Introduction to Non-Linear Optics and it is Applications
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Lecture - 48
Cross Phase Modulation (Contd.), Nonlinear Absorption

So, welcome student to the next class of Introduction to Non-linear Optics and it is Applications. Today we have lecture number 48, in the previous lecture we have started 1 concept call cross phase modulation, so today we will going to continue that.

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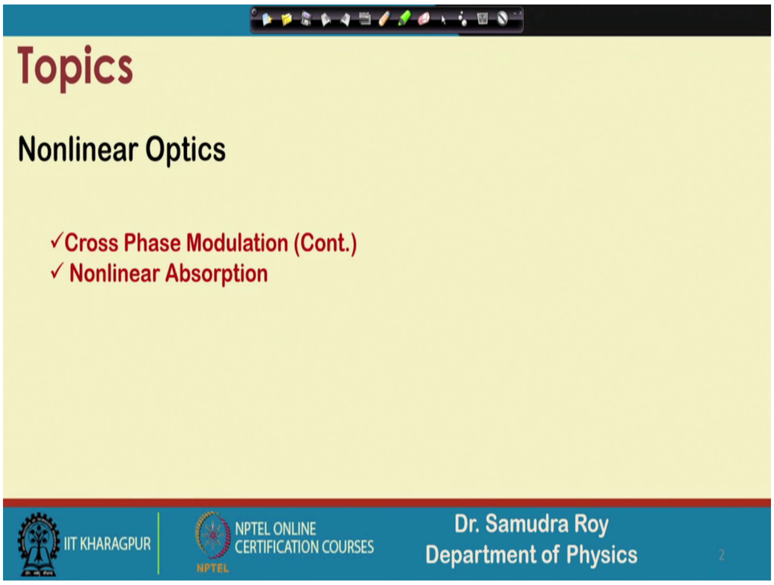

So, in the cross phase modulation is a process where we know that if 2 different waves or light beam are incident in a system. Then one beam will be affected by other one and vice versa. So, in cross phase modulation in the previous class we derive the differential equation of the cross phase modulation and the differential equation of the cross phase modulation we will going to solve today to find out. What should be the evolution of the corresponding amplitude and the phase under cross phase modulation.

So, this is extended version of the self phase modulation, so let me go back to the slide. So, this is the old slide and this slide suggest how this cross phase modulation is going to work.

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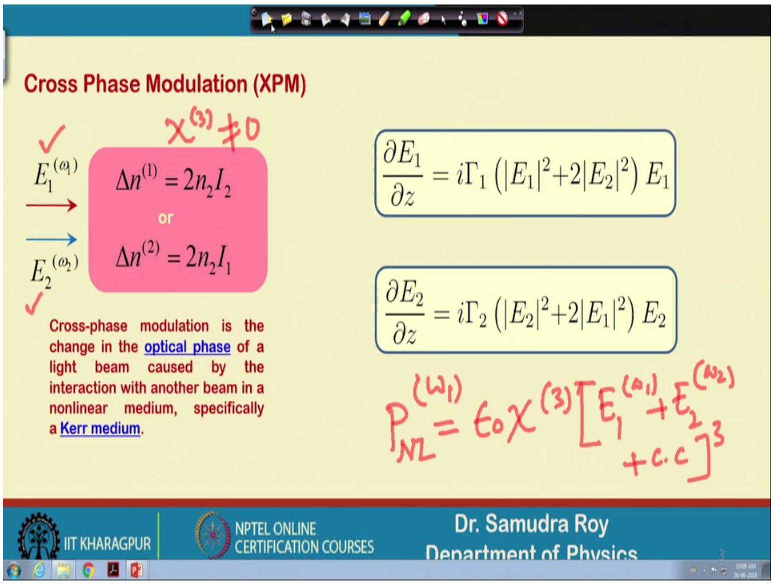

So, let me explain once again what is the feature of cross phase modulation, so cross phase modulation is the change in the optical phase of light beam caused by the interaction with another beam in a non-linear medium specially Kerr type of medium.

So, this schematic picture we already described in the last class again let me give you the idea what is going on. So, E 1 and E 2 are the 2 waves that is propagating together into the medium and this medium is non-linear and third order nonlinearity is there. So, chi 3 is dominating here chi 3 is not equal to 0.

When they are propagating together then non-linear interaction will take place p non-linear will be epsilon 0 chi 3 and then multiplication of E 1 which is the frequency of omega 1 plus E 2 which is a frequency of omega 2 plus complex conjugate of these 2 fields and then cube of that. So, in this way we calculate the p non-linear. Once we calculate the p non-linear then different frequency component will arise one particular case one can have, where in the p non-linear we have a frequency component omega 1 or frequency component omega 2 and how these 2 frequency will come we can readily see with this term.

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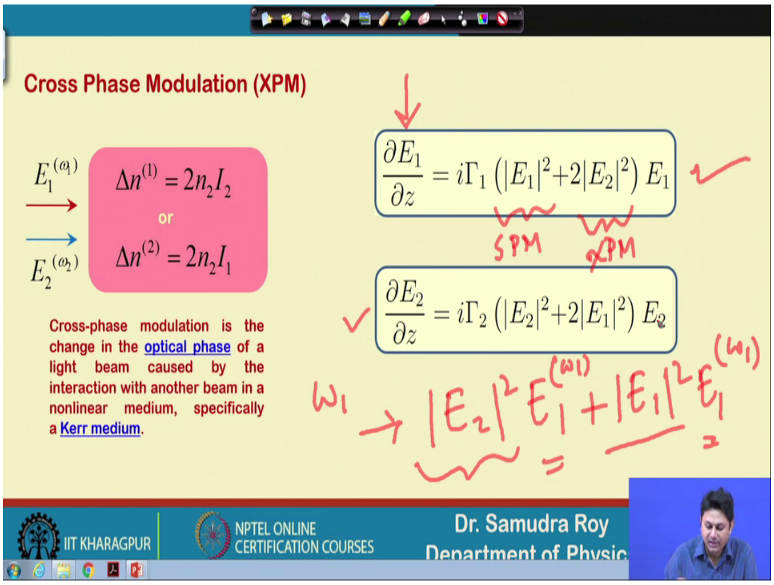

So, 1 term so in order to have omega 1 frequency in the p non-linear essentially we have E 2 square E 1, this term will come on top of that there was self phase modulation term. So, E 1 mod square E 1 term will also be there.

So, if you calculate the corresponding frequency mod of E 1 E 2 square should not have any kind of frequency. So, E 1 term will contain 1 frequency which is omega 1 here also mod of E 1 square will not going to have any kind of frequency because, this is a mod square but E 1 term again will give rise to one frequency omega 1. So, this entire term will have a frequency component of omega 1. So, if we look the differential equation shown here, then you can see that the similar kind of expression 1 2 term should be here because, of this degeneracy factor why these 2 time is here we have already explained in the last class.

But the thing that you remember that when I am looking for the cross phase or self phase modulation term for E 1, there should be a mod of E 1 term here and also a mod of E 2 term is there; the first term mod of E ones term is corresponded correspond to self phase modulation and the next term is cross phase modulation.

So, this 2 term basically there with having a frequency component omega. So, this a naturally phase matched equation so there will be no delta k kind of things because, it will cut from both the sides that you should also remember and you will have not only one term or one equation you will also have another equation for E 2, the same thing will be done for another wave which is propagating along with E 1 with the different frequency say omega 2.

So, we will get a similar kind of expression for E 2 only E 1 and E 2 will replace here and you will get a set of coupled equation, so that is the derivation we have done in the previous class. So, today what essentially we do we just try to solve this and find out what is the effect on amplitude and phase.

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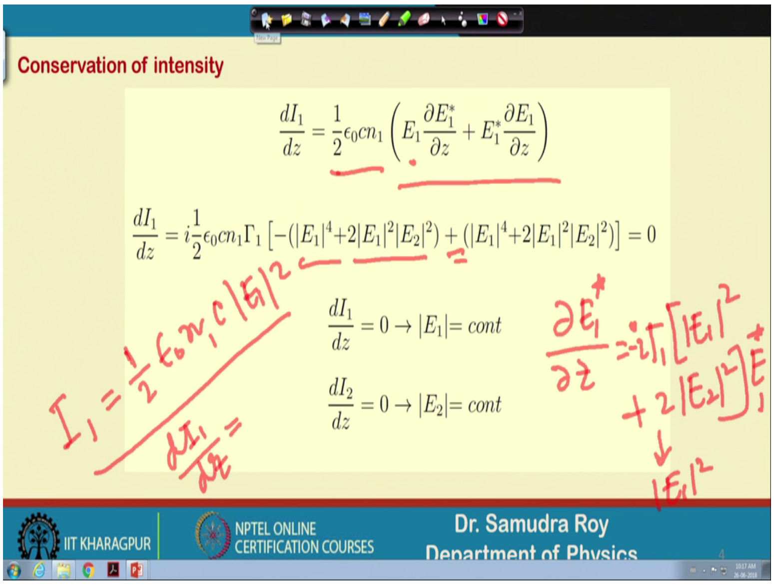

So, let me go back to the next slide, so first thing that if I look these 2 equation carefully delta E 1 del z delta E 2 d z, this 2 equation are coupled and we know that when there is a coupled equation there is a possibility that energy will transfer from one to another and in our third harmonic generation or second harmonic generation process we find that energy is coming from fundamental wave to third harmonic wave or fundamental wave to second harmonic waves. So, energy is moving from one wave to another one wave is acting as a pump and another wave is gaining the energy from that pump. In this case also we have a coupled equation 2 coupled equation the other one is this one and another is this one these 2 coupled equation we have.

So, this 2 coupled equation can exchange the energy really they will going to exchange the energy or the energy will remain conserved that we will going to find in our next slide. Well if I considered the energy in terms of intensity, then the intensity our old equation that we always use the intensity, let me write it here somewhere intensity of wave 1 is equal to half epsilon 0 n 1 c mod of E 1 square. This is the relationship between the field amplitude and the corresponding intensity we are using this equation almost every day, so that I can convert the field amplitude to intensity if you make it more to have the physical to have a more appropriate physical meaning. So, now if this is my intensity and I have 2 equation in our hand.

So, the next thing is to find out whether intensity of a particular wave say E 1 will remain conserved under cross phase modulation or not. So, process is I just need to make a derivative with respect to z of this quantity whatever is written and in the right hand side we will do the same half epsilon 0 n 1 c is a constant term, so it will not going to change so here it is and then mod of E 1 square we will have to make it derivative with respect to z and if we do that then we will find it is E 1 del E 1 star del z plus E 1 star del E 1 del z, so this is a standard form.

So, we have this once we have this the next thing this quantity from the cross phase modulation this quantity is known and we know that this is equal to I gamma 1 mod of E 1 square plus 2 mod of E 2 square bracket close E 1. This is the differential equation we derived in the previous slide, so what we will do you will just put this things here in this equation. If you put this things here in this equation you can see that in the first equation there is a star, so if I make a star of that the right hand side will be the complex conjugate thing.

So, I will be replaced by minus I and this E 1 there will be a star and now if I multiplied that entire thing with E 1. So, this E 1 star E 1 will give rise to another term mod of E 1 square and if I put that then we will have mod of E to the power 4 because this will multiplied by this term and this things will be multiplied by this term. So, we will have another 2 E 1 mod square E 2 square E 2 mod square.

In the similar way if I do the same thing for other term I will get the same thing except this plus sign we will have a plus sign here. So, we essentially find that dI dz if I do this calculation in detail dI dz. So, this term the first 1 and the second 1 will cancel out and we will have the conservation of individual intensity that is interesting in the previous calculation for third harmonic and second harmonic.

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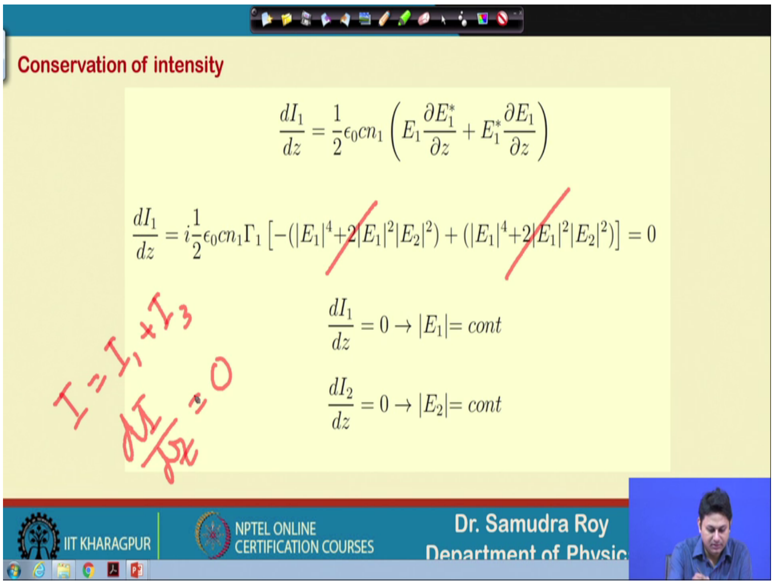

If you remember the total intensity was given by the fundamental wave plus a third harmonic wave and the total intensity was conserved in the system this was 0. But for self phase modulation or cross phase modulation, but we find that the individual intensity will remain conserved; so that means, intensity remain conserved means the amplitude of the fields will not going to change. So, what is the physical meaning of that let us understand once again.

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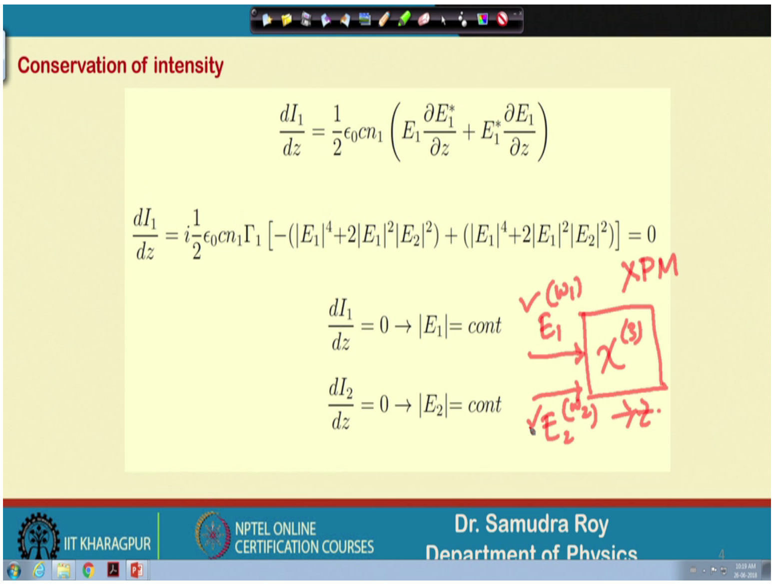

So we have say this is a block with nonlinearity chi 3, I am launching an electric field E 1, I am launching another electric field E 2 E 1 is containing a frequency components say omega 1 and it is containing a frequency component say omega 2 both 2 are launched here.

Now, our question is whether the amplitude of these 2 things will going to evolve over this distance z or not and we find that at least under cross phase modulation cross phase modulation, the individual amplitude will remain conserved. So, there will be no change of amplitude only thing that will going to change should be the phase because, we already have a coupled differential equation in our hand, so we will going to find that how the phase will going to change. So, first thing we find that in cross phase modulation the amplitude is not changing or in other way the energy remain conserved, not only that the individual energy of each wave is also conserved.

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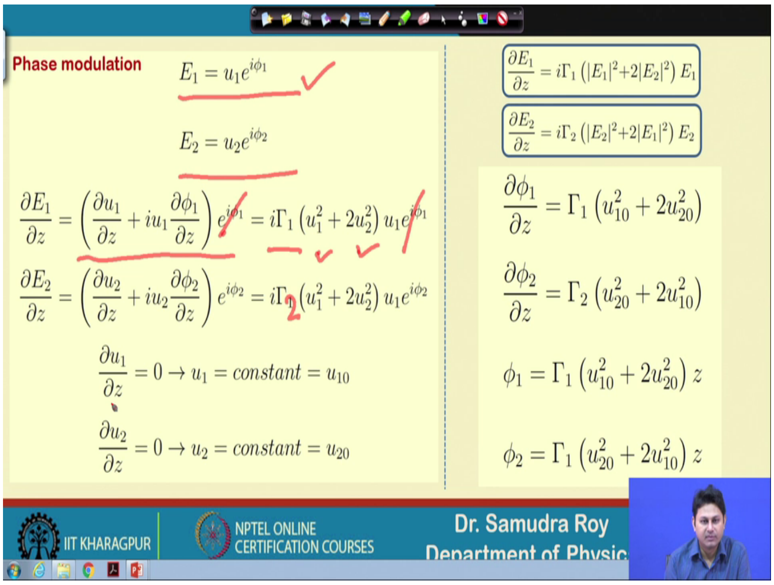

So, once we have the knowledge that individual wave are conserved, the next thing that will going to do is to find out what will be the effect on phase. So, phase modulation we try to understand under cross phase modulation. So, in order to understand the evolution of the phase or how the phase will going to change, we need to divide our total electric field the launched electric field into 2 part.

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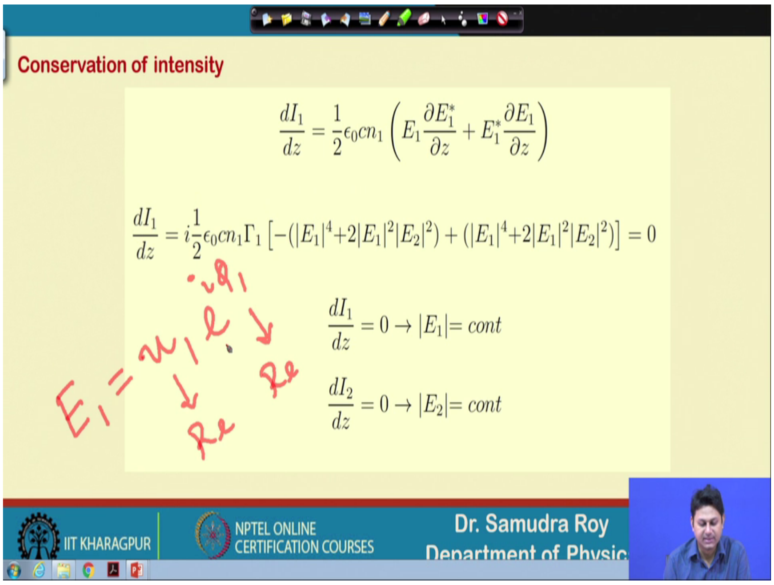

So, here this is the previous slide, so total electric field E 1 this is a complex this is called a complex amplitude, because it should have a amplitude and also a phase. So, if I divide this into 2 part it will be u 1 E to the power of I phi 1, this is the standard way to divide any complex amplitude.

Because we know that E 1 is a complex quantity once E 1 is a complex quantity then what we do we just divide into 2 part the amplitude part and the phase part if you look carefully here we divide this into 2 part u 1 and I phi 1, u1 is a real quantity and phi 1 is also a real quantity and they are written in the standard way the way we always write a complex term it is Re to the power I theta is something like this. Well once we divide the complex amplitude into 2 part real amplitude and the phase part real amplitude and the phase part.

We supposed to have 2 equation in our hand when we apply these 2 electric field to our coupled equations. So, this in the right hand side if you look in the boxes we have our equations, these are the coupled equations we derived d E 1 is equal to I gamma 1 mod of E 1 square plus 2 mod of E 2 square multiplied by E this is our equation. So, what we will do now we have the field in terms of real amplitude and phase. So, we will going to put these things here. So, when we put this things here, so d E 1 d z the derivative of E 1 with respect to z will be this quantity because, it is divided into 2 part E 1 and E 2 the power of I phi one both are function of z. So, d u 1 d z plus I mod of u mo[d]- u 1 d phi 1 d z E to the power of I phi 1 just making a derivative with respect to z of this term.

In the right hand side we have I gamma 1, so I gamma 1 will be there then mod of u 1 square mod of E 1 square is nothing but u 1 square which is amplitude of that, 2 of mod of E 2 square is similarly 2 of u 2 square where u 2 square u 2 is a amplitude of the field E 2 and then the total field E 1 E 1 is u 1 E to the power of I phi 1.

So, from this equation we can see that E to the power I phi 1 E to the power I phi 1 will cancel out. By the way in the similar way you can calculate the equation the amplitude and phase equation for E 2. So, the amplitude and phase equation of the E 2 will come exactly in the same way and it will have the almost similar form except few changes so here is a mistake. So, this coefficient for E 2 will be phi 2 this is again typing mistake it was written gamma 1.

So, it should be gamma 2 so if you look this equation carefully. So, E to the power I phi 1 E to the power I phi 2 will cancel out, then the rest of the equation in the right hand side we have a real part and imaginary part, in the right hand in the left side in the right hand side we have a real and imaginary part also; but you can see that u one is real u 2 is real the square of these 2 are; obviously, real gamma was is real u 1 is real, only thing is everything is real with a multiplication of i, so right hand side is completely a imaginary term. Now in the left hand side we have a real and imaginary term, so obviously if I match these 2 things. So, we know the theorem that if a complex quantity is equal to another complex quantity the real term must be equal to the real term of the left hand side.

The imaginary term in the left hand side will be equal to imaginary term on the right hand side. So, here we are using the same principle so that means duz du 1 dz has to be 0 because, there are no real part in the right hand side of this equation. So, from here we readily find that u 1 is a constant, that means there will be no change of the amplitude of u 1 and whatever the value we have in the input it will remain conserved.

So, this is not a new outcome because in just in the previous slide, we have done the same thing that the intensity of a individual wave will remain conserved under cross phase modulation, so here we are doing a similar kind of stuff. In the similar way we find that u 2 is also conserved or constant, so amplitude will not going to change what about the phi equation that is interesting. So, in the this right hand side we write the other equation by equating the complex the imaginary parts and you find the d phi 1 dz is gamma 1 u 1 square u 2 square.

So, u 1 u 2 was conserved, so that is why write u 1 0 square 2 u 2 0 square and phi 2 is gamma 2 u 2 square u 2 0 square plus 2 u 1 0 square of that, so we have a differential equation of phi and if we solve and it is non 0; obviously, if we solve we find that phi 1 is this quantity which is a constant multiplied by z. So that means, phi 1 will going to change linearly with respect to z. So, similar thing we have already figure out when we are discussing about the self phase modulation, how the phase of the pulse will going to modify under self phase modulation.

Here we are having a similar kind of expression in our hand only thing the additional term here is due to the cross term or the other wave. So, we find the phase is modulated by the pulse itself as well as the pulse that is launched together to the system. So, we will have a phase equation which we derived quite easily and find that under self phase modulation how the phase will going to modulate and it will change linearly with respect to z.

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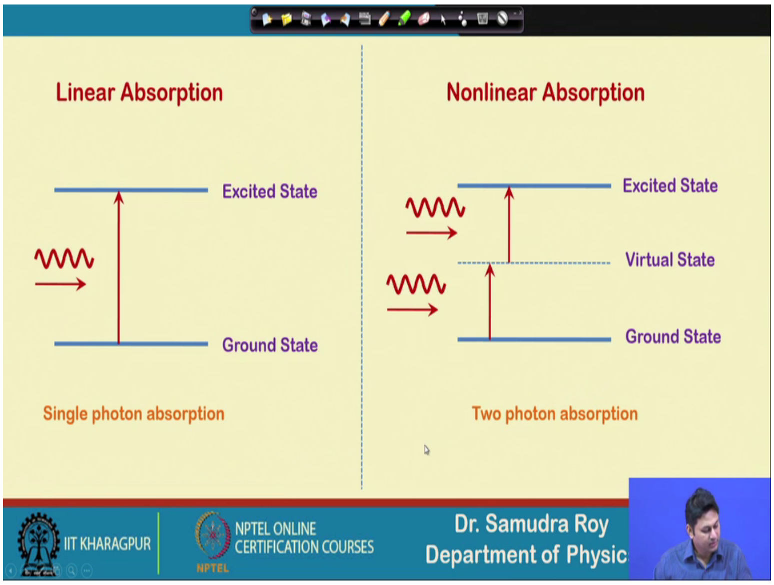

Well next let us try to understand 1 important thing and that is the non-linear absorption. So, in the linear absorption is essentially a single photon kind of absorption. So, today we will going to learn the non-linear absorption, but before that let us give a very very rough idea about the linear absorption. So, when a linear absorption is there so light will going to absorb with a single photon phenomena and then it goes to the molecule goes to a the atom goes to the excited state.

In non-linear absorption normally 2 photon are involved as shown in this figure. So, there is a virtual state 2 photon are incident to the system and they will absorb and some kind of excitation will appear here since 2 photon photons are involved this is related to the intensity square of the system.

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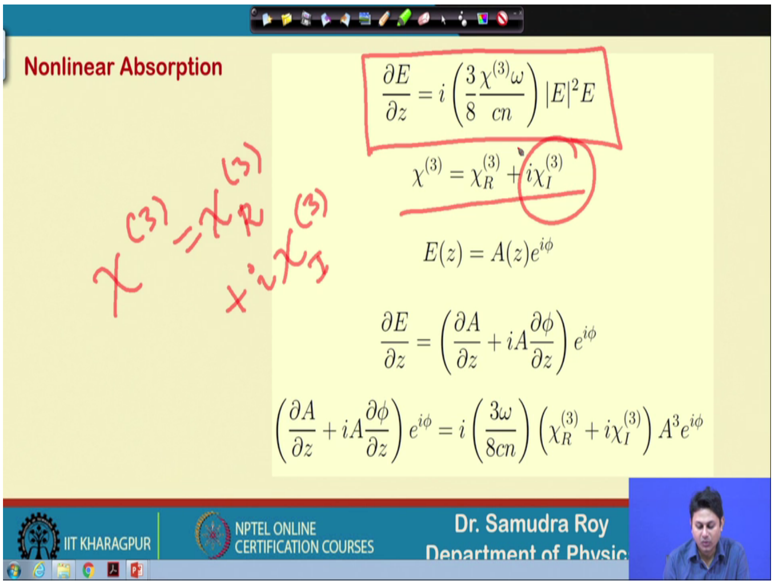

So, we will we will see that how this things will happen, so this is the equation through we start with the equation that we derive for self phase modulation. So, this is the equation that we have derived for self phase modulation this is a naturally phase matched equation and once we have a naturally phase matched equation. So, there will be no E to the power I delta chi kind of term, so only the exponential terms are there.

So, now what we will do we just write this susceptibility third order susceptibility into it is complete form which is the real and imaginary part. So, far we are dealing with only the real part. So, now what we will do that we will introduce the imaginary part also this is the complete form of the third order susceptibility, it is a real part and also imaginary part.

For first order susceptibility also we have done the treatment, where we write it is a real and imaginary part and find that once we introduce the imaginary part. Then basically this give rise to some kind of loss in the system here a similar kind of effect we will going to find. So, we now divide the entire thing into real and imaginary.

So, write a total form of susceptibility and now we put this susceptibility term to here. So, now our equation is chi in place of chi 3 now I write chi R 3 plus I chi I 3; total field again divided into 2 part amplitude and phase here I write the amplitude a which is the function of z and also phi is supposed to function of z also and now d dE dz if I do the derivative because, in the left hand side I need to have this quantity it is nothing, but da dz plus I a d phi dz E to the power I phi.

This is the same treatment that we have done in the previous slide, so just make a derivative just take a amplitude and phase part and make a derivative and you will get these things. Now we put this quantity here in the left hand side like this and in the right hand side we write everything and also expand the susceptibility term here, mod of E 1 square is simply E cube mod of E 1 square in multiplied by E is simply E cube E to the power I phi.

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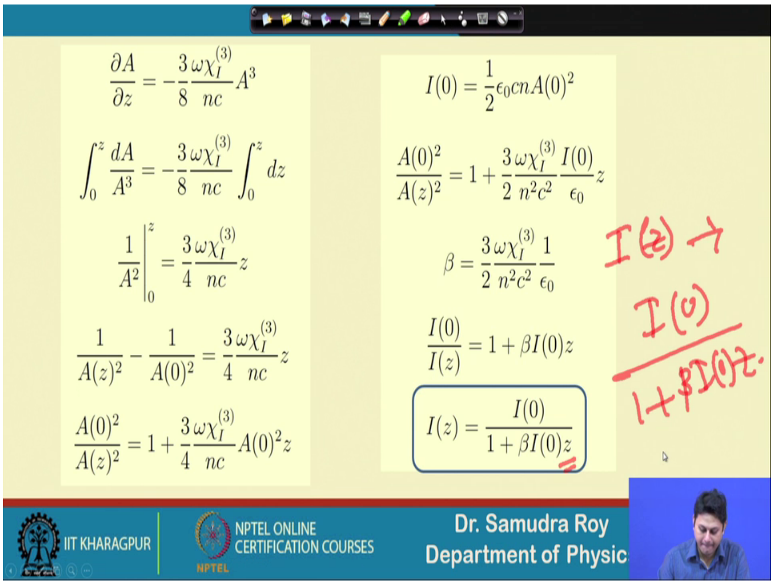

Then once we have this expression, now the next thing that we can do we can divide into real and imaginary part and once we divide into real and imaginary part you can see that this real part E to the power I E to the power I will cancel out and this part E to the power dA dz will now have one equation in previously it was 0.

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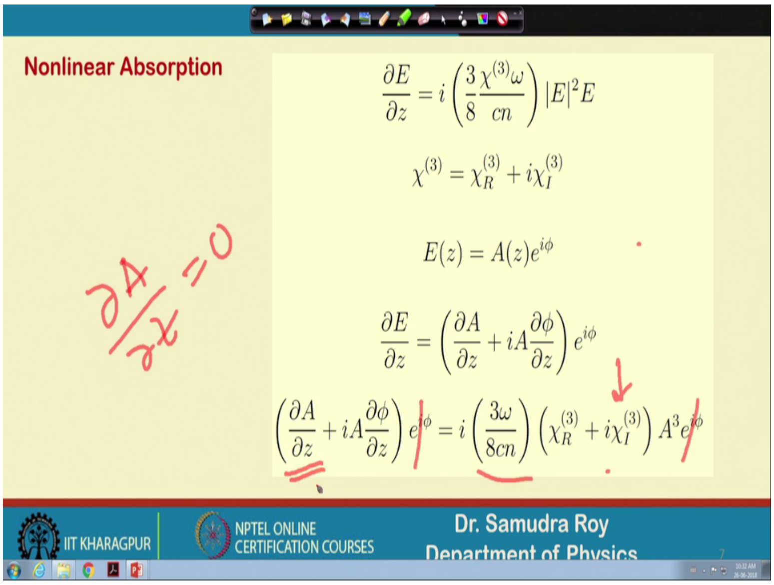

If you remember for self phase modulation or cross phase modulation because, amplitude was not changing; but as soon as you introduce this term you can see that this there is a contribution for this term and we will have this equation in our hands. Amplitude there is a differential equation of the real amplitude and it seems that it will going to change with z. So, the next thing is that to just integrate this differential equation to solve this equa[tion]- differential equation by directly integrating. So, if you directly integrate this differential equation, so it will be da a cube and right hand side this is a constant integration 0 to z.

So, this integration gives you 1 by a square with the limit 0 to z then 3 by 4 and then omega the right hand side is just multiplied by z, do some simple algebra and you will basically have an expression like this a 0 square divided by a z is equal to some quantity.

So, this is the evolution equation of the amplitude. So, now if I try to write in intensity in terms of intensity this a 0, I can replace in terms of intensity and also this ratio I can write in terms of intensity also. So, this ratio basically in terms of input intensity is something like this and now I defined a coefficient beta which is 3 by 2 omega chi 1 3 into square c square and 1 by epsilon like this and this ratio is simply the ratio of the intensity and then if I make a slight calculation then I will have an expression like this.

So, this suggest that if we look carefully the expression, this expression suggest that due to the presence of the imaginary susceptibility term third order susceptibility term which is here in term in inside the beta the intensity will going to change and it will decay.

So, I z is decaying, so it will be decaying like I 0 divided by 1 plus beta I 0 z, if I increase z sin[ce]- than the denominator will going to increase and you will find the intensity will also decrease.

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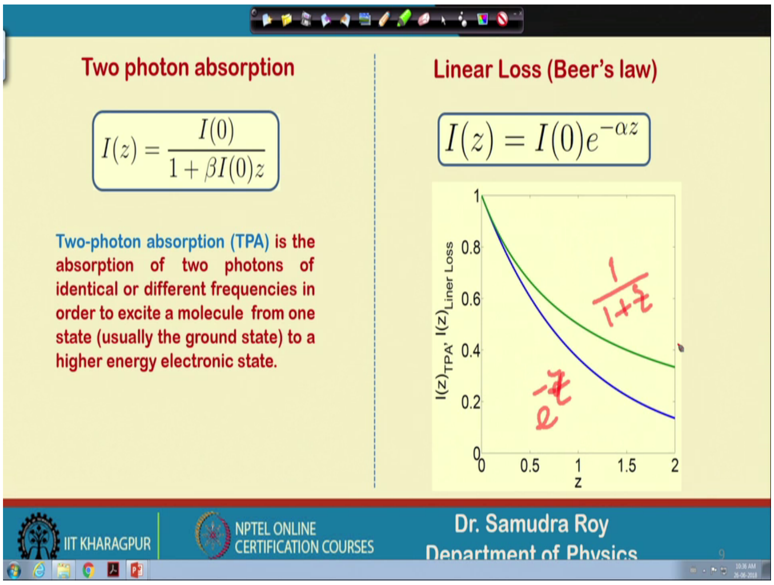

So, 2 photon absorption we find an expression which is this. So, in 2 photon absorption is the absorption of 2 photon of identical or different frequencies in order to excite the molecule from 1 state to the higher energy electric state. So, in that case intensity is all changing and the change of intensity is following this rule this rule, but also we know the linear loss rule which is normally call the beers law.

In beers law we have an expression like this which is exponential decay, now if I compare these 2 decay both the cases the intensity decaying it should it should be something like this here this is a exponential decay which is a little bit faster and this is corresponding to 1 divided by 1 plus z this is the decay. If I write this this is the expre[ssion]- this is the schematic is just to show the idea that how these 2 decays will be there.

So, it is the exponential decay, so E to the power minus z and this decay is 1 plus 1 by z this is the form of the decay both the cases, if I increase z the amplitude or the value will going to decay, but the nature of the decay is not particularly same exponential decay will decay faster with this form where as in other form the 2 photon absorption form the decay will be there, but not as deep as the exponential term.

So, with this note so let me conclude today's class. So, today we have learnt 2 important thing 1 is the cross phase modulation, the phase is going to modulate the amplitude will not remain conserved and next thing is the non-linear absorption or the 2 photon absorption where the susceptibility terms has a imaginary component or the third order susceptibility. If the third order susceptibility have some imaginary component what happened that due to these things we have some sort of absorption, so this absorption is non-linear in nature and that is why it is called the non-linear absorption. So, let me conclude my class here.

Thank you for your attention and see you in the next class.