Introduction to Non-Linear Optics and its Applications
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Lecture - 47
Third Harmonic Generation (Contd.)
Cross Phase Modulation (XPM)

So, welcome student to the next class of Introduction to Non-linear Optics and its Application. Today we have lecture number 47 and in the previous lecture, we tried to understand under pump depletion condition; how third harmonic wave should evolve. In order to calculate that thing; we assume that there should be some quantity that is a constant of motion.

And we find that if this constant of motion is there then from that we can extract few information and we find a differential equation of u 1 and u 3; where u 1 and u 3 was the amplitude of the corresponding fundamental and third harmonic wave. So, equation is little bit modified; so we will start from that point.

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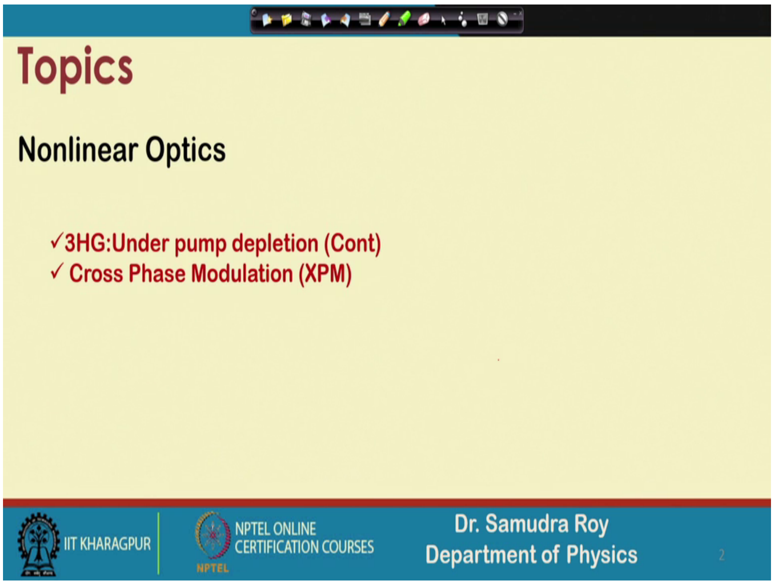

So, in today's lecture; so we will the third harmonic generation under pump depletion condition this calculation I will like to finish today. So, the continuation of that thing and then we study something called cross phase modulation which is very interesting.

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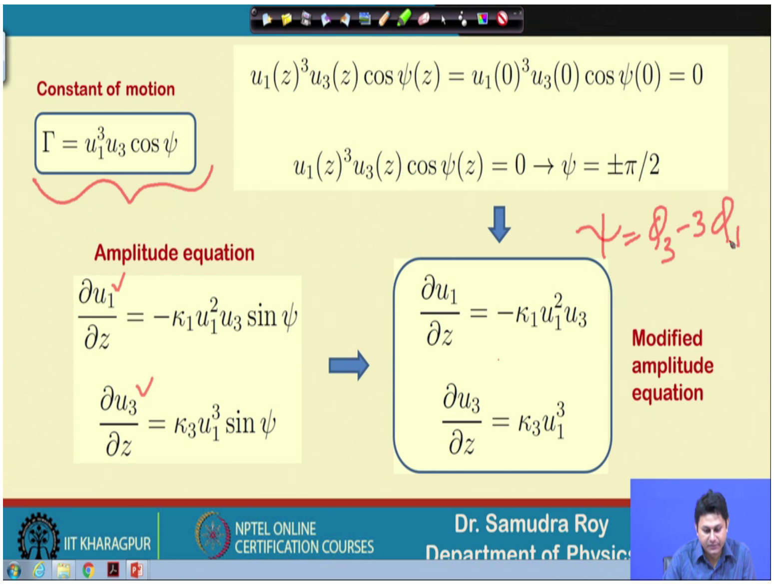

Well, so this is the expression that we had in our last class. So, constant of motion as I mentioned there is one quantity that is seems to be not changing with respect to Z and that is this. So, amplitude equation is a equation that gives you how the amplitude u 1 and u 3 amplitude of the fundamental and amplitude of the third harmonic will going to evolve; they are coupled to each other. So, that is why you can see in the right hand side there are coupled equations. And this constant of motion give us an additional information regarding the relationship regarding the value of psi, which is which is the relationship between the phases of these 2 waves remember psi is equal to phi 3 minus 3 phi 1.

So, there is a phase relationship psi basically gives us a phase relationship. So, this phase relationship suggest that if the value of these quantity is pi by 2; once this is once we know the value of this is pi by 2, we have a modified expression in our hand as far as the amplitude of the fundamental wave and third harmonic wave are concerned. Once we have this equation; next thing is to solve this that is the main issue here how to solve this coupled equation?

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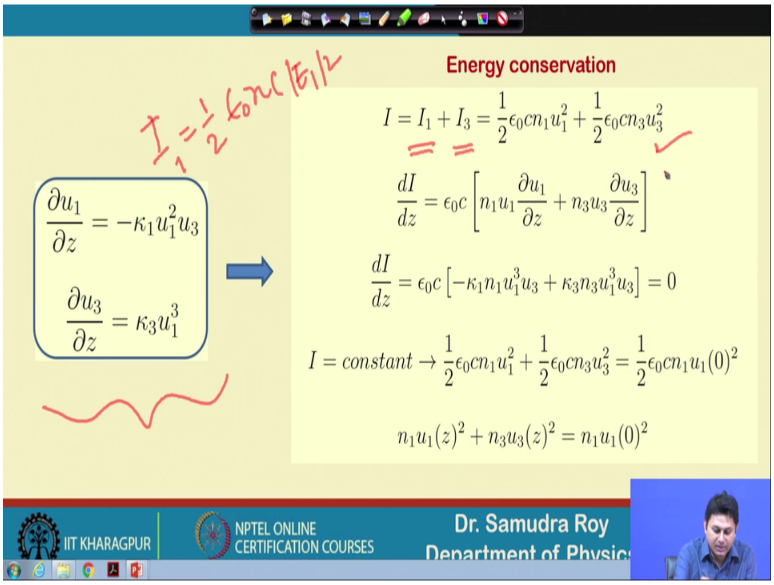

So, in order to solve this; we need to find few more things that what this equation suggest. These 2 equation whatever we have here; can gives a very important information and that information is the intensity or the total energy of the system remain conserved. So, if there is no loss these 2 equation contain does not contain any kind of loss; only thing these 2 equation suggest that some of the energy can come from u 1 to u 3; that means, third harmonic will generate and third harmonic will generate as the as a result of the as the decrement of the fundamental waves.

So, fundamental wave basically give some come kind of energy to third harmonic wave. So, that the third harmonic wave will going to evolve. So, there is some sort of energy exchange between these 2 quantities u 1 and u 3. So, they are coupled with each other that is why; so, even though the energy is changing from one to another, the total energy or the total intensity if I write in terms of intensity should remain conserved. So, there should be any kind of loss of intensity or any kind of gain of intensity with this process; if I say this is a total intensity. So, total intensity is I 1 plus I 3, intensity of the fundamental wave plus intensity of the third harmonic wave.

So, I can write it is half epsilon 0 cn 1, u 1 square because this expression every day almost every day we are using this expression. So, by that time I believe you are now in a position to appreciate this expression; if I it is 1, it is square. So, u mod of u 1 square is nothing, but u 1 square mod of u 1 square is nothing, but u 1 square.

In the similar way mod of E 3 square should be equal to u 3 square. So, these 2 equations we have and once we have this; these things then the next things to find out whether the I will going to change or not with respect to z; we make it derivative over that. So, once we make a derivative; so I have the derivative with respect to u and the derivative with respect to u 3 and u 1 like this. And now the derivative with respect to z; d u 1, d Z I can put the value from here. In the similar way, del u 3 del z; I can put the value from here once I put this value this from here and put is put this here.

Then we will readily see that this quantity will be 0; that means, I the total intensity remain conserved. If the total intensity remain conserved then again we can write that half of epsilon 0 c n 1, u 1 square at any Z point plus half of epsilon 0 c n 3, u 3 square at any Z point should be equal to whatever the intensity we have in the input. So, these quantity is intensity at input and you can see that this quantity only have u 1 square and this 0 suggest that it is at Z equal to 0; why 1 only one term is here? Because there was no third harmonic wave at Z equal to 0 point; so, that is why we have only one term.

So, from this equation I can have relationship between u 1, u 3 and u 1 0 which is the amplitude of the fundamental wave at Z equal to 0 point which is non zero. So, n 1 u 1 square plus n 3 u 3 square is equal to n 1 u 1 0 square where n 1 u 1 0 is a amplitude at Z equal to 0 point. So, this is an additional equation or additional relation we have which help us to calculate or the solve this differential equation. So, let us this coupled differential equation rather.

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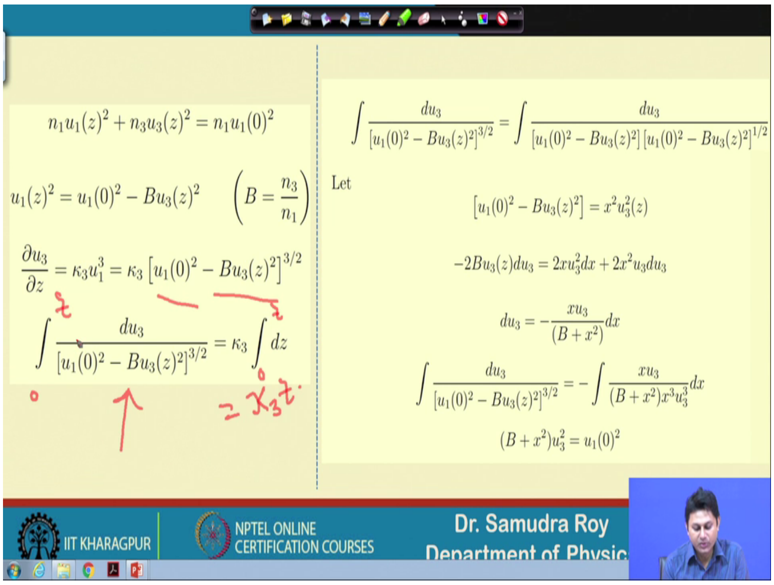

So, let us try to find out how to solve this coupled differential equation and what kind of expression we have under pump depletion condition; the evolution of the third harmonic rather. So, we have this expression this is the expression that we have just derived; now from here we can write u 1 in terms of u 0 and u 3. So, u 1 square is u 1 0 square; I just divide everything with n 1 and put this u 3 to that side. So, I will have this and the ratio of n 3 and n 1 I write this constant quantity as B. So, my u 1 square is now equal to u 1 0 square minus B u 3 square.

Now, go back to our equation for this third harmonic wave. So, if I go; so this is the equation I am looking for. So, I need to solve first this equation; so, here u 3 is here while in the right hand side we have u 1. So, if I somehow able to change this u 1 in terms of u 2 in terms of u 3; then we can solve this. So; that means, we are try to decoupling it and this decouple process I can use this relation that we essentially we are doing. So, del u 3 del z is equal to kappa 3 u 1 cube and just above we just write u 1 in terms of u 3. So, if I now replace this u 1 with this u 3; we will have u 1 0 which is a constant term minus B u 3 square whole to the power 3 by 2.

Now, you can see that this equation u 3 in the left hand side and right hand side also we have u 3. So; that means, we can do this integration and if we write this integral form and then it will be d u 3 divided by this quantity; in the right hand side it will be kappa 3 integration of dz. Now the next challenge is to just find out the solution or the integration of this quantity; how do how to integrate this? That is only challenge because in the right hand side this quantity is nothing, but kappa 3 multiplied by Z.

If I integrate to from 0 to Z both the cases I am integrating it from 0 to Z. So, this will be the limit, but before that we need to know how to how to basically solve this integration, how to integrate these things. So, we have in the right hand side the solution is basically here. So, d u 3 divided by u 1 0 whole square minus B u 3 z square whole to the power 3 by 2; I can write in the 2 part. One is this and another is this 3 by 2 I can write is 1 plus half in this form, this is the standard way to write this form. Once we write this things, then I took u 1 0 minus B u 3 square this quantity; that means, entire this quantity as x square u 3 z square of that.

I just replace or I just replace these things to another variable x and write in this way. Now if I make a derivative of both the side and the left hand side I can write minus B; u 3 del u 3 is equal to in the right hand side because both are the variables now. So, 2 xu 3 square dx plus 2 x square u 3 du 3; from here we can find interesting expression that d of u 3 is xu 3 divided by B plus x square dx. So, my entire equation is now transferred to dx. So, du 3 this quantity is simply xu 3 divided by B plus x square and in the denominator also we have this quantity u 3 minus B; u 3 Z square whole to the power half.

This quantity is nothing, but cube of this quantity because if I write to the power half and then it should be this entire things is 3 by 2. So, this 3 by 2 basically come here; so, du I replace when I replace here du. So, du will be replaced by this part and the rest part is basically whatever the thing we have in the denominator and in denominator we have x square u 3 x cube; u 3 cube. Now if I look carefully these things; so B plus x square then u 3 square is equal to u 1 0 square.

From here, I can write these things straight way; if I write this B this side then B plus x square if I take common u 3 square; then B plus x square u 3 square is nothing, but u 1 square. In the numerator you can see that we have B plus x square multiplied by u 3 cube. So, if I write B plus x square u 3 square to u 1 0; so this is constant term and another u 3 is sitting here, but in the numerator we have another u 3; so that will cancel out.

So, if this is cancelling out; so we will have 2 here and this I can replace and it will be simply u 1 square. So, the total expression the total integration becomes quite simple.

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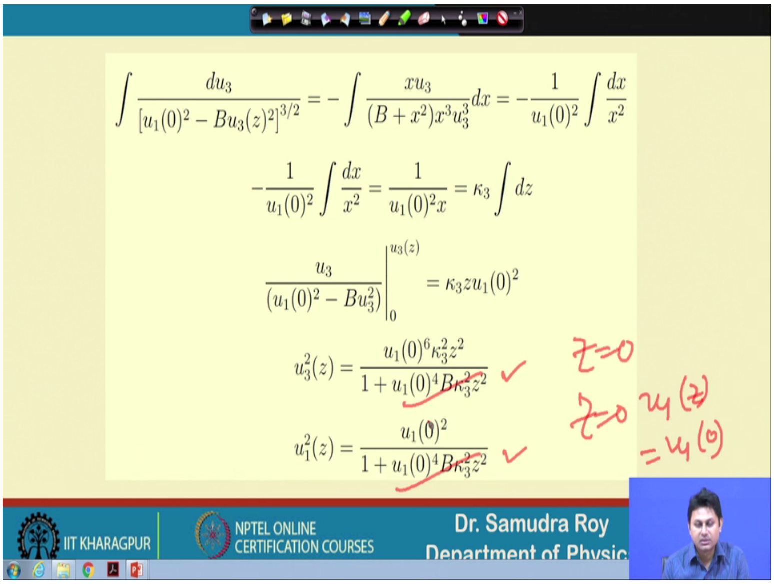

And here in this slide we can see that integration of du 3 divided by u 1 0 square minus Bu 3 z whole to the power u 3 z square whole to the power 3 by 2 with the proper with the proper transformation, I can write this in this simple form. So, now du 3 can be replaced by dx; where the transformation was if you remember that u 1 0 square minus B u 3 square is equal to x square, u 3 square that was the transformation I used. So, now we are in a position to solve this straight way. So, if I now integrate this things it will be simply this negative sign will absorb and we will have simply one divided by u 1 0 square x; that should be equal to kappa 3 integration of dz that we have from the beginning.

Now, we replace this x in terms of u 3; so if I replace this x 3 in terms of u 3; in the right hand side it is simply k 3 kappa 3 zu 1 0 whole square with the limit 0 to u 3 in the left hand side. And if you do a simple calculation and take u 3 one side then you will have an expression u 3 like this. So, this is your the your the solution; this is the solution of u 3 and this solution you can do by just simply doing this integration and this basically give us how the u 3 or the third harmonic will going to evolve.

Once you know what is your u 3; then readily you can calculate what is your u 1 because we have an equation in our hand, we have a relationship between u 1 u 3 and this is the equation I am talking about. So, this equation you will have; now you know what is your u 3. So, you just put your value of u 3 and you can readily find out what is your u 1. So, once you do that; so, I want the student to do that calculation by your own hand and check whatever is written here is a correct thing or not. Once you do that you will have an equation like this equation like this.

So, now if I put the boundary condition and check what is happening; as soon as I get the expression of Z. Then the next thing is to find out what is boundary condition and if you look very carefully about this u 3 equation; the boundary condition suggest that at Z equal to 0 point Z equal to 0 point, u 3 should vanish. It is really the case? Yes it is really the case; u 3 if I put Z equal to 0, this quantity will not be will vanish. So, it will be 0 and in the numerator we have Z equal to 0.

So, this quantity will 0 entire quantity will 0. Again another boundary condition is at Z equal to 0; u 1 z should be u 1 0. So, u 1 z solution is also in our hand; so if I now put Z equal to 0 you can see that this quantity will not be there. So only u 1 0 will surviving; so, we will have both the boundary condition satisfied with whatever the expression we have in our hand. So these 2 are the expression of the evolution of u 1 and u 3 under pump depletion condition for third harmonic generation.

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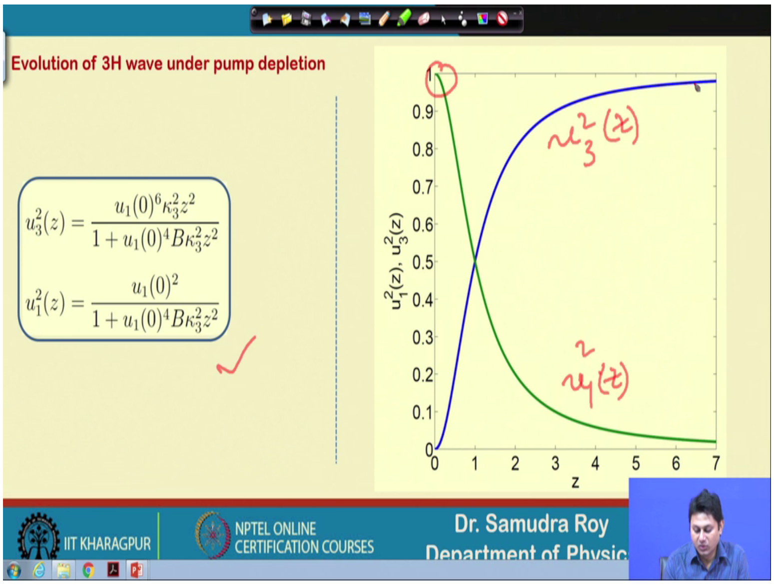

So, now the next thing is to plot these 2 equations as a function of Z. If I plot these 2 equation as a function of z; you can see that this kind of figures are here. This is a quite physically justified figure because this green line suggest how u 1 is going to evolve; this is for u 1 evolution of u 1 u one is a function of z, it will going to evolve like this rather I am plotting u 1 square. And the blue line for u 3 square z. so, u 3 is going to evolve in this fashion.

So, quite this is a quite expected thing that when I launch the fundamental field here at Z equal to 0 point; the total energy was here which is the fundamental wave. And gradually what happen is fundamental wave is decaying the energy or intensity is decaying and as a result what happened? This energy will use to feed the third harmonic wave; so, third harmonic will going to evolve, but it will not going to evolve to in a infinite way.

So, it will evolve and go to a saturation point which is quite justified because we know that when it is vanishing; so there is entire power is now converting from this to this. So, it will not going to increase more like we have in the previous statement; it is increasing is a parabolic way, it will not be like a parabolic way it will go and then saturate. So, once we calculate the pump depletion condition then only we can have this things in our hand.

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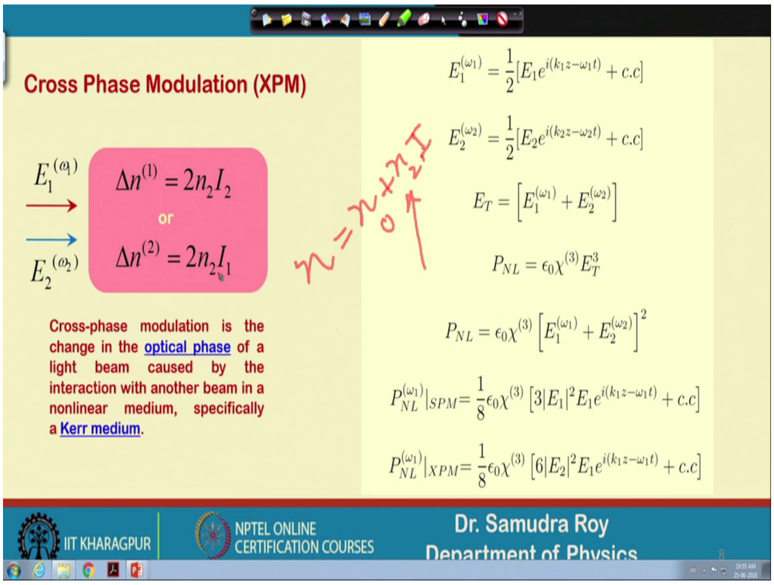

Well, now the next thing I like to discuss is the cross phase modulation. So, far we are dealing with third harmonic generation and the discussion of the third harmonic generation is almost finish. So, we now go to another important phenomena which is called a cross phase modulation; well the cross phase modulation is nothing, but the extended version of self phase modulation; in the self phase modulation what happened if I launch an electric field.

This electric field will modify the refractive index by its own; that means, if I launch an electric field; if you remember the expression refractive index this a Kerr effect was n 0 plus n 2 I. So, I is a intensity of the launched light; when the light is launched the intensity of the light basically leads to the change of refractive index and this change of refractive index is coming through this Kerr coefficient n 2 if n 2 is non vanishing it has a significant value.

Then we have a significant change of refractive index and this is happening because the pulse itself is modifying its refractive index. But what happened if in that particular system we launch not 1 wave, but two wave as shown here E 1 and E 2 with frequency omega 1 and omega 2. So, what happened for this 2 wave that the refractive index is also modified and this modification will be 2 fold; one modification will be due to its own way with this. And on top of that what happened? Because of the presence of other wave also the fundamental wave if I say; this is a fundamental wave this fundamental wave will experience a change of refractive index due to the presence of other wave.

And here the change of refractive index is shown is 2 n 2 n I. So, you can see that n 2 n I is a is a change of refractive index, but the refractive index will change for fundamental wave; that means, fundamental wave in one suffix suggest that it is for E 1 wave. So, E 1 wave will going to see the refractive index and change this change will be done by other wave and it is vice versa. Also for other wave; E 2 wave the refractive index n 2 will going to change and the reason is the other wave or the fundamental wave I 1.

So, this is that is why it is called cross phase modulation; the phase modulation is still there, but it is crossing over, the one is we are basically manipulating light by another light. So, by definition if I write the cross phase modulation is the change of optical phase of a light beam caused by the interaction with another beam in a non-linear medium or specially a Kerr medium. Because in Kerr medium we have n 2 not equal to 0; so, that is why for Kerr medium it is more evident. Now, the calculation wise it is straightforward; so, E 1 and E 2 are the 2 waves that we have. So, I can write this 2 as a plane wave; then total electric field is E 1 omega plus E 2 omega. So, non-linear polarization when I calculate it should be E T cube; so E T is a total.

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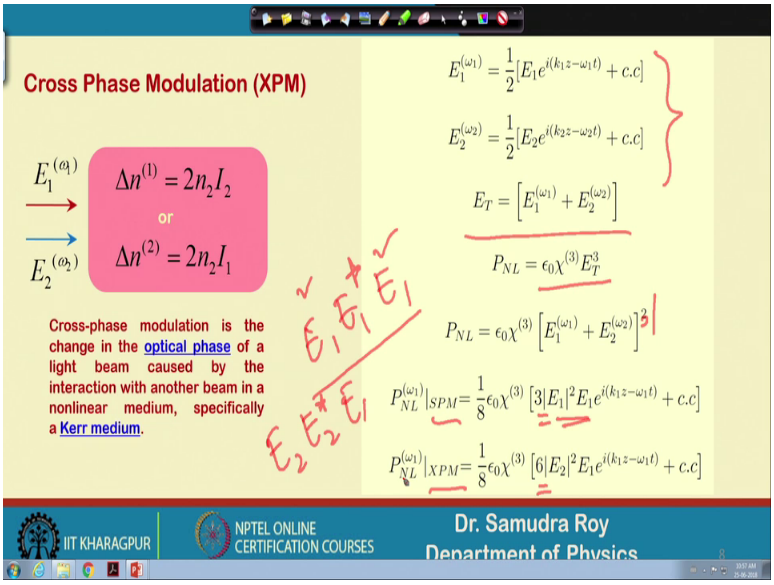

So; that means, I am making it; so sorry here should be a cube it is not square it is a mistake. So, once we make a cube of these things; so, there will be many terms. So, the terms that will contain 3 E 1 square E 1; so, we know that this degeneracy concept that if these 2 fields are different then we have a quantity 6 degeneracy factor; if anyone is same then it is 3. So, you can see it is E 1, E 1 star and E 1 these 3 combinations can give rise to this mod of E 1 square E 1. So, this E 1 and E 1 are same; so that is why we have a degeneracy factor 3 here.

For this case we have E 2, E 2 star and E 1; 3 distinct fields are there that is why the degeneracy factor 6 is sitting here. So, we can calculate the self phase modulation and cross phase modulation term with P non-linear.

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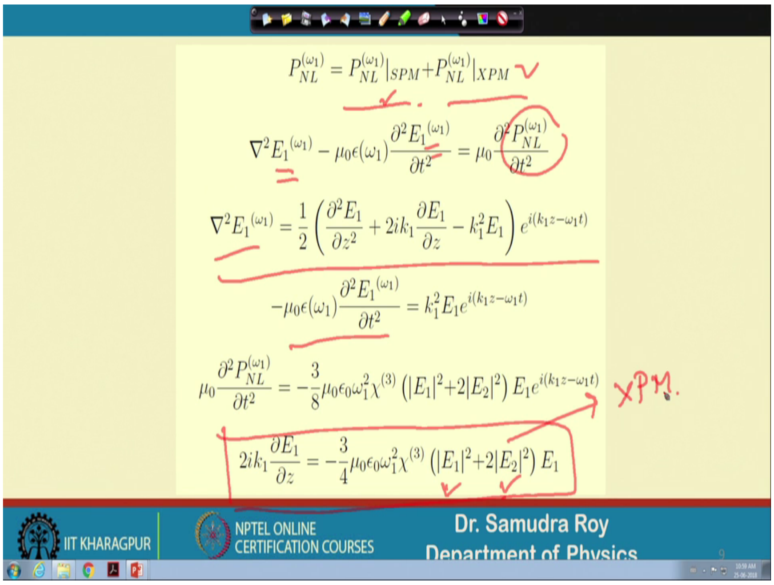

And once we have this P non-linear term in our hand, then we can readily solve the non-linear Maxwell's equation. This equation we have solved several time only thing is that in the source instead of having P self phase modulation; we have self phase modulation plus cross phase modulation term; both together.

So, now these two things I will put my non-linear Maxwell's equation, this is a old process that we have been doing. So, if I want to find out the evolution of only E 1 waves; so it will be E 1, E 1 and this is our source term which contain both cross phase and self phase modulation. This treatment is almost similar the way we calculate the self phase equation for self phase modulation only another additional term is there.

So, we calculate this quantity and then we calculate the next one and eventually we have an expression like this. Interesting thing is that instead of having one term here; we now have two terms and this second term is basically the contribution of cross phase modulation. The treatment is as I mentioned straightforward for self phase modulation we are doing the similar kind of treatment, here we are doing the same thing and basically this leads to a straightforward expression and this straightforward expression is so.

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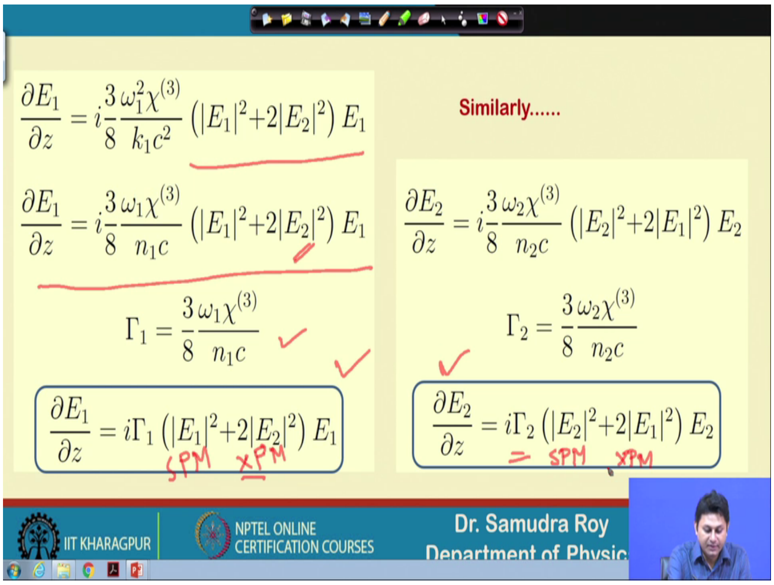

Let me go back to here we have 2 of i of 2 of i of k 1; d E 1 d z and minus 3 by 4 and their quantity. So, if I write this 2 and k 1 down stair and mu 0 epsilon 0 like 1 by c 0s; 1 by c square and if I manipulate few things then we can readily find this expression. So, this is not a very new thing; so this kind of things we have done. So, here this is a thing you can see that 1 by k 1 term is here 3 by 8. So, minus i can be absorb, minus 1 can be absorb this i term; important thing is what we are having here. We are having mod of E 1 square plus 2 of E 2 mod 2 E 2; W 2 mod of E 2 square E 1. So, finally we will have this expression and in this expression again; this is the term which is basically the self phase modulation term.

And if I write this coefficient as gamma then we have a straight relatively straightforward expression in our hand. And this is expression for self phase modulation and cross phase modulation both this is basically the term related to cross phase modulation XPM and this is for self phase modulation, SPM; one is XPM and another is SPM. And since this term is crossed E 1 and E 2; they are crossed that is why it is cross phase modulation, E 1 and E 1 is same this for self phase modulation.

Exactly in the similar way; exactly in the same way one can calculate the self phase modulation for E 2 also; so, this is for E 1. So, another way will still there E 2 and if I write this E 2 expressions. So, everything will same only this coefficient will slightly change because in coefficient we have omega 1 and n 1, now it should be omega 2 and n

2 as is written here and E 1 square is replaced by E 2 square and E 1 E 2 is replaced by E 1.

So, here this is my cross phase modulation term and this is my self phase modulation term SPM. So, cross phase and self phase modulation term is still coming here and this is E 2 field and we have a set of equation which basically gives us the idea of self phase modulation. So, with that note I would like to conclude my class here.

So, today we learnt very important thing how under pump depletion condition; the third harmonic will generate and how this will going to evolve. That is the first thing that the first part we have done and the second part we understand what is the cross phase modulation? So cross phase modulation is nothing, but if I launch two different web length, two different light strong beam; so, one beam will be modified; refractive index of one beam will be modified by another beam. And because of that we will getting 2 differential equation, which seems to be a couple differential equation and this 2 equation basically governed; how the phase will going to modify.

But in the next class, we try to find out because of this cross phase modulation; is there any change in amplitude or not or the amplitude or the total energy will remain conserved or not. And then we try to find out how the phase will going to modify as the name suggest is a cross phase modulation so; obviously, the phase will going to modify. So, this kind of things we will check or do in the next class; so, with that note let me conclude here.

Thank you for your attention and see you in the next class.