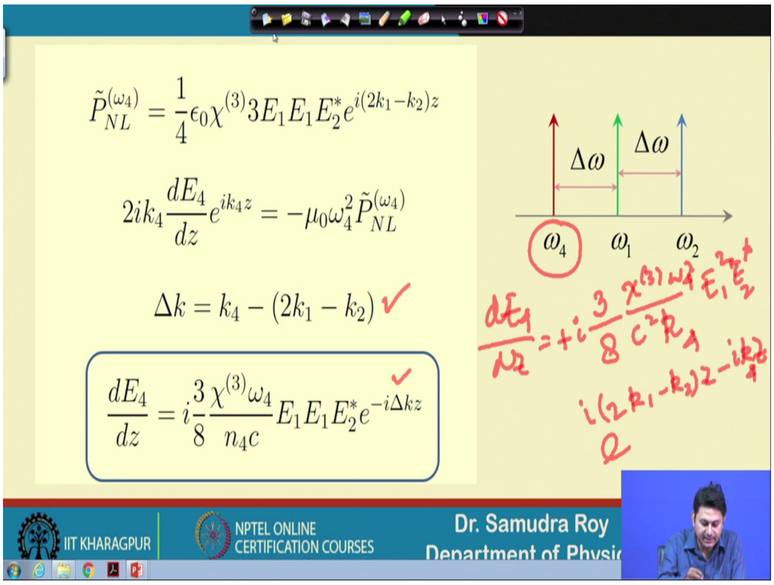

So, I will write 2 i k here in the denominator and the other things are the similar thing so, I will write d E 4 d Z is equal to minus of minus of so, this minus will be represented replaced by i. So, this is already 3 by 4 term is there so, I should have a 3 by 8 term because of this half and, then I will have mu 0 epsilon 0 mu 0 epsilon 0 will be C square, which is here and we have k 4 here and in the numerator we have kappa 3 and omega 4 square.

And then the rest of the term E 1 square E 2 star and exponential of this term is already there 2 k i minus k 2 i z and in the left hand side, we have a k 4 so, k 4 will go here so, I will have minus of i k 4 Z. Now, if I write k 4 minus 2 k i plus k 2 is our delta k, then it should be minus of delta k and rest of the term you can manipulate, because k 4 is omega 4 divided by C multiplied by n 1 so, 1 omega 4 we will cancel out so, we will eventually have this equation.

So, this is the evolution equation this is the evolution equation of the cross phase the cross frequency term or the omega 4 term so, cross talk frequency 1 cross talk frequency or omega 4, one can generate and this is the corresponding field equation or the field evolution equation.

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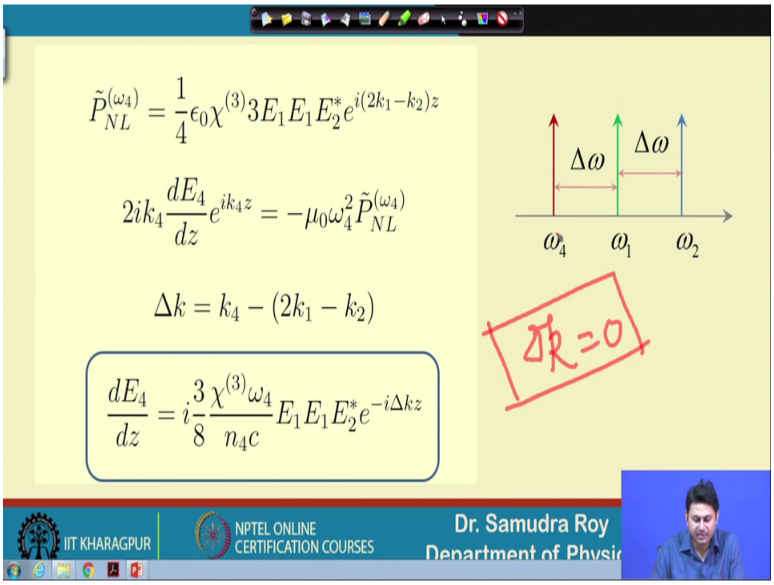

Now, the next thing is if I want to excite these things how do I excite that so, delta k is a phase matching so, delta k has to be 0 that is the next step, because I want to excite this things in order to excite in order to increase the efficiency what we need to do we need to make delta k equal to 0; that means, we need to put absolute phase matching condition.

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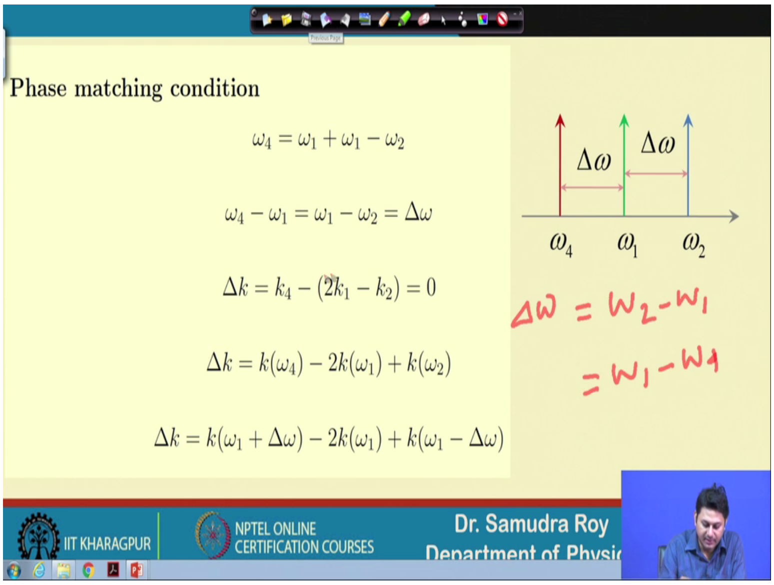

So, if I do then we will have exciting things. So, omega 4 is equal to omega 1 plus omega 1 minus omega 2 here, we have an additional information or additional thing, that is the difference between omega 2 minus omega 1, which is equal to delta of omega is equal to the difference between omega 1 minus omega 4. So, these differences are same; that means, all the frequencies I can write in terms of omega 1 and delta omega.

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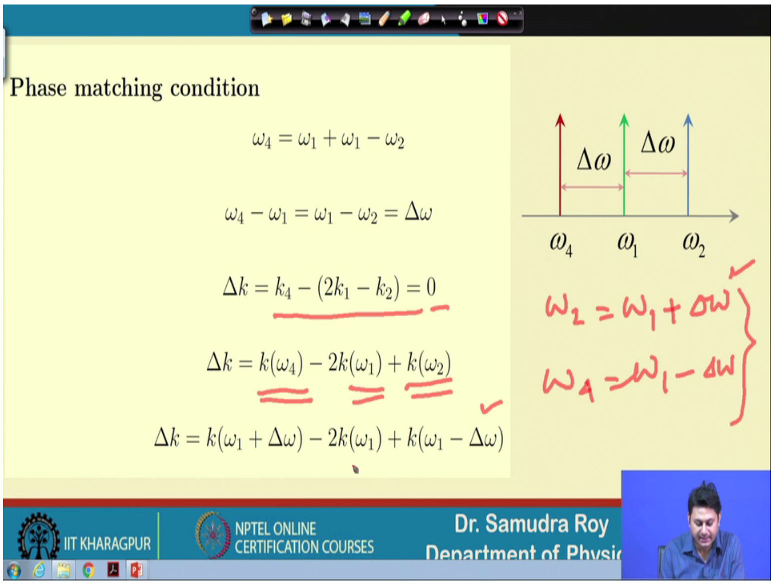

So; that means, here as I as it is written so, omega 1 omega 2, I can write as omega 1 plus delta omega and omega 4 I can write omega 1 minus delta omega. So, in this way I can write the frequency omega 2 and omega 4 in terms of omega 1 and the related frequency difference which is delta omega.

So, delta k is our phase condition the phase matching condition is delta k equal to 0 so, delta k is k 4 minus 2 k 1 minus k 2 so, delta k k 4 is what k 4 is the frequency propagation constant at frequency omega 4 k 1 is a propagation constant of the frequency at omega 1 and k 2 is a frequency propagation constant at frequency 2. So, once we have these term in our hand, then the next thing is we just try to find out delta k. So, I right delta k is equal to change the omega 4 as omega 1 minus this term k omega 2, I write k omega 1 plus delta omega here and omega 4 I write k omega 1 minus delta 4 this and k omega I write just omega so; that means, in terms of omega 1 I write my delta k.

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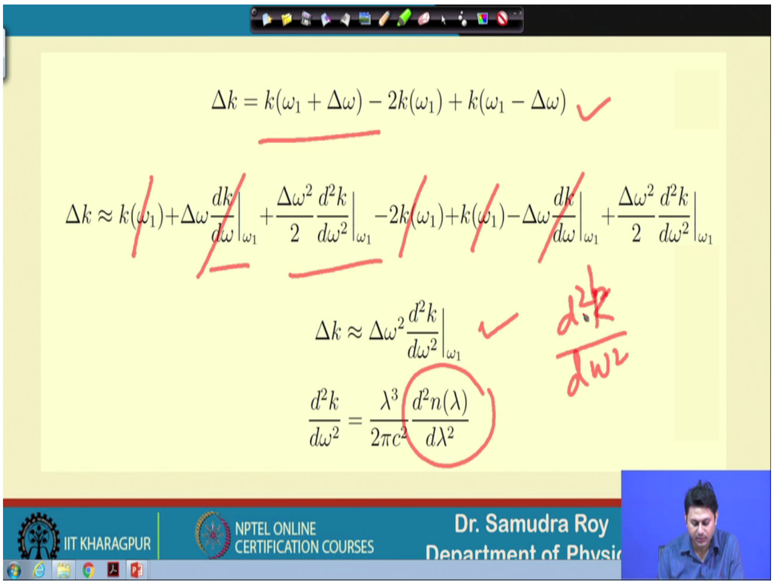

Once I write the delta k in terms of omega, next thing that we will do we just expand is in Taylor series. So, if I do then k of omega 1 delta omega can be expanded at k of omega 1 delta omega d k d omega at omega 1 and delta omega square divided by 2 d 2 k d omega square omega 1.

So, we just expand this function as a Taylor series and, we just take up to second order term. And then if I do that for this minus thing also, I will have a compact expression you can see that k of omega 1 will cancel out with this k of omega 1 and 2 k of omega 1 which is a negative sign, delta omega d k d omega is also cancelling out and, eventually we have delta k equal to delta omega square d 2 k d omega square at the frequency omega 1.

Now, d two k d omega square is a quantity called dispersion, which is related to the refractive index deviation with lambda like this. So, d 2 K d omega square is called the dispersion term how the k vector or the propagation constant is change with respect to frequency, this is equivalent to how the refractive index is change with respect to wavelength or frequency. So, there is a relationship between d 2 k d omega square is with the wavelength like this.

So, I suggest the student to please try to do this calculation by yourself and, check it whether you are getting this expression or not. So, the question the thing is if delta k equal to 0; that means, d 2 k d omega square is equal to 0; that means, d 2 and d lambda square is equal to 0.

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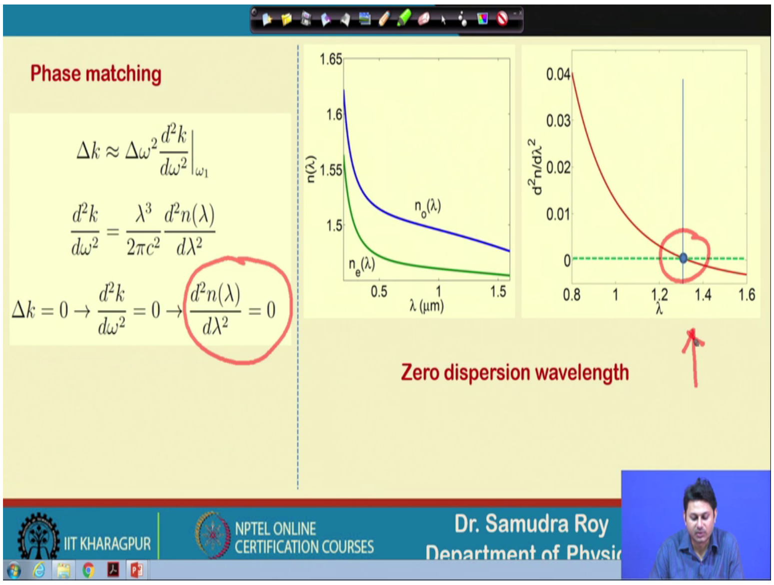

So; that means, in order to generate in order to generate the phase matching absolute phase matching in order to have the absolute phase matching, we have a frequency we need to launch the omega 1 with a frequency such that d 2 and d lambda square is equal to 0.

Now, this is a plot of n as a function of lambda in the left hand side this is for k d p crystal. So, I can plot this things also with d 2 and d lambda square the derivative of these things, when you plot the derivative of this quantity you find there is some specific frequency where, we have d 2 and d lambda 0. So, this is a frequency where d 2 and d lambda 0 below this frequency, we have a positive value of d 2 and d lambda and negative value of d 2 d lambda after this frequency.

So, one case it is called the normal dispersion in other case it is called anomalous dispersion, we will come to this later, but 0 dispersion wavelength is possible, if I launch the wave at 0 dispersion wavelength what happen that, we can excite the phase matching condition will be valid at that point and we can excite our cross talk frequency, whatever the frequencies we were talking about here which is omega 4. So, we can excite the frequency omega 4.

So, with this note I will like to conclude here. So, today we learnt how to generate any frequency crosstalk frequency we take as an as an example and, what should be the corresponding field evolution equation and, if I solve this or if I partially solve this, or if I want to find out what is the condition phase matching condition, we find that if I launch the pulse exactly at 0 dispersion point, then there is a possibility that I will generate that frequency.

But for fiber optics communication this is harmful because, if you generate this cross talk frequency it will interact with the original pulse and, you will lose some sort of energy or you will suit some sort of that the clarity of the signal will lost.

So, normally we do not want to generate any kind of cross talk frequency, in that case the simple way that we just launch the input frequency far away from the 0 dispersion. So, that the delta k become non-zero, if delta k become non-zero and high what happen that we should not have any kind of phase matching and the cross talk frequency is not going to generate.

So, with this note let me conclude here, in the next class we will go further and study more about the four way mixing mainly parametric amplification will be our main concern. So, thank you for your attention and see you in the next class.