Introduction to Non-Linear Optics and its Applications
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Lecture – 49
Four Wave Mixing

So, welcome student to the next class of Introduction to Non-linear Optics. In the previous class, we learnt about the cross phase modulation and then studied the non-linear absorption. So, today we will have today the lecture number is 49 and we will have in our lecture today one important concept called 4 wave mixing.

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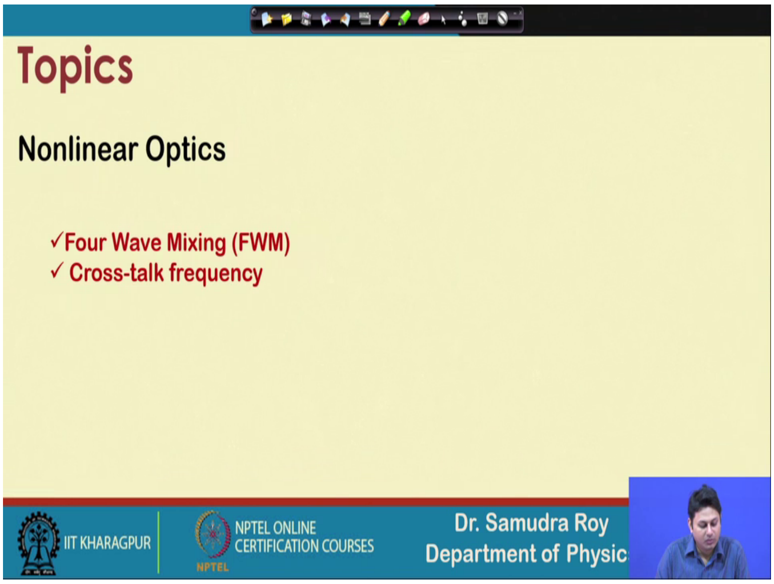

And then learn another thing called cross talk frequencies; so, what is 4 wave mixing and cross talks frequency let us start.

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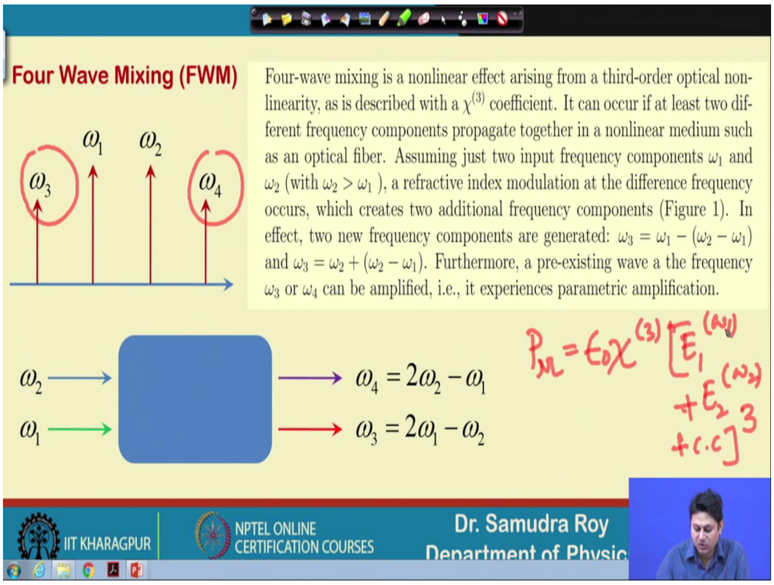

So, 4 wave mixing is a interesting phenomena in non-linear optics. So, what is the meaning of 4 wave mixing let us try to understand. So, here the definition or the idea of the 4 wave mixing is written. So, 4 wave mixing is a non-linear effect arising from third order nonlinearity, as described with a chi 3 coefficient. So, if our chi 3 coefficients are there in a system what happened the non-linear polarization will be coming as a function of e cube. So, it will be e e and e there will be 3 e related to that term.

Now, if 2 frequencies are there at least 2 frequencies are there omega and omega 1 and omega 2 may be more than 2 frequency there we will see that. Then these 2 frequency omega 1 and omega 2 can combine themselves to generate another 2 frequency omega 3 and omega 4 in this fashion. If omega 2 is greater than omega 1 then additional frequency component as shown in this figure 1

The additional frequency component omega 4 and omega 3 can be generated. So, let us try to understand this P non-linear is equal to epsilon 0 chi 3 and 2 waves are there so; that means, E 1 omega 1 plus E 2 omega 2 is there; obviously, the complex conjugate will be added with that and then we make a cube of that once we make a cube of that. So, there will be a different kind of frequency mixing. So, we will have 3 omega 1, we will have 3 omega 2 we can have minus of 3 omega 1 minus of 3 omega 2 the complex conjugates of that 2 term and also the combination of these 2.

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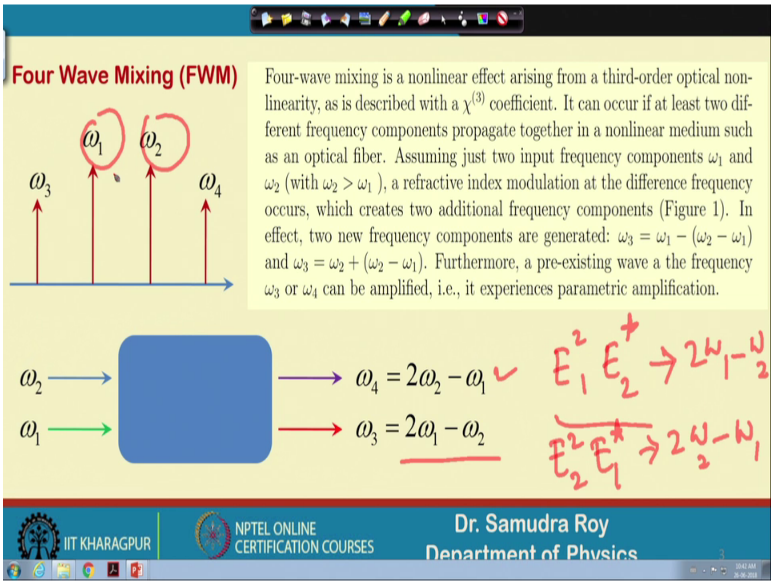

So, one combination maybe one combination maybe E 1 square E 2 star; so, E 1s it is. So, E 1 square E 2 star basically give raised to one frequency 2 of omega 1 minus omega 2 which is this one. So, this combination basically give raise to a frequency omega 3 and also another combination can be possible which is E 2 square E 1 star which give us 2 omega 2 minus omega 1. So, another frequency is there.

So, we start with 2 frequency omega 1 and omega 2; the combination of these 2 omega 1 and omega 2 can give raised to 2 different frequency; one is this omega 4 and another is this. And the relationship between this frequency is something like that if you write this things here, then we can write a more simpler expression.

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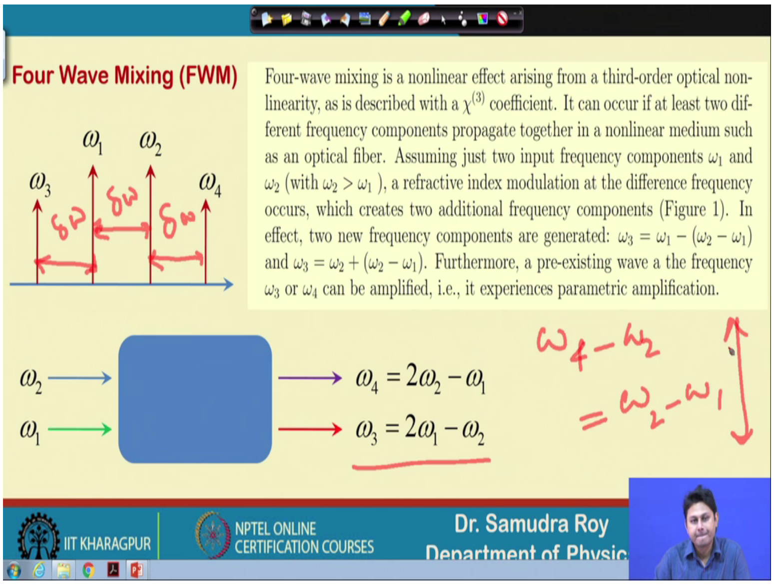

And this expression is omega 4 minus omega 2 is equal to omega 2 minus omega 1.

So, if I have a difference here delta omega which is the difference between omega 1 and omega 2, then I can generate a similar kind of frequency beyond omega 2 with addition of delta omega and also another frequency with delta omega frequency less than omega 1 which is the lowest frequency. So, we have a frequency omega 1 and omega 2 omega 2 is greater than omega 1. So, if I make a difference between omega 2 and omega 1; we can generate omega 4 and if we also generate another frequency this side with omega 3 with this relationship.

So, this is essentially called the 4 wave mixing because we are basically mixing 4 different waves or the 4 different waves are generating; omega 1 and omega 2 are there, and then we generate another 2 wave omega 3 and omega 4. By the way there are different kind of possibilities are there, this is one specific possibilities; I am talking about and in this case if omega 1 and omega 2 are already there in the system.

So, what happened that we can able to amplify this signal. So, this is called some sort of parametric amplification. So, parametric amplification for second order effect we have already discussed. So, this is a different kind of parametric effect under 4 wave mixing. So, that we will going to discuss, but the thing is if I launch 2 wave omega 1 omega 2 in third order under third order nonlinearity, there is a possibility that it will going to generate 2 different frequency omega 4 and omega 3 with following with this relationship with the same same frequency apart.

Well next we will try to understand this in detail. So, first the degenerate 4 wave mixing.

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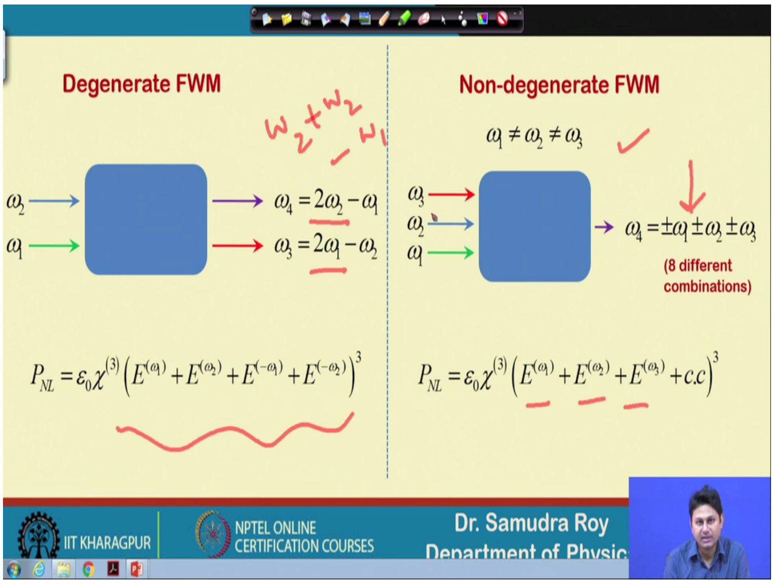

In degenerate 4 wave mixing as I mentioned omega 1 and omega 2 are used, they are used twice because when I calculate this omega 4 calculate this omega 4, you can see here I used 2 omega 2 and 2 omega 1; that means, omega 2 plus omega 2 minus omega 1.

So, I used 2 difference. So, in input I have omega 1 and omega 2 when I generate omega 4 I am using omega 2 twice and omega 1 once. So, that; that means, this is a degenerate case. But more general is non degenerate case which is shown in the right hand side you can understand easily. That if instead of launching 2 wave if there are 3 waves are in the system omega 1 omega 2 and omega 3 which are not equal to each other as shown here, then there is a possibility that they will going to generate a combination of all these 3. So, omega 4 will generate and omega 4 will be the combination of all these 3 frequencies. So, plus minus omega 1 plus minus omega 2 plus minus omega 3; this 8 different there will be 8 different combinations. So, 8 different combination one can generate.

So, here the P nonlinearity term shown how this from P nonlinearity term how this frequency mixing is mixing is happening because of the launching of 2 waves one is omega 1 and omega another is omega 2. But in this case we are launching omega 1, omega 2 and omega 3 when I make a cube of that we you will have 8 different combination

If omega 1, omega 2, omega 3 are distinct they are not same mind it they are not same that is why it is called non degenerate 4 wave mixing. So, we have a generate 4 wave mixing we have a non degenerate 4 wave mixing; in non degenerate 4 wave mixing if omega 1 and omega 2 are same, then it will just convert to degenerate 4 wave mixing. So, degenerate 4 wave mixing is a special case of non degenerate 4 wave mixing, where 3 waves are mixed to generate a fourth wave, but if these 3 waves are not identical; if not distinct there if they are 2 of them are identical, then we will have a degenerate 4 wave mixing and we will get some specific frequencies with that.

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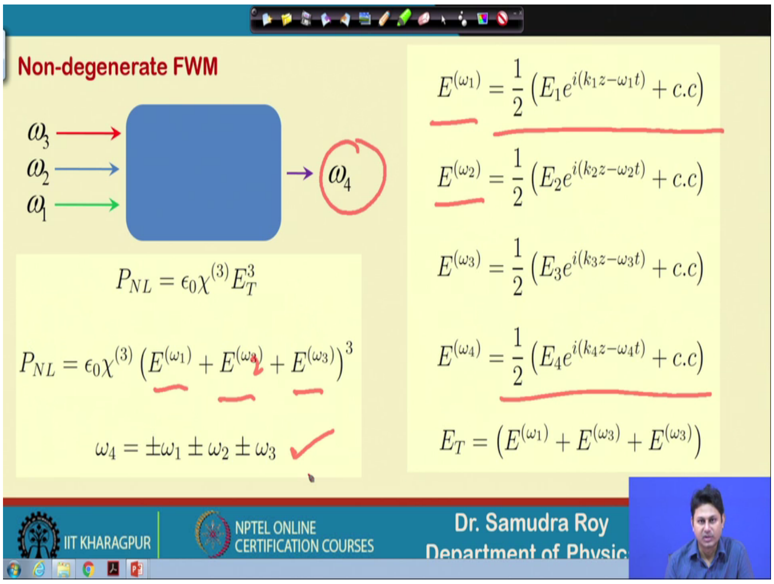

Once we have the idea of degenerate and non degenerate frequency. So, let us try to find out how mathematically we can understand that or how the treatment is done. So, for non non degenerate 4 wave mixing, which is a most general case omega 1, omega 2, omega 3 is launched and as a result I am generating omega 4.

So, the 4 waves can be represented in this way. So, E of omega 1 is a wave of omega 1, it is half E 1 e to the power i propagation constant k 1 omega 1 t plus complex conjugate this is a standard way the plane wave form to represent a wave we have been using this for last almost all classes I am using this notation. So, in the similar way I can write the electric field for omega 2, omega 3 and also generating field omega 4.

So, inside the system if I want to find out how omega 4 is generating. So, what we need to do that there are 3 waves here. So, this 3 waves I write this plus this plus this that is E omega 1 plus E omega 2 plus E omega 3 and if I make a cube of that as I shown in the previous slide, that there will be a combination of this frequencies because complex conjugate terms are also there. So, omega 1 is there; so, minus omega 1 is also there, omega 2 is there, minus omega 2 is also there omega 3 is there minus omega 3 is also there.

So, now if I write this six term essentially there are six terms omega 1, omega 2, omega 3 here we have omega 2 this. So, again make a mistake. So, if I have 3 terms then essentially the complex conjugates are also there. So, six terms are there and if I make a cube of that, and if I consider omega 1 omega 2 omega 3 are not same then we will have a combination of omega 1, omega 2, omega 3 and this combination give raised to a forth frequency called omega 4, which is plus minus omega 1, plus minus omega 2 plus minus omega 3 in general it maybe plus plus plus it maybe minus minus minus there will be eight different combination one can expect.

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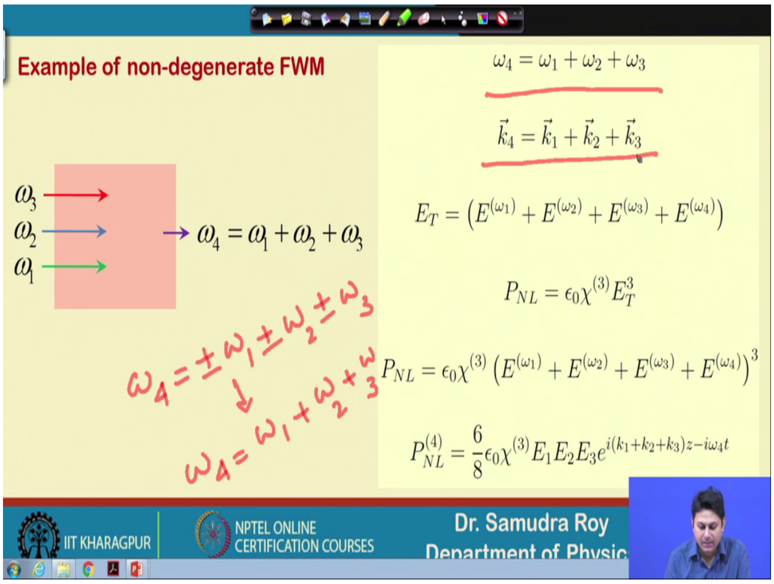

So next let us try to find out some example of non degenerate 4 wave mixing as I mentioned there will be eight combination of that, but from this eight combination if I take only one combination that all waves are all frequencies are added to generate omega 4 then this is a specific combination. So, mind it what we are trying to do. So, we find that omega 4 is essentially the combination of omega 1 plus omega 2 plus minus omega 3 there will be 8 specific combinations, if omega 1 omega 2 omega 3 are not equal to each other they are not identical to each other they are distinct; then from these we can take only one combination and try to understand what is going on. So, omega 3 is get just omega 1 plus omega 2 plus omega 3.

So, this is our frequency; So, the k vector can also be written in this way. So, in order to have the phase matching condition, we have the wave vector the momentum matching in this way the total electric field I can write in this way also.

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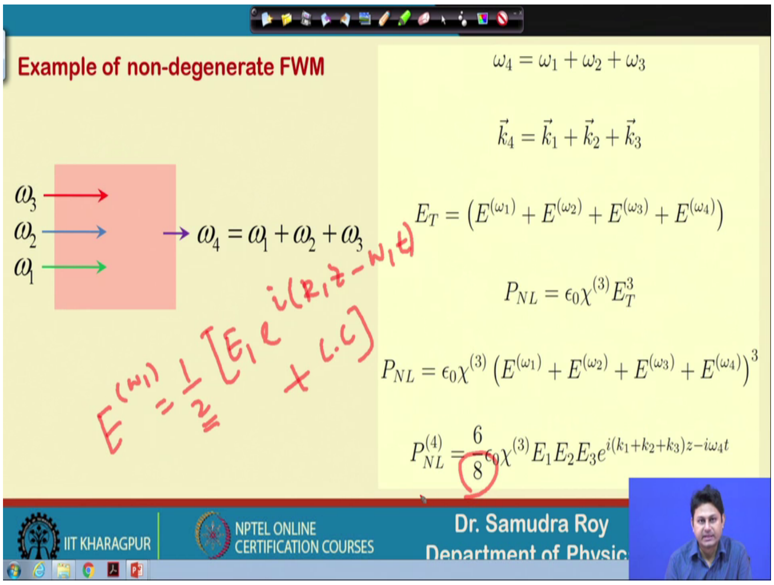

The total electric field of omega 1 plus omega 2 plus omega 3 plus omega 4 because all the 4 waves are now inside the system, and I need to find out the differential equation of each individual fields.

So, that in order to do that treatment we need to take care of all 4 fields and P non-linear is now written is epsilon 0 chi 3 total field cube, where total field is the 4 combination of 4 field which are generating inside the system. When I make the cube of all this things, then there will be different combinations one can expect, but mind it our aim is to find out the frequency omega 4 which is the sum of all these things. So, from here we can readily find out what should be the non-linear polarization component having the frequency omega 4, and we write it at P non-linear in bracket 4; the bracket 4 suggest that we are taking the frequency component omega 4 here.

So, how the combination one can get? So, omega 1 plus omega 2 plus omega 3 so; that means, E 1 E 2 E 3 that is the amplitude of this 3 waves that should be multiplied; exponential term E 1 k 1 k 2 k 3 will be there and I will have a frequency component omega 4, because if I multiply this 3 things then I will have a frequency component omega 1 omega 2 plus omega 3; so, this is essentially omega 4. Also the degeneracy factor one can consider the degeneracy factory we know that if this 3 distinct fields are associated to generate the corresponding polarization, then one 6 multiplication should be there.

So, here 3 fields are distinct; so, this 6 term will appear because of that fact and also you remember here it is not written. So, already we have written in the previous slide that E of omega 1 is half of E 1; E to the power of i k 1 z minus omega 1 t plus complex conjugate of that. So, this half term basically give raise to this 8.

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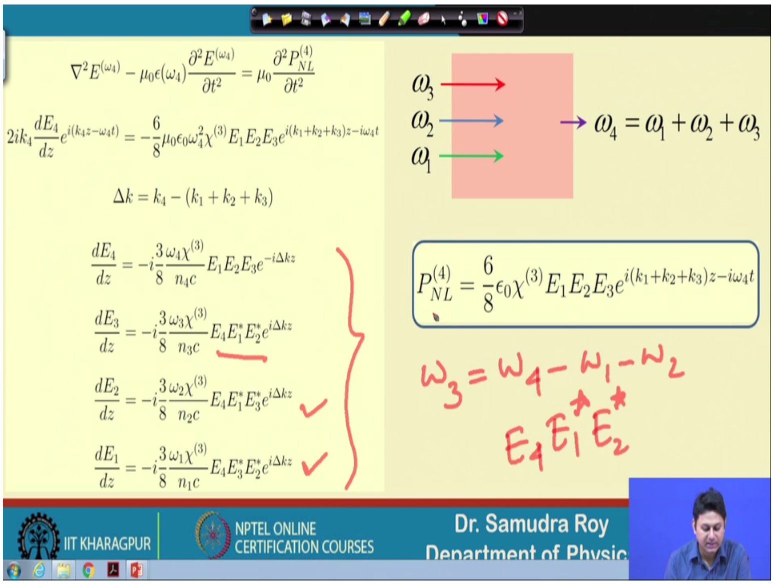

So, I now I have P non-linear term in our hand and if I manipulate the non-linear Schrodinger equation for E 4 term. So, it will be simply it will be simply this quantity. So, this is the non-linear Maxwell's equation that we have been using to generate the evolution of different fields. So, for E 4 fields how our aim is to find out how this E 4 or the field containing the frequency omega 4 will going to evolve, we will just use this non-linear Schrodinger equation; non-linear Maxwell's equation.

So, this non-linear Maxwell's equation basically is a source term. So, we have already calculate this source term P non-linear 4, which is basically give raised to this field omega 4; and then if you calculate the way we have we have been calculating the derivatives. And making a derivative and then do all these things you will come to this expression in a straight away and then you will have an equation for electric field containing the frequency omega 4 this.

How this things are coming if you do yourself you will find out quite easily it is already done here in this slides, I am not going to detail because this calculation is quite trivial. But important thing is that the source term here in the source term E 1, E 2, E 3 which basically give raised to the field omega 4, but from that also you can all calculate the other field evolution of the other fields like E 3, E 2 and E 1 because all these 3 4 fields are there. So, essentially we should have the evolution of other field also.

So, now for other fields if you look carefully this term will going to change and it should be because once we find E 3 the non-linear polarization term will contain a term. So, omega 3 is if I write this is my equation, then omega 3 is basically omega 4 minus omega 1, minus omega 2. This minus omega 1 and minus omega 2; let me write is once again. So, omega 3 suppose is equal to omega 4 minus. So, let me write it omega 3 is omega 4 minus omega 1 minus omega 2.

So, if I want to find out the corresponding frequency component, the field will be E 4 for this minus sign; it will be E 1 star and for this minus sign it will be E 2 star. This will be the combination of the field in P non-linear. And if you look carefully it is exactly the same thing we are having here for dE 3 dz. In the similar way d E 2 dz I will have a differential equation like this and d 1 dz; we will have a differential equation of this.

So, for 4 different fields I have the differential equation in our hand, only for one cases I calculate the P non-linear term for omega 4, but you can do that for other frequency terms and you can calculate quite easily by just inspecting I can calculate this, and this is basically the evolution equation under 4 4 wave mixing this is the evolution equation of 4 fields that is generating.

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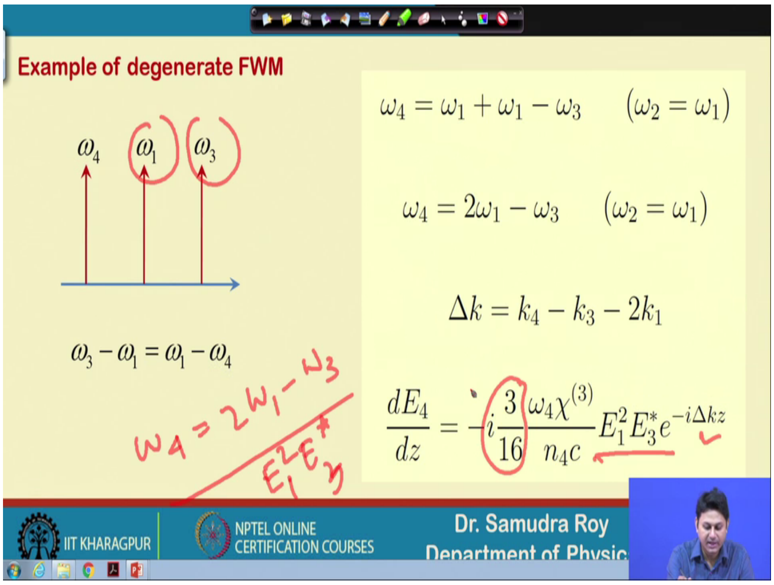

Well, we can go further we can have an example now you can have an example of degenerate case. For degenerate case the equation one can also derive, but for degenerate case 2 electric fields is now associated not all the 4 electric fields. So, these 2 electric fields can give raised to omega 1 and omega 2 is the these 2 are the frequencies this will going to generate a third frequency omega 4, and omega 4 this is a special from here we can see that omega 4 is 2 of omega 1 minus omega 3 this is a special combination. And for the special combination once we calculate once we calculate the evolution of the electric field corresponding to omega 4.

So, I will have here the frequency component 2 omega 1 minus omega 3. So, the field should have E 1 square E 2 of star E 3 of star sorry because it is omega three. So, this quantity the phase term will come as usual only thing that you note that since E 1, E 3 and again E 1 is used so; that means, there are no 4 distinct fields are associated with that. So, degeneracy factor will be not 6, but 3. So, when we use the degeneracy factor 3. So, this quantity will going to change slightly.

In the previous slide if you look carefully in the previous slide it was 3 by 18 now it should be 3 by 16 because the degeneracy factor 6 here is will be changed as 3. Because I am I am making 2 same fields here to generate our P non-linear terms well.

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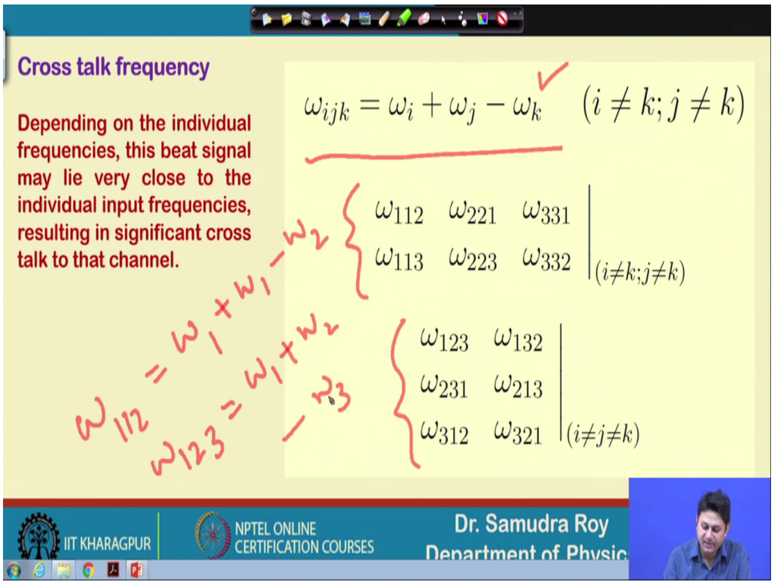

Now, we will briefly describe about the cross cross talk frequency what are the cross talk frequency. As I mentioned there are several kind of frequency mixing can be possible. So, depending upon the individual frequencies the this is called beat frequencies. So, some beat frequencies are such that they are very close to the launching frequency.

So, because of that they can generate some kind of cross phase or cross talk problems, and this is happening in optical 5 wave where WDM systems are there, wavelength, division, multiplexing process, where different frequencies are moving together as a different channel. But if the 4 wave mixing are there what happened that different frequencies component will going to generate and from this different frequency component there are few frequency components which are very close to the launched frequency component and as a result what happened we will have some kind of distortion in the signals; so, here if you look carefully.

So, let us try to understand. So, with term omega i j k is equal to omega i plus omega j minus omega k with i not equal to k, j not equal to k. With this condition this we write this there is a specific frequency condition or the specific frequencies now how many frequencies is possible if ijk are 1 2 3? There should be 12 different frequencies if i is not equal to j and j is not equal to k, i is not equal to k and j is not equal to k. So, I have six combinations omega 1 1 2, omega 2 2 1, omega 3 3 1, omega 1 1 3 and so on.

So, these are the six combinations that we have and another combination we also have if all these 3 frequencies are distinct. So, ijk are not same to each other. So, they will have another six combination. So, this total 12 combination one can have with this equation if I change ijk as 123. So, as an example what is the meaning of the omega 11 2 frequency what is the meaning of that? It means it is omega 1 plus omega 1 minus omega 2 this is the frequency I am talking about.

So, here 2 frequencies omega 1 and omega 1 are same omega 2 is different. So, i is equal to j here, but I and j both are not equal to k that is the condition we have. But if I write one 2 3 omega 1 2 3 it is simply omega 1 plus omega 2 minus omega 3 and here ijk all are different and this is another combination of the frequency, where these 3 frequency are non degenerate.

So, degenerate condition and non degenerate condition both conditions are there and based on that we can have 12 different frequency if I define my frequency in this way why we are defined this frequency in this way we will try to understand that.

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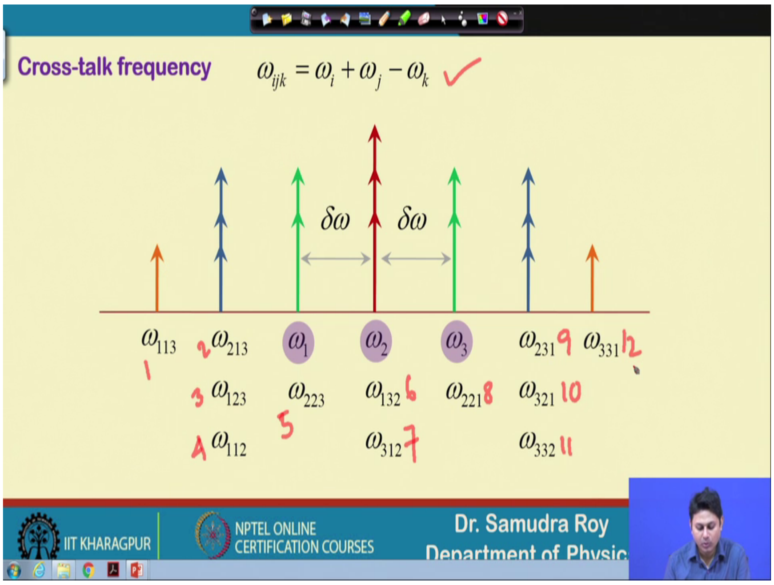

Because this frequency will give raised to some frequencies which have very close to very close to the launching frequency. So, this is a very important figure and if you understand then figure, then you will readily understand what is the meaning of cross talk frequencies. So, our frequency definition is this thus omega ijk is omega i plus omega j minus omega k, omega 1 omega 2 omega 3 is the 3 fundamental frequency that is launched into the system, where the spacing between omega 3 and omega 2 is delta omega, and omega 2 and omega 1 is delta omega these are the spacing between these 2.

If this is the case then I will have 12 different frequency combination and in this frequency plot if I plot this 12 different frequencies, then it will be plotting one can plot in this way. So, first let us try 1 2 3 this blocked these sided frequencies are the fundamental frequencies and from this fundamental frequencies I can generate omega ijk which is the combination of this frequencies. So, how many combination we have in the previous slide.

It was 12. So, I can see that there are 12 frequency if I start from here this is 1 frequency this is 2 frequency, this is 3 frequency this is 4 frequency this is fifth this is sixth seventh eighth 9 10 11 and 12. This 12 frequency will be distributed in the frequency location in this way how this things will be distributed let us try to understand quickly in our last slide.

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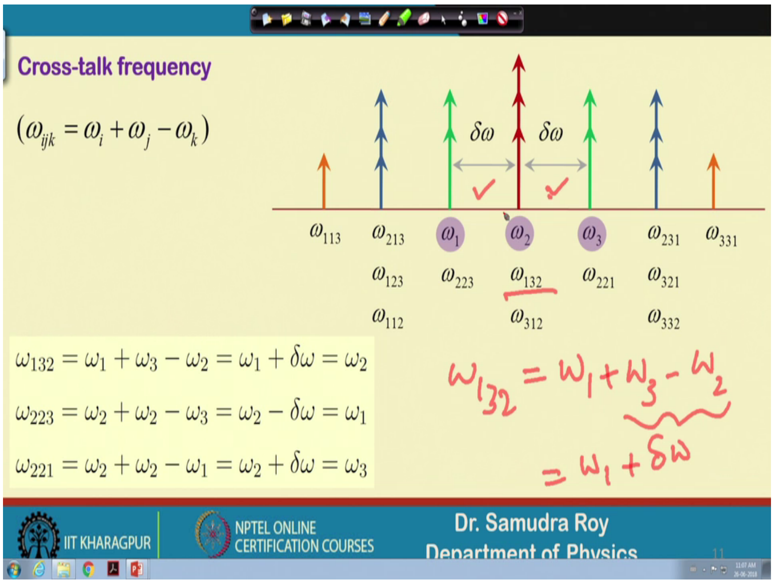

So, this is the matrix of the frequencies and now, we try to understand how this frequency are located. For example, let us start with omega 1 3 2 this frequency how this frequencies are generating? Omega 1 3 2 is essentially omega 1 plus omega 3 minus omega 2.

Now, omega 3 minus omega 2 is means this is this delta omega. So, I write this as omega 1 plus delta omega, because delta omega is the spacing between omega 3 and omega 2. So, this omega 1 plus delta omega means this is my omega 1 and if I add delta omega, this frequency basically fall on here on top of omega 2. So, omega 2 frequency is nothing, but omega 1 3 2 frequency, the combination of these 3 frequency basically give us a frequency which is omega 2.

So, if omega 1 omega 2 and omega 3 these 2 these 3 frequency are the difference between these 2 frequency are close enough or if they are not exactly same, but very close enough, then what happened that this frequency omega 1 3 2 will fall very close to omega 2 frequency. Now it is falling exactly on omega 2 frequency because we consider this delta omega same for both the cases if they are not same, but close to each other then what happened this omega 1 3 2 frequency will fall exactly or very close to this omega 3 frequency, and that basically creates the problem of cross talks.

You can do for all other frequency by yourself here, few examples are shown. For example, omega 2 2 3 is omega 2 plus omega 2 minus omega 3 omega 2 minus omega 3 is minus delta omega and it is subtracted from omega 2. So, essentially it leads to a frequency omega 1. So, it will fall on omega 1.

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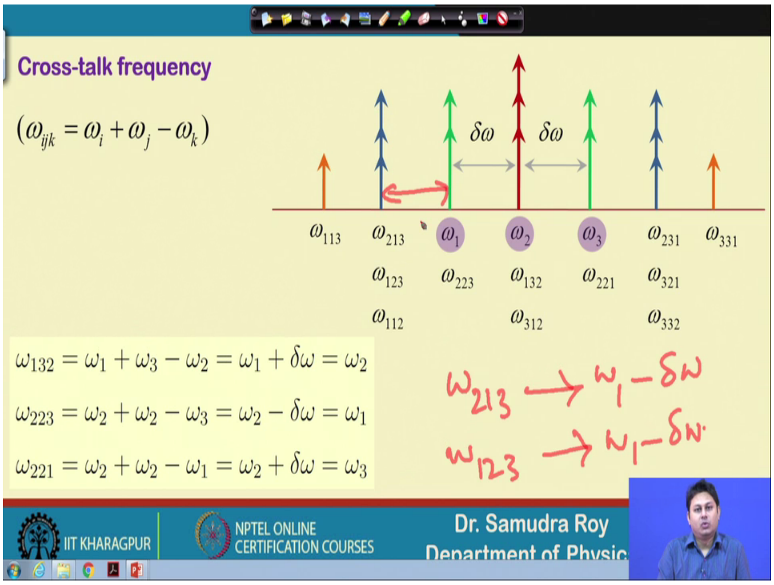

In the similar way you can calculate that omega 2 omega 2 1 3 frequency 2 1 3 will give raised to omega 1 minus delta omega frequency, and also omega 1 2 3 frequency give rise to omega 1 minus delta omega frequency which falls in this side of omega 1.

So, right now we like to conclude because we do not have much time. So, today we you have learned very important concept one is 4 wave mixing. So, we will learn more about the 4 wave mixing in the coming classes and also learn a specific example of cross talk, and how this cross talk create the problems we discuss and how this cross talk frequencies are generated also that is discuss. So, with this note let me conclude the class here. So, in the next class we I will discuss more about the 4 wave mixing.

So, thank you for your attention and see you in the next class.