Introduction to Non-Linear Optics and Its Applications
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Lecture - 46
Third Harmonic Generation (Contd.)

So, welcome student to Introduction to Non-linear Optics and Its Applications course; today we have lecture number 46. In the previous lecture, we have started important concept third harmonic generation which is basically the continuation of the third order effect.

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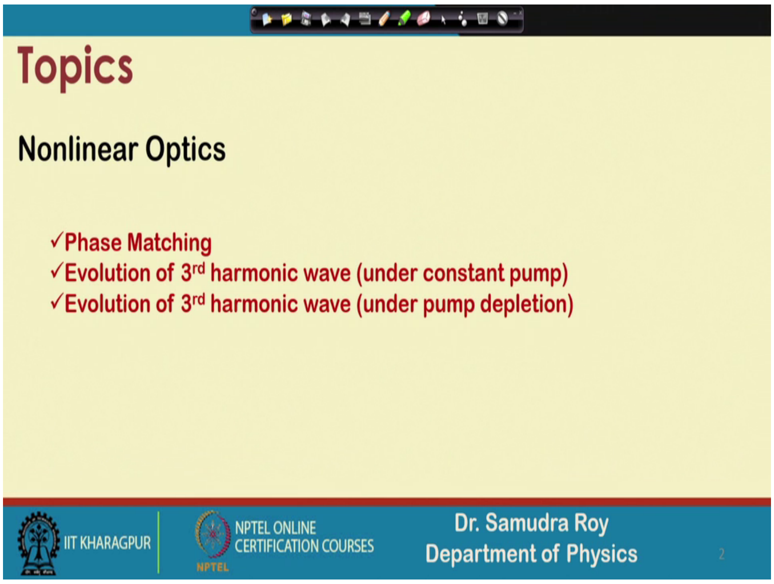

And these third order effect also contain few other important phenomena; self phase modulation is one of them. We also discussed this self phase modulation in detail, but today we will going to learn more about the third harmonic generation process; where if you launch an electric field with frequency omega then it will give rise to another frequency of 3 omega.

It is exactly the same process that we have in second harmonic generation, where it was a second order effect. And we are getting a frequency of 2 omega here we will going to get a frequency of 3 omega instead of 2 omega, because it is a third order effect. How we are getting this 3 omega frequency? We have discussed, but the evolution of the third harmonic wave is still there; so today we will going to learn that. And also the corresponding phase matching conditions. So, what should be the phase matching of third harmonic generation? That we will, going to learn which is very much similar to the second harmonic generation process.

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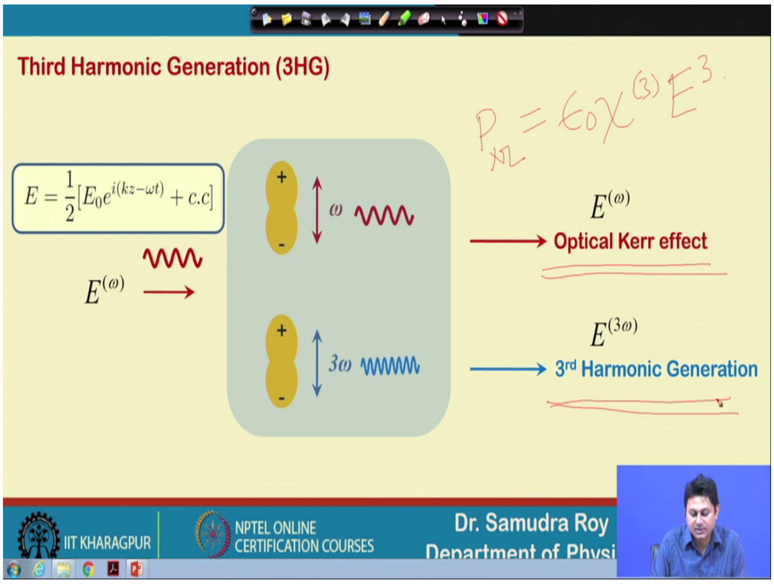

Well in third harmonic generation the as the figure shows this is a old figure I am using several time that when you will launch in electric field with this particular form, then what happened that this electric field will enter into the system. And we know that for chi 3 effect; P non-linear is equal to epsilon 0; chi 3 and then E cube. So, when it is E cube; then the electric field will be total electric field here whatever the electric field is launched will be cube. And when it is cube then we know that there will be 2 different frequencies that can have into the system one is omega and another is 3 omega.

So the frequency that is related to omega is giving us optical Kerr effect and the frequency which containing 3 omega that gives us third harmonic generation. So, optical Kerr effect leads to self phase modulation and change of refractive index are there because of this effect. So, we have already discussed these issues; so, third harmonic generation is a part that we will discuss today as I mentioned ok.

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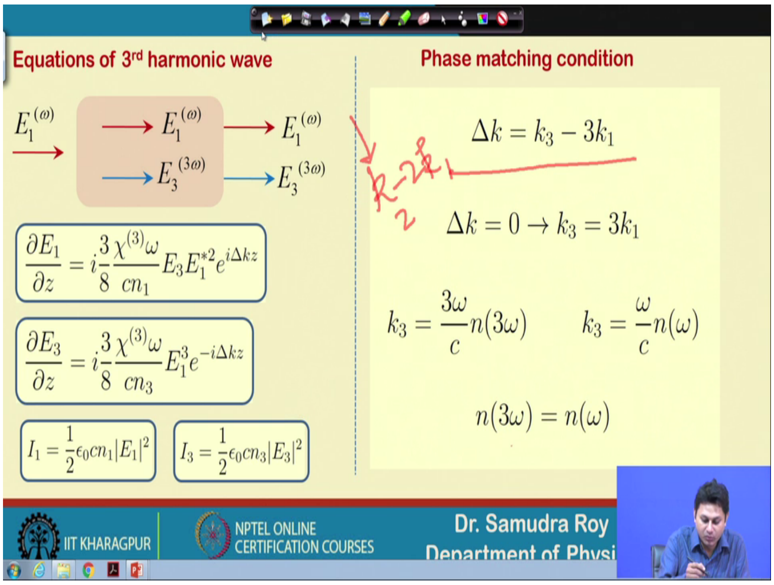

So, equation of the third harmonic wave that we derived in our last class; so the derivation is almost same the way we have derived the second order effect or the second harmonic generation. But only thing that you need to note here is the phase thing this is the phase term and for this phase term; we have k 3 minus 3 k 1. For second harmonic generation, if you remember it was k 2 minus 2 k 1; k 2 is the corresponding propagation vector for second harmonic wave and k 1 was the fundamental wave.

Exactly the similar way here k 3 is the wave vector for third harmonic wave and k 1 is the fundamental wave. So, here in the left hand picture you can see that electric field E 1 is launched with the frequency omega. Inside the medium where we have the third order effect E 1 and E 3 will going to generate; E 1 with frequency omega and E 3 with frequency 3 omega. And when they will going to generate there will be some differential equation that should govern the evolution of these 2 waves and these are the 2 differential equation which are coupled to each other as usual.

And they will basically tell this 2 equation basically govern; how E 1 and E 3 will going to evolve.

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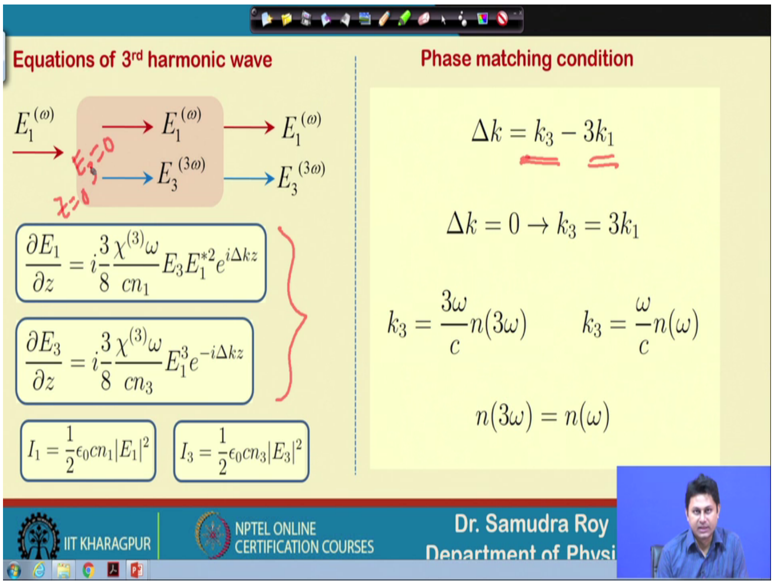

Please note that at the input at this point where Z is equal to 0; E 3 was not there, so E 3 is equal to 0. So, boundary condition is that when I launch the electric field E 1 at the input there was no third harmonic wave. Because, the wave will enter into the system and then the dipole will start oscillating at 3 omega frequency, and then gradually the third harmonic wave will generate.

So, that is why at the input they should not be any kind of third harmonic wave. Well the intensity and electric field relation is also given here; now let me go back to the phase matching equation. If you look carefully to these 2 coupled equation, you can see that there is a term sitting here in the phase. And this is a usual term that we always have in this kind of equations in second harmonic generation process also we are getting this kind of terms and for third harmonic wave also this term is there. Now in order to have a phase matching condition; we know that this delta k has to be 0. So, if this delta k; that means, the phase mismatch between the third harmonic wave and the corresponding fundamental wave are 0.

Then delta k is 0; basically gives us a condition that k 3 has to be equal to 3 k 1. Now k 3 is a propagation constant of the third harmonic wave; so, if I write k 3 see it should be 3 omega divided by c multiplied by n at 3 omega. Because the third harmonic wave contain a frequency of 3 omega; we write here 3 omega frequency; in this place. In the similar way, k 1 should be here again a typing mistake it should be k 1.

So, k 1 is supposed to be omega divided by c then multiplied by n of omega. Omega is a frequency of fundamental wave who is corresponding; whose vector is k 1 and n of omega is a refractive index at the frequency omega. So, now if I equate these 2 things (Refer Time: 08:09), you can find that we have an expression that n of 3 omega has to be equal to n of omega. It is a similar kind of expression that we have for second harmonic generation; only thing is that the frequency here is 3 omega in that case it was 2 omega.

So, now we need to find out the way how do make this n of 3 omega, to n of omega some (Refer Time: 08:34) in process can still be possible to do that.

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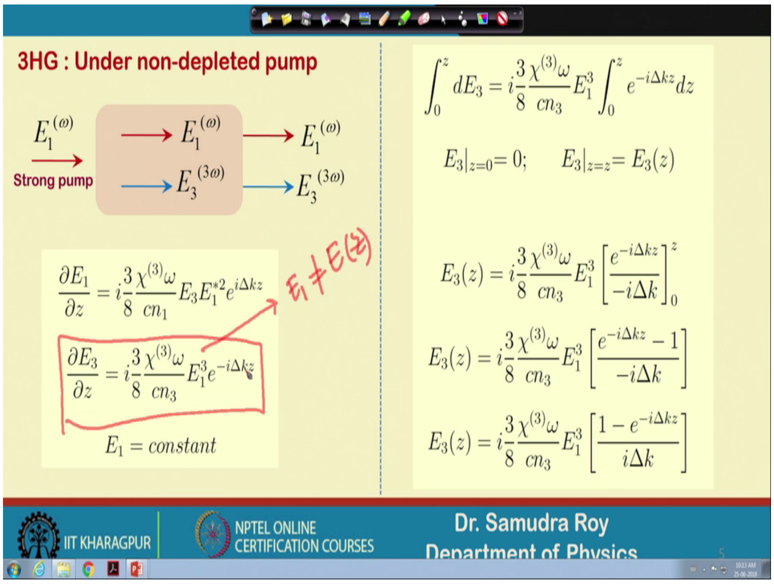

So, we will not going to discuss in detail because we have already discussed this phase matching issue in the second harmonic generation process. Now the next thing we need to know the evolution of the third harmonic wave. So, we know that the third harmonic wave is evolving.

So, this is the third harmonic wave that is generating. So, we need to find out the evolution of this wave; how it will going to evolve over Z? This is the distance, this is the distance and this distance is Z. So, we need to find out how this will going to evolve and to understand the evolution; we have 2 equations in our hand which are coupled to each other and this is our equations; these 2 our equations for E 1 and for E 2, E 3.

Now here the condition is the non depleted pump; that means, we launch a very strong pump and this strong pump basically generate, this is a source term to generate the E 3 wave or the third harmonic wave. So, since E 1 is very strong; so certain percentage of E 1 will be there to generate the frequency 3 omega; the field with the frequency 3 omega; so, we can consider E 1 as a constant.

So, once we consider E 1 is a constant then it is very easy to solve this equation. So, we know that from our second harmonic generation process, the study of our second harmonic generation process. Once we considered the pump is very strong and it is constant then the equation become quite simple and integration is very straight forward. So, now if I integrate this equation this is my equation now because I want to find out the evolution of E 3 and E 1 is constant this is not a function of z.

So, if this is not a function of z; then the integration is straight forward because other term is not containing any kind of z; only term that we have z is delta kz. So, once you integrate this quantity with the boundary condition that E 3 at z equal to 0 is 0 and E 3 at z equal to z is some quantity say E 3 z. Now if I integrate these things as I mentioned this is a constant it will come out from the integration only I will going to integrate this quantity; it will be as usual e to the power of minus i delta z divided by i delta z with the boundary condition that 0 to z.

We put the 0 to z and we can write it is e to the power minus i delta z minus 1 because at z equal to 0, it is 1 divided by i delta k with a negative sign. We can further modify this expression and we will have this equation.

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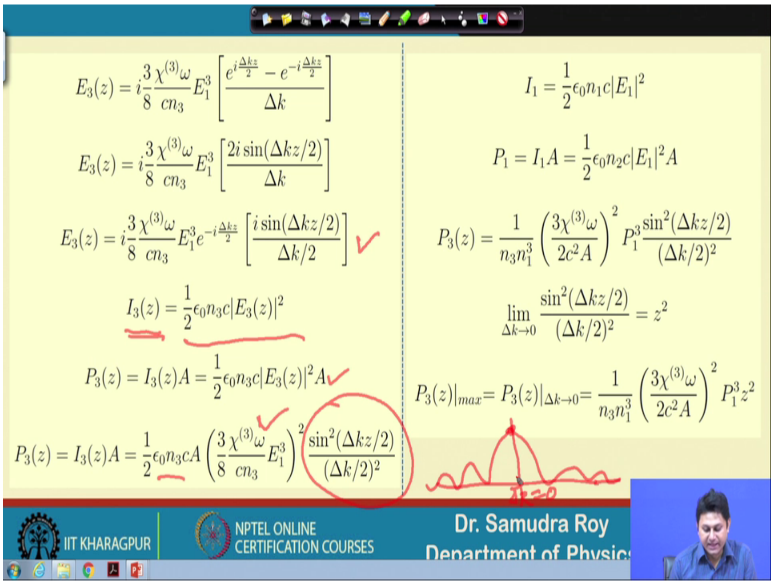

And then finally, we will come to a point which is quite similar than that we have in our second order effect; this term. This calculation is quite straight forward and I am not going to give you all the detail because all the steps are here. When you do this calculation by yourself you can verily understand that how this thing is happening, how this sign term is coming this small manipulation is there; so, this is a very old calculation that I am basically using.

Now after having the expression like this; so this is the P 3 mind it every time when we deal with the electric field; it is then converted to the corresponding power. We know the intensity of the third order wave is related to this expression; half of epsilon 0 n 3 c mod of E 3 z square E 3 z is now in our hand, this is our E 3 z.

So, once we know this is my E 3 z; so, I will make a mod square of that; when you do the mod square this exponential term will cancel out and only this term will be here. So, it is here and since I am calculating the corresponding power one additional half epsilon 0 n 3 c, this term will sit here with an area A because power is basically area multiplied by the corresponding intensity.

Now, this is a sink kind of function. So, sin x divided sin x square divided by x square; if delta k is not equal to 0. So, we will have a similar looking curve that we already got in our second. So, we will have the power change at this; so, this is at delta k equal to 0 point. So, at delta k equal to 0 point we will have a maximum power and then if delta k is not equal to 0; we have a window which is which is called the bandwidth. For this bandwidth we will have some kind of energy anyway; so this is or these things we already know.

So, now in the next thing is to find out over the corresponding efficiency; if I try to calculate the maxima of these things as I mentioned, the maxima will appear at delta k tends to 0 point. When delta k tends to 0 these sin x sin; square delta k z by 2 divided by delta k by 2 whole square, this function will simply be z square with the limit delta k tends to 0. So, delta k tends to 0 is nothing, but the phase matching condition.

So; that means, the maximum power that one can have is P 3 max with the condition that phase matching is there that mean delta k tends to 0. And if you look carefully there a few terms there, but the dependency is Z square which is important; that means, the power will the power of third harmonic wave will increase as a function of z, which a square form; that means, it will increase with z and this increment is in parabolic in nature; it will be a square form.

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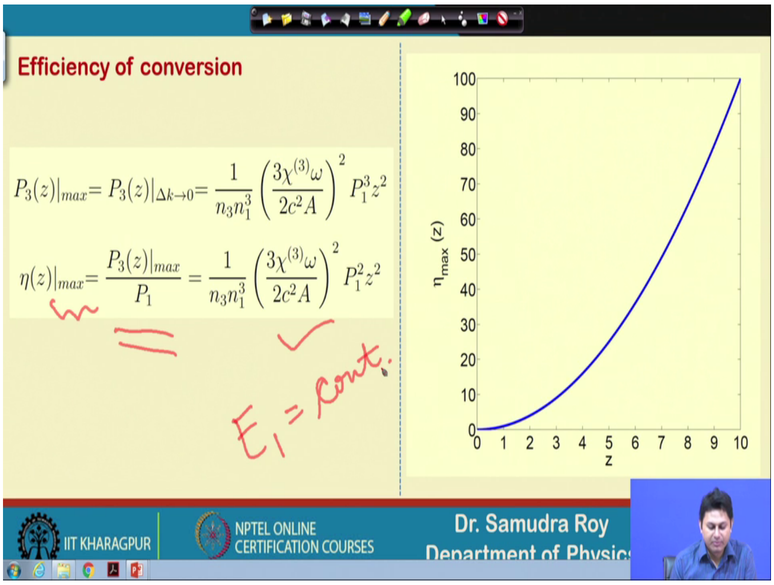

So, this dependency again we already know from our previous experience when we deal with the second harmonic generation. This is something the like the same thing that the it is evolving as a function of Z square. So, now look at this curve if I now plot the efficiency is nothing, but the ratio of the power that is generating divided by the input power.

So, this is the quantity we can define as efficiency and now we write efficiency max; that means maximum efficiency I am talking about. So, whenever I talk maximum efficiency; that means, I consider delta k is automatically 0. So, under that condition we have an expression like this.

And this efficiency suggest that the energy of the third harmonic will increase as a function of Z in this way. Now as I mentioned this is this is really true when the conversion efficiency is really very small. So, whatever the drawing we have here; so, please note that we have already reached to 100 percent efficiency and if I go z equal to 10. So, if this if I am normalise this fact then we can see that the efficiency as if it is reaching the value 100; that means, huge amount of energy conservation can be possible.

But this is; obviously, a non physical argument because efficiency cannot be reach monotonically cannot be increase monotonically because the one very important assumption we consider here is E 1 is constant. So, the energy E 1; E 1 is a field is constant is a important approximation that we consider. So, E 1 is not in the in the physical sense, it should not be a constant quantity; see it will also decay. If it is decay then we need to do a more careful calculation to find out how the power of third harmonic will going to evolve as a function of z.

So, for small efficiency maybe it is true, but if I increase the value of the efficiency; then we need to consider the condition that E 1 is also going to decay, but here in this approximation or this calculation we consider E 1 is a constant which is not a very much true for real case.

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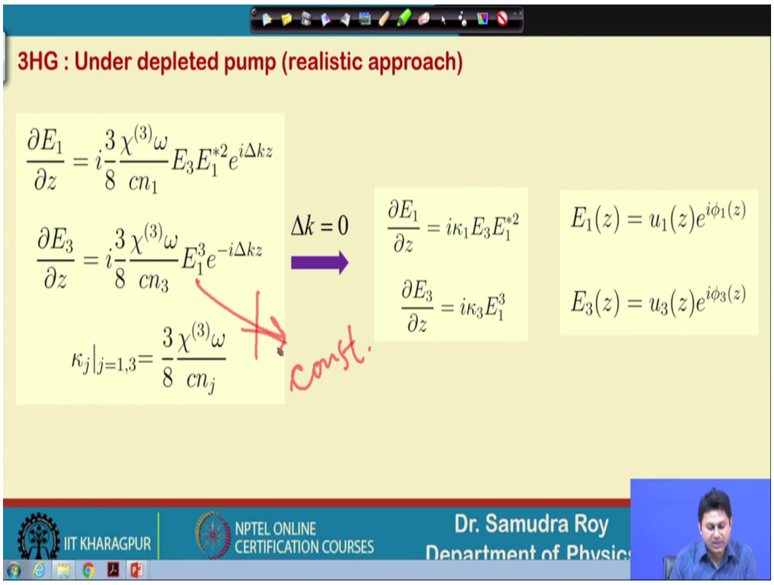

Now, we start the another condition that we consider that the electric field containing frequency omega which is the fundamental electric field or E 1 is no more a constant quantity. So, we will going to take care of both the equation together; in the previous case we consider in the previous case we considered this is a constant quantity, but we will not going to consider this anymore. So, we will consider both the equation together.

So, try to solve the coupled equation. So, to make life simple we consider delta k is equal to 0; that means, with the absolute phase matching condition what happened. So, this equation basically simplify with these 2 form; K i; K j is basically the coefficient that will contain all these terms 3 by 8, kappa 3 omega c n one this is a constant term. So, we remove this constant term it is smaller coefficient called kappa 1 and kappa 3.

This is a coupling coefficient also because this coupled one field to another one. Now once we have a simpler expression this; next thing we need to do e to calculate the amplitude and phi amplitude and corresponding phase separately.

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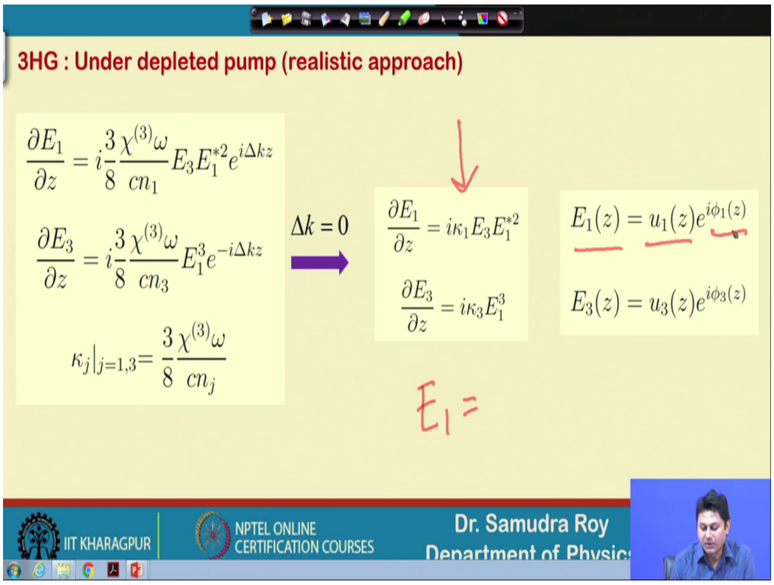

Because we know that E 1 is a called complex amplitude, E 1 has a complex quantity E 1 is a basically complex quantity; it should have an amplitude also a phase.

So, in order to understand how the amplitude and phase will vary; we just divide this total electric field which is a complex electric field into 2 part. One is amplitude part and another is the phase part and we know that any complex quantity can be represented it in terms of r e to the power i theta. So, exactly that we are doing here we write one amplitude part and another phase part for both the cases. Now here E 1 and phi 1 both are real because we just divide these 2 things to real and imaginary part; so, E 1 and phi 1 has to be real.

In the similar way, E 3 and phi 3 are real when E 3 and phi 3 corresponds to the field E 3. So, u 3; phi 3 is real and we divide the total field into amplitude and phase part and that is our form. Well once we have this form once we have this form.

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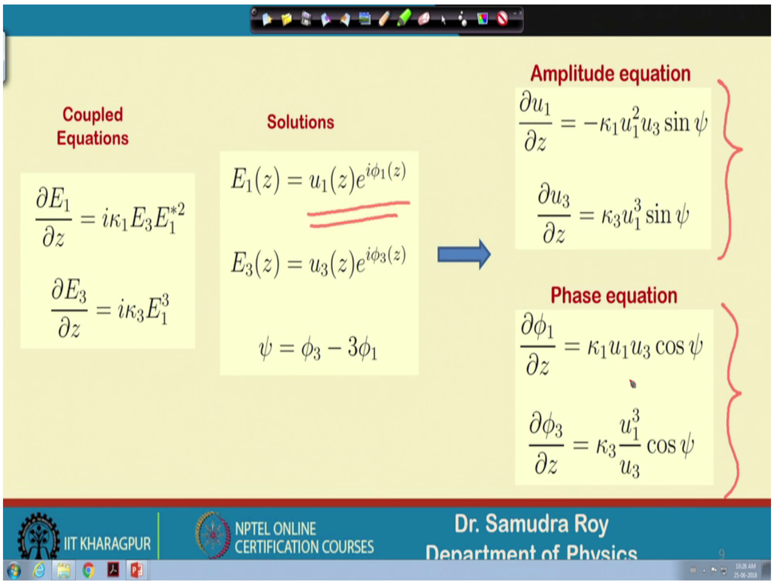

Next this is our equation coupled equation and this coupled equation basically; this coupled equation basically this is our solution, for this solution I mean I want to put this things into my coupled equation. When I put this thing into coupled equation and separate the real and imaginary part. So, we will have a set of equation amplitude equation like this and set of phase equation like this.

Please note that when you put this E 1; so, u 1 and phi 1 will be here and in the right hand side also you will have u 1 and phi 1. And now we will have a equation, where real part and imaginary part in the left hand side and right hand side also have a real and imaginary part. If I separate out the real and imaginary part then we will we can separate this amplitude equation and phase equation.

This is the treatment that we have already done in the second order effect. So, I ask the students please do that by yourself and try to find out whether you are getting this amplitude and phase equation by yourself or not; the expression is already given in this is slide. Well once we have the amplitude equation and phase equation; you may carefully look that there is a quantity psi here; psi is a phase difference.

And this phase difference between these 2 is defined by this; it is phi 3 minus 3 phi 1, it is something like K 3 minus 3 K; K 1 like kind of things. So, we will have a 2 compact equation phase equation and amplitude equation; the similar kind of treatment we have done in the second harmonic generation all the details are there.

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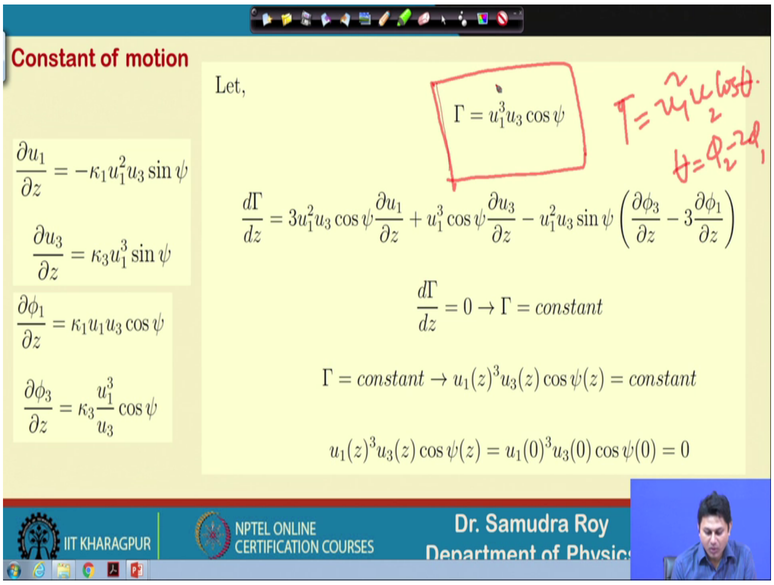

Now, once we have this 4 equation in our hand; then we can define one quantity which is which is constant over motion. And this constant over motion quantity is called the gamma; if you remember for second harmonic generation case, this gamma constant of equation motion was u 1 square u 2 cos of theta; where theta equal to phi 2 minus 2 phi 1.

It is exactly the similar kind of expression that we have right now, but instead of having u 1 square, we have u 1 cube. And u 2 is replaced by u 3 because u 2 was a field corresponding to amplitude of the field correspond the second harmonic; here we are dealing with third harmonic that is why the nomenclature is different it is now u 3.

So, once we have this expression we consider this as a constant; try to find out whether this is really constant or not. So, what we will do? We will just make a derivative with respect to z both the side; when you make a derivative with respect to z, then the right hand side you write 3 u 1 square because u 1, u 3 and psi both are function of z. So, first there will be 3 functions; so 3 derivative will associated with that. So, first is 3 u 1 square u 3 cos psi and the derivative corresponding to u, second term is derivative corresponding to u 2 and third is a derivative corresponding to psi.

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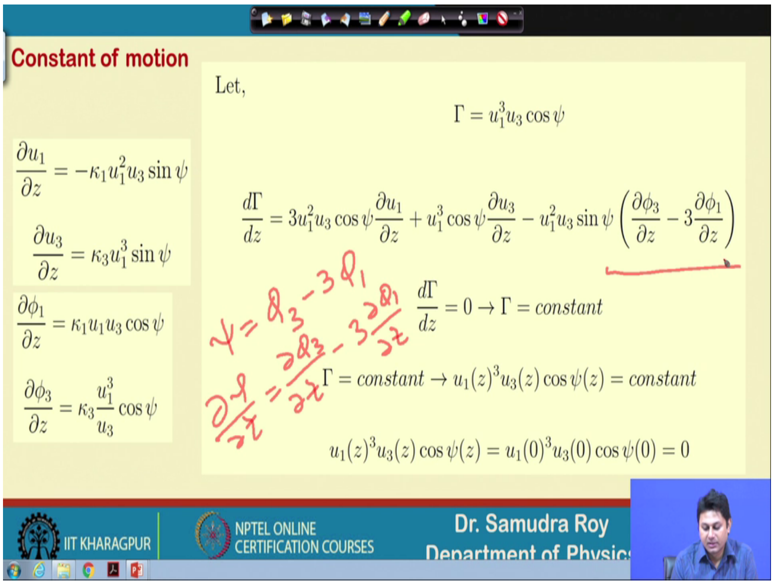

So, psi is equal to phi 3 minus 3 phi 1 so; obviously, del psi, del z can be represented as del phi 3 del z minus 3 del phi 1 del z; so, exactly these things are here.

So, we just expand the expression of phi. So, that we have all our terms like this this and this; all the 4 terms in the left hand side is given. If we put all these 4 term whatever we have in the left hand side; we can verily find that this quantity is a constant quantity. So, once we have this is a constant quantity then we can see that u 1, u 3 and cos z is a constant is a constant of motion; that means, if I change both all this things 3 functions or function of z, but they are constant over the z. That means whatever the value of this function at z equal to z; it should be same at z equal to 0. Because they are constant; so if I put z equal to 0; then u 1 cube u 3 in z, u 1 cube u 3 cos psi at z point is equal to u 1 cube u 3 cos psi at 0 point.

So, if I put this 0 then one term is appearing here which is u 0; u 3 0. Now u 3 0 is nothing, but 0 because u 3 is amplitude of the third harmonic.

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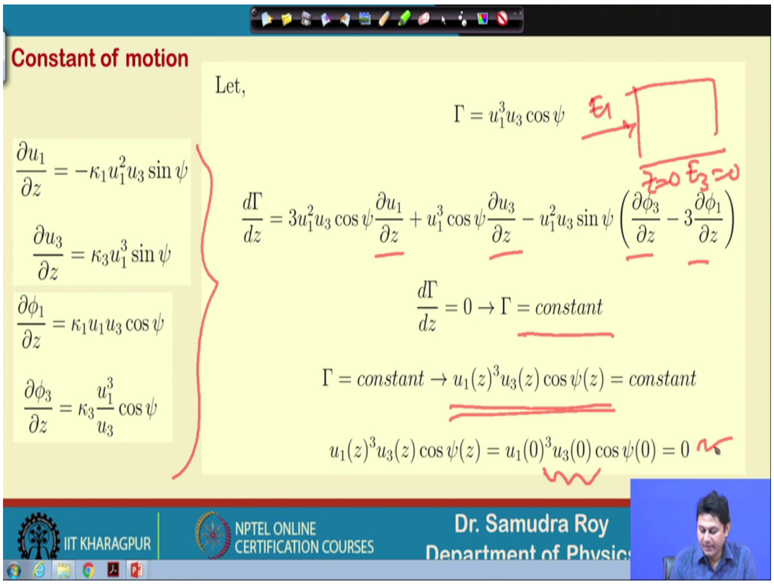

And we already mentioned that when will launch an electric field E 1; there was no at Z equal to 0; there was no E 3. So, E 3 is also 0; so at Z equal to 0; E 3 0 means u 3 at Z equal to 0 is 0. So, entire quantity here is 0. So now we have an very interesting expression.

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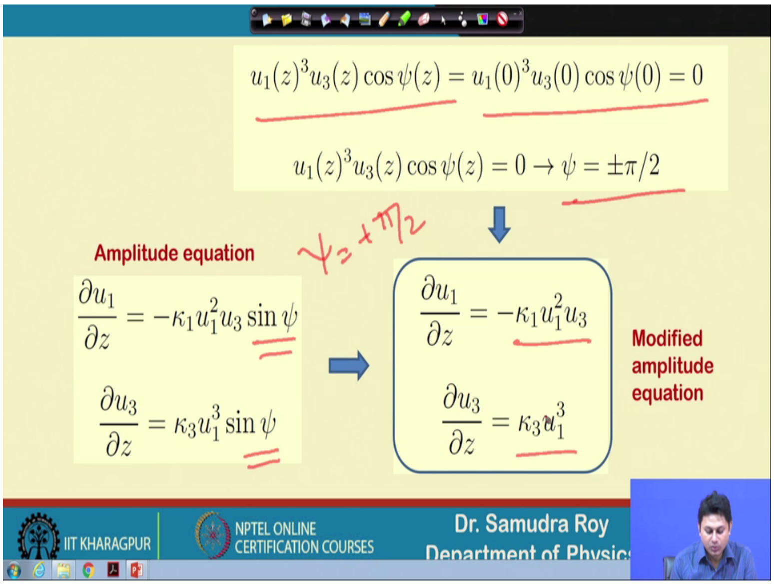

This is a this is exactly the same treatment that we are doing we have already done in our second harmonic generation. Well; so, this is the expression that we have and from here we find the quantity is 0, u 1 and u 3 are changing. So, cos of psi has to be the phase relationship has to be some constant value and if it is 0. Then it leads to one fact that psi is equal to plus minus of pi by 2; then only we have this quantities equal to 0. Once we put psi equal to say plus pi by 2; let us put plus pi by 2 first, then this amplitude equation is now simplify with this and this.

So, this phase term of psi term will vanish because sin of pi by 2 is equal to 1. So, this term will just vanish and in state of this term we have 1. So, we will have two things in our hand, two expression in our hand and this is the modified amplitude equation. So, using this modified amplitude equation. We will try to find out how u 1 and u 3 will going to evolve and that we will do in the next class.

So, let me conclude because our time constant is there. So let me conclude my class here. Thank you for your attention and see you in the next class to understand the third harmonic generation in more detailed way.

Thank you.