Introduction to Non-Linear Optics and its Applications
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Lecture – 05
Basic Linear Optics (Contd.)

So, welcome back student to this Introduction to Non-Linear Optics and its Application course. So, in this particular course for last few classes we learn the basic optics which is very important because this is building block of non-linear optics. So, we start studying about the optics in anisotropic system optics means special the light propagation. So, we will continue this discussion.

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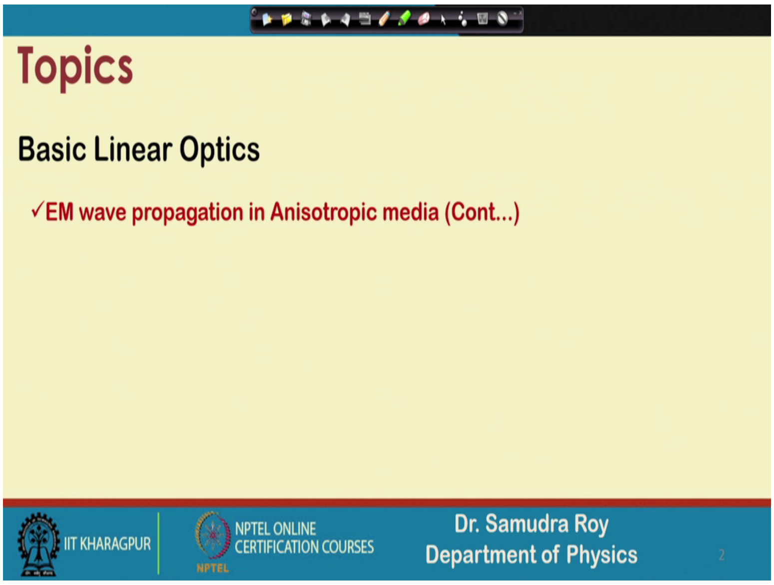

So, let us go back to this slides, so electromagnetic wave propagation in anisotropic media it is continued.

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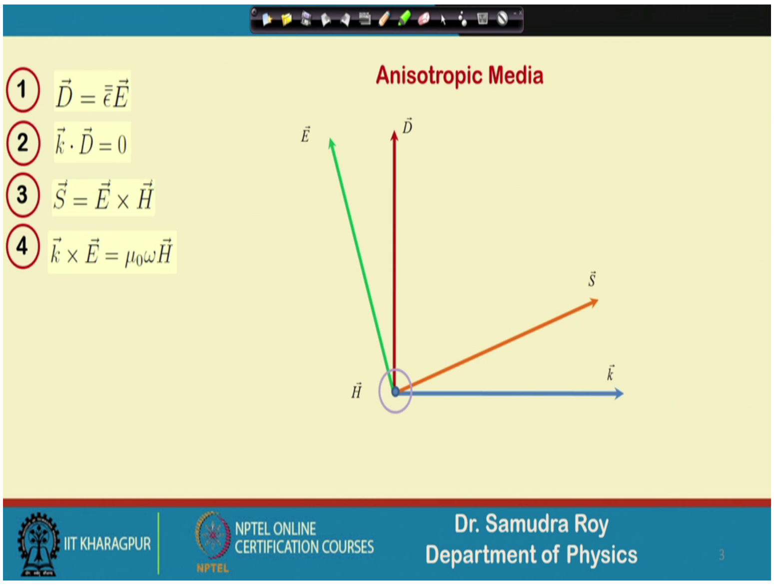

So, this was our starting point in an isotropic system the relationship between D E S and k in general is like this that E and D are not parallel. So, that is why S and k are not parallel, but D and k will be perpendicular and E and S will be perpendicular there is a angle between vector k and S we call this angle is the work of angle.

We will calculate the amount of angle in terms of the refractive index along different direction. So, that is the process we are doing. So, this we have already explained in last class.

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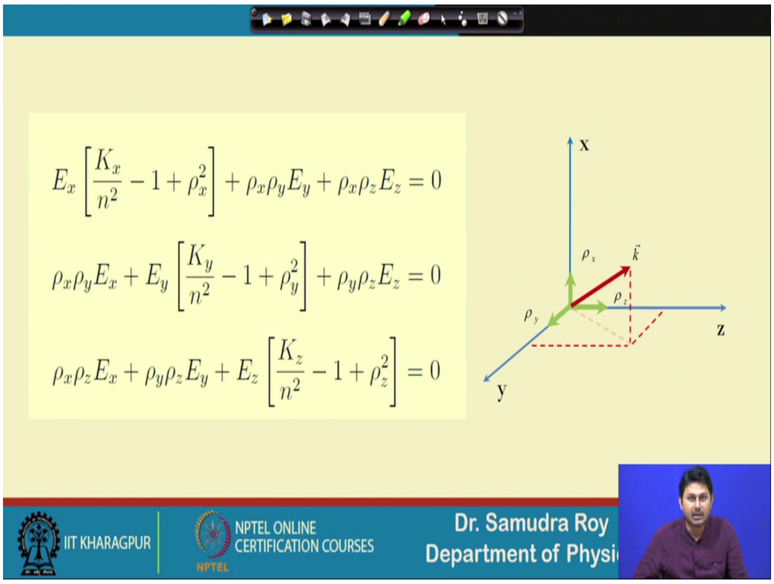

And then we find these 3 equations and these 3 equations are the relationship between the kx, ky, kz, with E x, E y, E z. From directly Maxwell's equation we find out these 3 equations and now we try to solve these 3 equations under different condition.

So, in general if I launch E k vector in arbitrary direction then these 3 equation will be simultaneously there. So, after that what we did in our last class we put some kind of condition.

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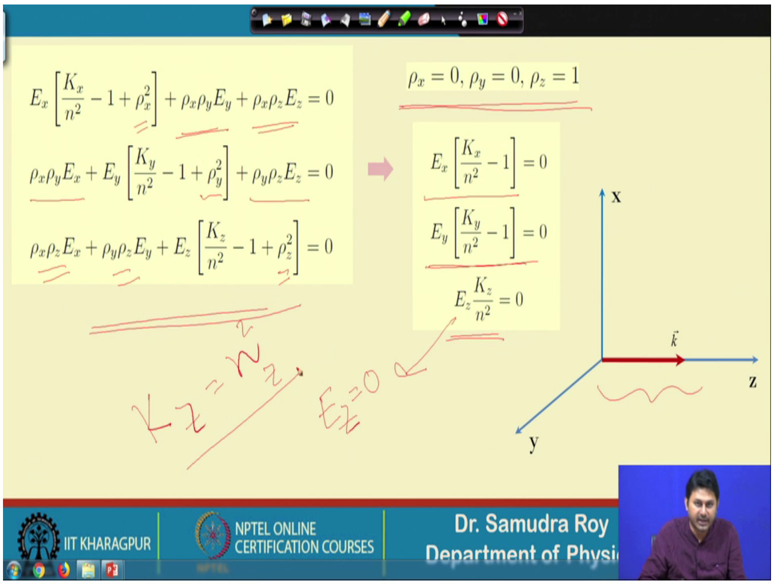

So, the condition here is this. So, let me explain once again so these are the 3 equations; these are the 3 equations that we have derived. So, these are the 3 equations that we have already derived. So, these 3 equations are now put under some condition and the condition is here I show the same figure that we have done in the previous class that if I will if I put my k in such a way that it is along z direction.

So, now, my k is in z direction if that is the case then the vector unit vector associated with all the direction cosine associated with k vector is now rho x, rho y, rho z and they are having value 0 0 1 because I am launching k in z direction if that is the case then in this mother equation this 4 fundamental 3 fundamental equations I will going to put the value rho x, rho y, and rho z which is 0 0 1.

Once I put rho x and rho y 0 my first equation will be this term will vanish, this term will vanish, and rho x this term will also vanish. We will have E x multiplied by K x n square minus 1 and here it is this is my first equation under the condition rho x equal to 0, rho y equal to 0, and rho z equal to 1. Next I will do the same thing in the next equation if I put rho x equal to 0, rho y equal to 0 then this term will not be here this term again will not be here because this is multiplied by rho x which is 0.

But here rho y is again 0, and I will have my second equation and the second equation is E y multiplied by K y by n square minus 1 is equal to 0 this is the second equation under condition that E is launched. And finally, we have other equation so rho x, rho z will be 0 this will be 0 now here rho z is equal to 1.

So, 1 plus and 1 minus will cancel out so I will eventually have these equation, these equation suggest that when I will launch my k vector in this direction z direction there will be no z component of the electric field; that means, my electric field has to be in the plane of x y. So, x y will be the plane which is perpendicular to the k. So, E vector E vector should be in that plane because E z from here one solution we can have from this 3 equation one solution we can have and the solution here we can we can able to figure out very easily that it has to be 0.

Because K z is not equal to 0 and n square is not equal to 0 K z is related to refractive index K z is equal to n of z square of that. So, it is related to refractive index. So, it will never be 0 n z means that the refractive index along z direction. So, it will never be 0, refractive index will never be 0 and if this is the case then E z has to be 0; that means, E there will be no z component of the E. So, E has to be in x and y direction along I mean this plane of x and y.

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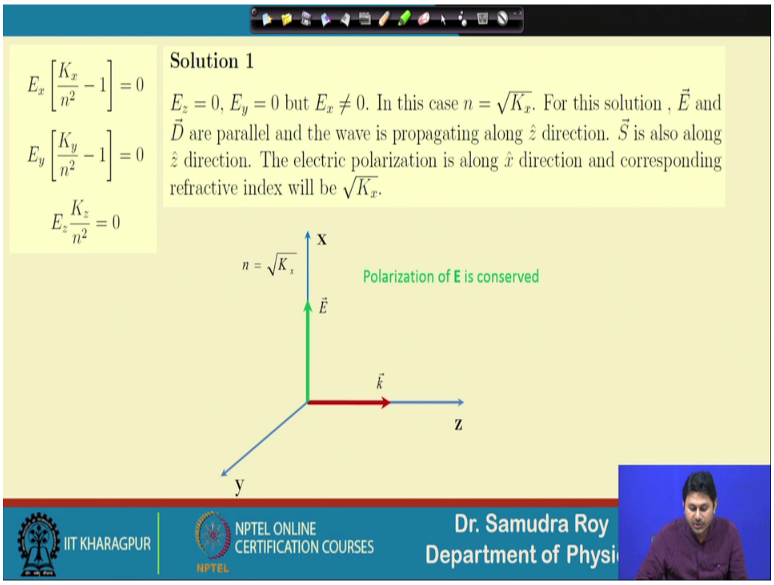

So, the solution 1 as I mentioned in the last class the solution one may be one solution is along if I put E along y direction only; that means, E x component is there E x component is the only component that is here.

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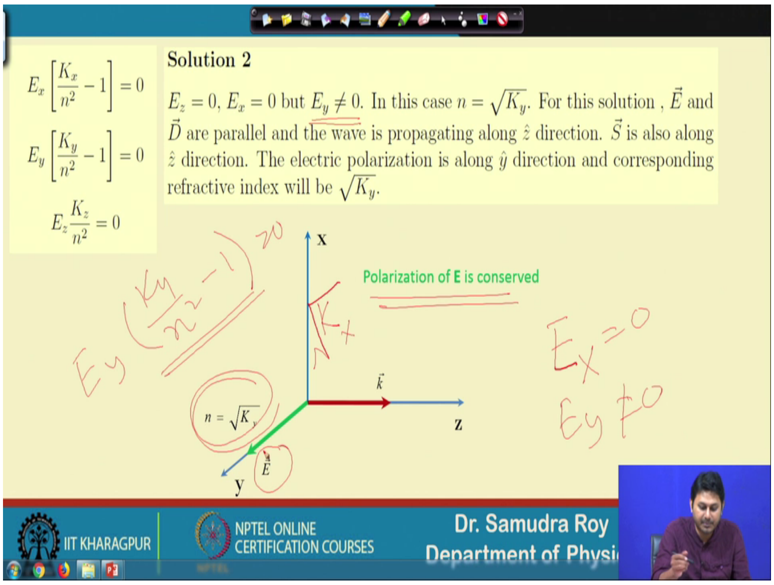

So, E x component the only component ok, this is now another slide ok. This is the previous slide.

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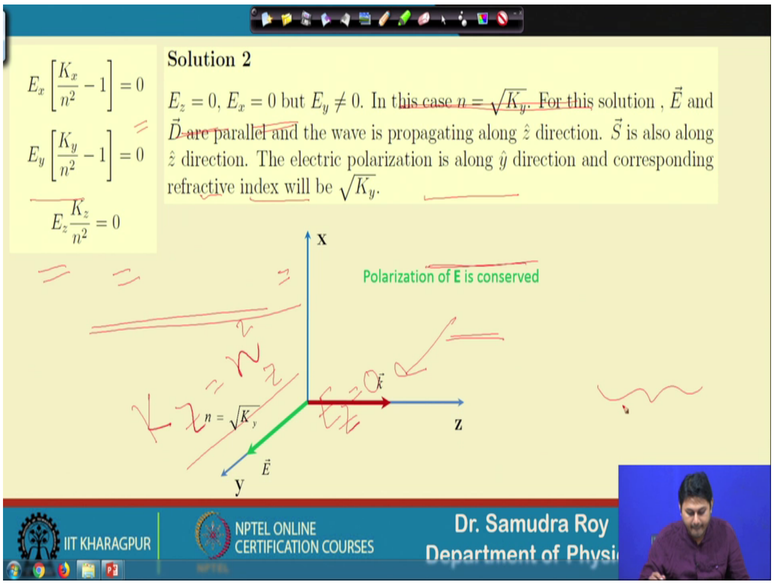

So, solution 2 here is one solution is E x equal to 0 another solution is E x equal to 0 and E y is not equal to 0 if E x is 0, but E y is not equal to 0. If E y is not equal to 0 then I will have a solution here and in this case polarization which remain conserved that is one issue.

Second thing is that from this equation if I write E y multiplied by K y by n square minus 1 that was my equation and I am saying that E y is not equal to 0 that essentially means that K y by n square minus 1 has to be 0 if K y by n square minus 1 is 0; that means, K y is equal to n square; that means, n the refractive index along y direction is K y.

In the previous case we find the refractive index along y direction is K x root over of that. So, here one important thing one should note that the refractive index experienced by the electric field is different in y direction and when it is polarized along y direction and when it is polarized along x direction.

So, that is the important thing here because refractive index may not be same in y direction and x direction and that is the property of the isotropic system that is indeed the property of the isotropic system.

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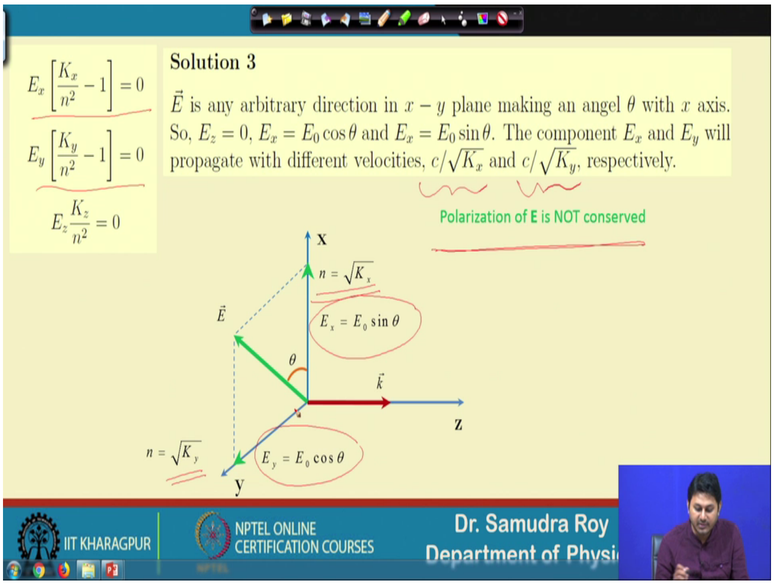

Now, we will come back to solution 3 that is rather more important and that is I am now launching the E vector any arbitrary direction there is no y component along no x component along. So, there is both components are non-zero, so this is a very interesting solution.

So, let us try to understand this so E x these 2 equation is 0 and in the previous case we consider either E x is equal to 0 or E E y is equal to 0 now we consider both are non zero; that means, I am launching an electric field in the plane of x and y as the figure suggest and this component E x component is now written by E x E 0 some amplitude into sin theta and E y component is now written E 0 some cos theta, so these two components are now not equal to 0.

So, they both are non zero if that is the case then we will find that the refractive index here is n x and the refractive index here is root over of K y here it is root over of K x E is still propagating along z direction. So, what will happen here? That is interesting thing now these things will the polarization of E is it conserved here the answer is no because it will not going conserve.

In the previous case it was conserved because when I launch the electric field only along x direction it will experiencing only one refractive index, and that is root over of K x another condition was if I launch the electric field along y direction it will be experiencing only one refractive index and that refractive index was root over of K y.

But here what happened? The two component of the electric filed will be experiencing the refractive index root over of K x and root over of K y; that means, two component will experiencing the refractive index and these two refractive index are different. What is the consequence of that? The consequence consequence is the consequence is or rather the consequences are that the velocity of this particular component will be c divided by root over of K x because we know that the velocity is in a medium is c divided by the refractive index.

So, it will be c divided by K x and the velocity of these components y component will be c divided by root over of K y. So, the velocity of the 2 electric field component is now will not be same it will be different and if this is the case then what happened let us see?

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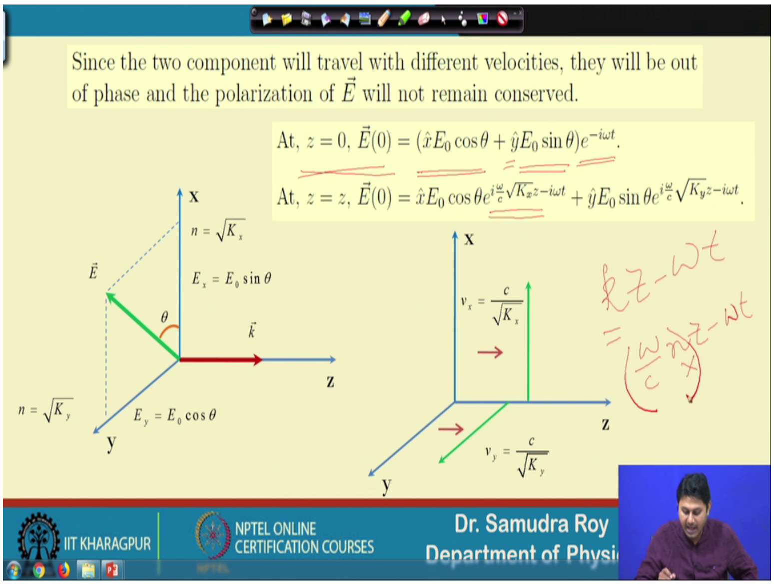

So, what happened? That the one component will travel with a faster speed and another component will be travelling, will be will be travelling in the lesser speed. So, if I write my total electric field if I write my total electric field at z equal to 0 it will be the component x component E 0 cos theta multiplied by unit vector S it will be E y sin theta y component multiplied by the unit vector y and then E to the power I minus omega t this is the phase associated with the field.

Now, what happened if I increase the distance? That means, now the electric field is moving along z direction because K is along z direction. So, what happened? That the one component x component there will be a refractive index and this refractive index or the velocity c y root over of K x. So, if I write the corresponding phase this phase will be i of omega c root over. So, we know that the phase K z minus omega t.

Now, here the value of K will be omega c then multiplied by n x z minus omega t K will be replaced by omega c multiplied by n x I write n x because the refractive index along this is differ. If I now replace n x by root over of K x then here we have the term omega c root over of K x which is nothing, but the propagation constant along x direction, z minus omega t.

In the similar way for y I will have another term and this is this exactly the same term, but root over in place of root over K x I have root over of K y so; that means, if I launch the electric filed any arbitrary direction, but in xz xy plane that has to be because E z component always be 0 when I launch the electric field k along z direction. Then E is perpendicular to the k, but we find that for arbitrary orientation the polarization of the system polarization of the electric field with not remain conserved; that means, the polarization will be changed.

And what why it is that? Because the component of E x and the component of E y of the launched electric field will be experiencing two different refractive index, so these two different refractive indices now appearing in this equation in terms of k or the phase. So, the phase will be different for these two components and once the phase is different for these two components. So, this phase mismatch leads to the non conservation of the polarization.

So, polarization of the if E this E vector is now will be changing. So, there will be some change in the polarization the orientation on all these things will be changing once these things will move at different speed the y and x component ok. So, one important thing will learn that here many things are there that if I launch the electric field along k along z direction that is this in as shown in this figure.

There are three possibilities three solution one is if I launch the electric field along x direction and along y direction these two are. So, called trivial solution and we find there is no change of polarization, but there is another possibilities that I launch E in arbitrary direction in x y plane in that in that case the E x and E y both are nonzero. This components are both non zero, if that is the case we find that they will travelling they will be travelling at 2 different speeds and as a result what happened? That the polarization of the state of the electric field will be not remained conserved. So, we will have a system where the polarization will go to change ok.

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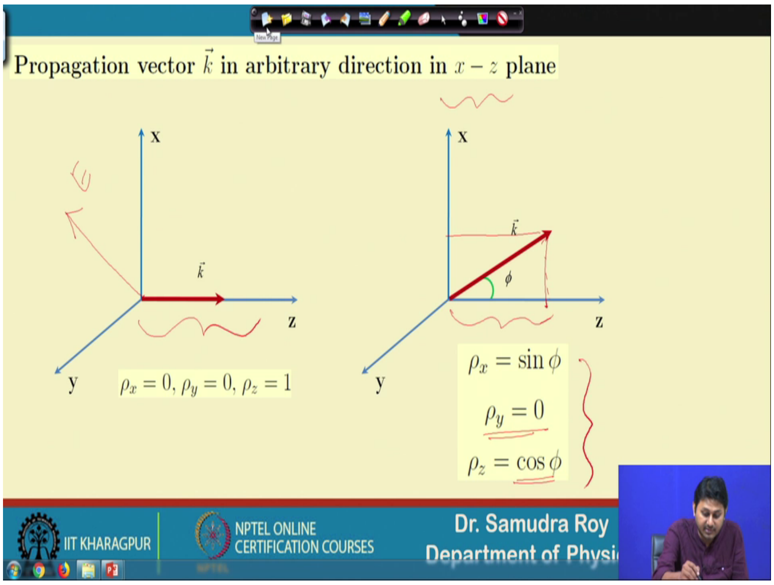

Now, go back to our slides and then find another condition this is a very interesting condition and we will try to discuss these issues also. So, what happened here in the previous case so, far we are deal with the orientation of the electric field under the condition that k is launched along z direction.

So, launching direction of k is was remain same so there was no change in the direction of k; k if the k was in z direction we find that E there will be the value of E and this E field was always perpendicular to the k; that means, even if I launch the electric field in an anisotropic system where the k vector and E vector are not supposed to be perpendicular to each other, but here if I launch k along z direction they are indeed perpendicular.

However, their polarization state will not remain same if E is launched in any arbitrary direction like this as shown in the previous slide. But now, what we will do we will completely change the orientation of the k vector; that means, are electric field is now moving in different direction and this different direction is in as the heading suggest it should be along x z plane for the simplicity let us put y along x z plane if it is along x z plane.

But not in z direction it will be any arbitrary direction and then the rho vectors; the rho values rho x, rho y, rho z values will have the direction cosines rather will have this. So, the rho x will be sin phi, the rho y will be 0, and the rho z will be cos phi.

So, if I put a I mean if I put the vector component here and here if I put the vector components then I will have this value is mod of K multiplied by cos. So, mod of K was omega c omega divided by c multiplied by n and the rest value was rho z and rho z is cos phi in the similar way here rho z is sin phi and rho x is 0 because there is no component along y direction.

So, this is the case now it is interesting because rho x, rho y, rho z previously there was two 0 values and that was rho x was 0, and rho y was 0, and rho z was 1. But here we have value of rho x which is non zero value of rho y value of rho z which is non zero only the value of rho x is 0. So, it will also modify the equation and this that 3 equation we derive. So, let us check what kind of modification it will produce so ok.

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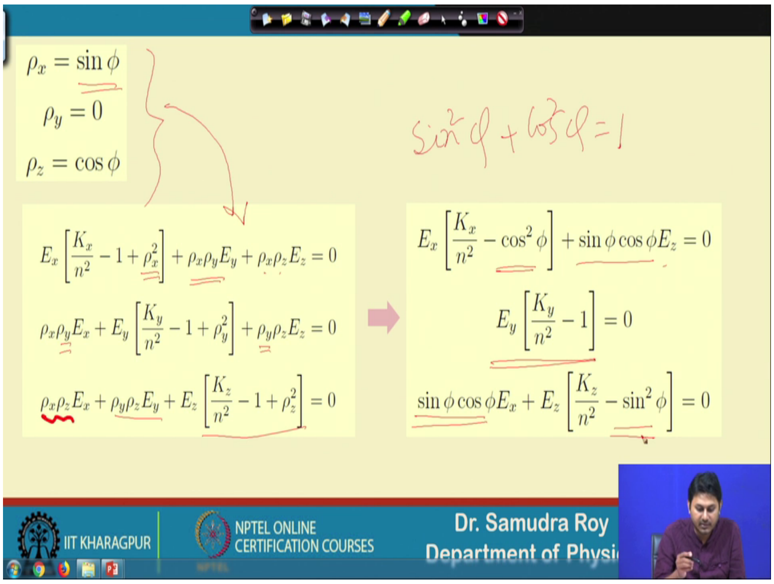

This is the total picture right now this is the total picture. So, now, my rho x, rho y, rho z is this if I put this rho x rho y rho z these 3 model equation this is the most general equation of rho x K x and K y. Then what happened? That these values now replaced now rho x will be replaced rho x square rather will be replaced by sin phi when I replace this as a sin phi. So, 1 minus 1 plus sin phi it will be cos square phi sin square phi plus cos square phi is one. So, I am using this.

So, sin square phi plus cos square phi is equal to 1, these value if I take minus common the 1 minus sin square phi. So, 1 minus sin square phi cos square phi so that is why it is written as minus of cos square phi. Then rho x, rho y, and rho x, rho z, rho x, rho y is 0 because rho y is 0 rho x rho z is sin phi cos phi and then I have E z. Next equation rho y is equal to 0, if rho y is equal to 0 if I put rho y here 0 and if I put rho y here 0, so these 2 components will not be there and rho y again if I put 0. So, I will have one equation E y is this?

And finally, the last equation again I will put rho x rho y whatever the value we have, so rho x rho z and this quantity and this quantity if I put these things. So, I I will have sin phi cos phi and this sin square phi. So, I have 3 modified equations when I put rho and this modified equation can leads to some kind of solution a general kind of solution. So, let us try to find out what kind of solutions we will have ok.

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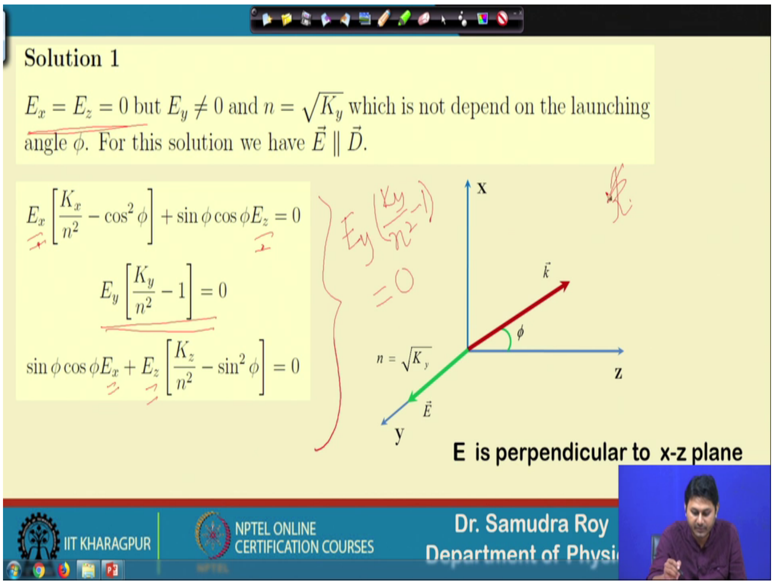

So, first solution what will be the first solution first solution these are the 3 equation this is interesting this is very interesting. That I have 3 equations in my hand and try to find out the solution try to find out the value of E x E y and E z. So, what here the special thing? k is not along z direction, k is having k is in different direction having an angle phi. And then what we will do we try to find out the solution.

The first solution here we are assuming is let us put K x, K z both 0 E x E z both 0 if E x and E z both 0 then this equation and this equation are consistent because if I put E x here 0 E y 0 then this things is 0. So, right hand side is 0, left hand side is 0 no problem in that. Here also in this equation if I put E x equal to 0 and E z equal to 0 I have left hand side is equal to 0, which is right hand side.

So, these two equation will satisfy with the solution E x equal to 0 and E z equal to zero, but another equation here in our hand and we assume we assume that E y is not equal to 0 because if E y is 0. Then these are the trivial solution all are 0. So, 0 left hand side will be equal to 0 right hand side. So, there is no I mean there is no kind no solution we have.

So, now, for these solutions the special solutions we have only one equation in our hand and that equation is E y multiplied by K y divided by n square minus 1 is equal to 0 this is the equation I have. So, from this equation if E y is not equal to 0; that means, K y like the previous case D K y rather sorry. So, let me erase this so big K y will be equal to n square.

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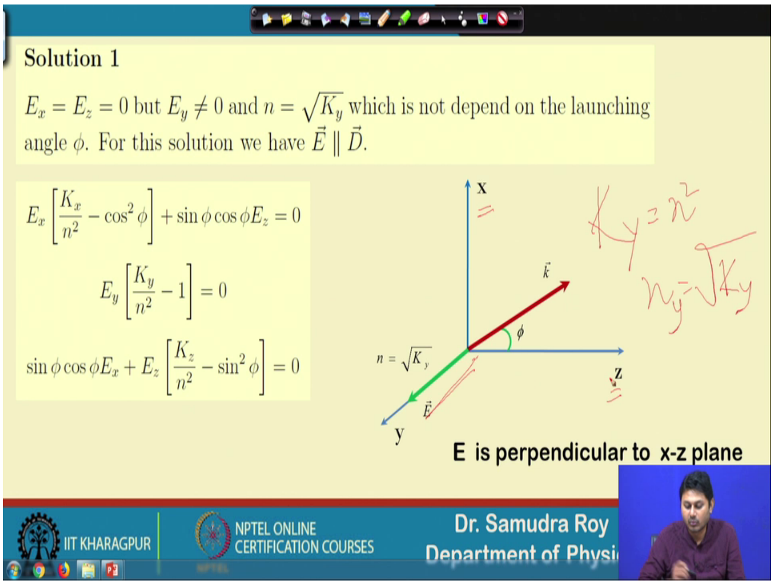

So; that means, refractive index along y direction is root over of K y this is the refractive index along this direction. So, if I look the solution carefully these E is along y direction now because these component and these components are not there. If that is the case then what happened? This is now experiencing this component will now going to experience the refractive index which is root over of K y that is one issue.

Second very important thing even though I am launching the K not in z direction, but any arbitrary direction the electric field along y direction will still be perpendicular to k vector; that means, I am launching the k vector along z direction and we find that electric field along y n E is there which is perpendicular to the k vector in an isotropic medium which is normally not the case.

But here we find that even if I launch the electric field in some different direction which is in x and z plane, but not in k direction, the perpendicular direction along perpendicular direction the electric field the solution can be there the electric field can be in y direction. And it will be experiencing refractive index K y and it will propagate without any kind of problem; that means, in the polarization will remain conserved and it will propagate without any problem that is one issue.

Second thing is that k and E they are perpendicular to each other even in this case. So, in anisotropic system it is not necessarily that always that k and E will be not this is the most general case that they are not perpendicular to each other, but some special cases like this they may be perpendicular to each other.

So, this is the thing so far we have done. But the most interesting thing we will come in the next class that what happened? If there is no field here so electric field is along the x and y direction if the electric field along x and y direction what happened because this is the most interesting thing here.

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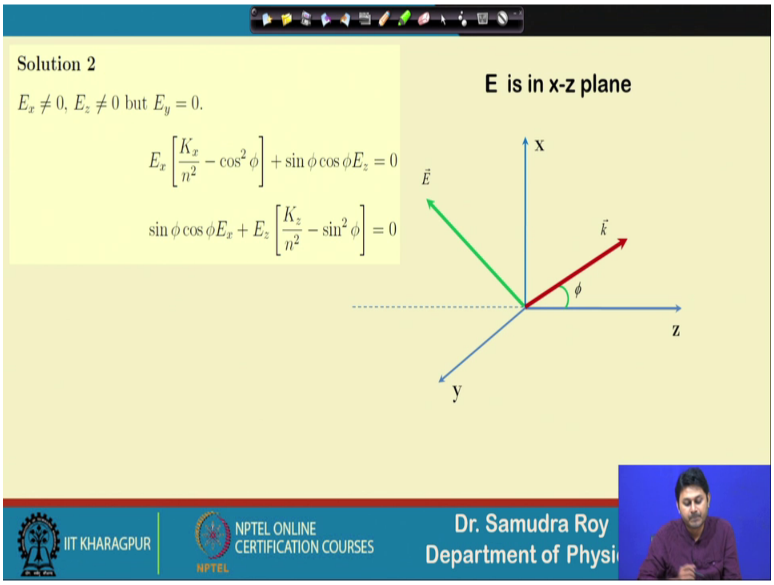

And then we find that they are not following the case. So, this is the problem that will going to solve in the next class. So, just I am giving you the hint, or the introduction of the problem the problem suggest that in the previous case the solution was E y was not equal to 0, but E x and E z was 0.

Now, I convert I just invert the solution; that means, now I consider now I consider that E y is 0, but E x and E z is not equal to 0. In this under this condition now we have 2 equation in my hand because other equation will vanish automatically E y equation is not be there because E y is this.

So, now, we need to solve these 2 equation under the condition E x and E y is both not 0; that means, E x and E z components are there, but E y component is not there, if I try to understand with this figure I am launching the electric field along this direction which is x z plane and E is also in x z plane. Because E x component and E E x and E z these two components are non zero that makes it has to be somewhere in the plane. This is maybe one orientation of E, but it has to be in that particular thing.

So, with this note I would like to say then in the next class we will solve this very interesting problem and try to find out that how the E and k vector which was so far perpendicular is really in the for solution 2 will be perpendicular to or not. If it is not perpendicular then how to measure the angle between these two that is the most important thing and that is the thing we are doing that to find out the angles between whatever the vectors are there in anisotropic system. Because if you know the angle between these two we know in which direction they are flowing that is one issue.

Second issue how this angle is related to the refractive index? So, if the refractive index is given to you we can calculate the angle the walk of angle the angle between the vector k and S or the vector D and E how to calculate that we will find out in the next class? So with that note so let me conclude here. So, see you in the next class.

Thank you for your attention.