Introduction to Non-Linear Optics and its Applications
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture - 45
Third Harmonic Generation (3HG), Energy Conservation

So, welcome student to the next class of Introduction to Non-linear Optics and its Application.

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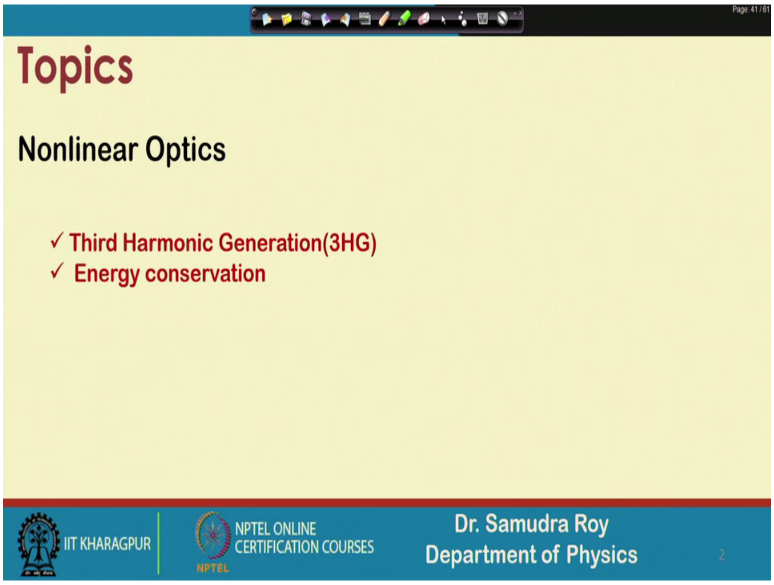

So, today we will have lecture number 45 and today we will start a topic related to third harmonic generation and corresponding energy conservation. So, so far we have been discussing about the self phase modulation and find that when the non-linear effects are there, there are 2 kinds of effects that happens one is the self focusing and another is the self phase modulation.

Self focusing is a special phenomena, where the self focusing self phase modulation is a temporal phenomena or this is a phenomenon where the light can be considered in the distribution of the light, the phase of the light will going to change. So, today we will going to find out another fundamental effect of chi 3, which is the third harmonic generation.

So, it is a technically exactly the same the treatment wise it is exactly the same that we have done in our previous class for second harmonic generation, where we launch a light of frequency omega and we are getting a frequency 2 omega this a different kind of frequency mixing, here we will going to have exactly the same thing I am launching a frequency omega and going to have a frequency 3 omega, provided some phase matching conditions are there.

So, we will derive the equation the equation of third harmonic and then find that because of the third harmonic generation whether any kind of energy dissipation will be there or the energy conservation will be still valid for third harmonic generation or not. So, today we will have 2 topic third harmonic generation and the corresponding energy conservation.

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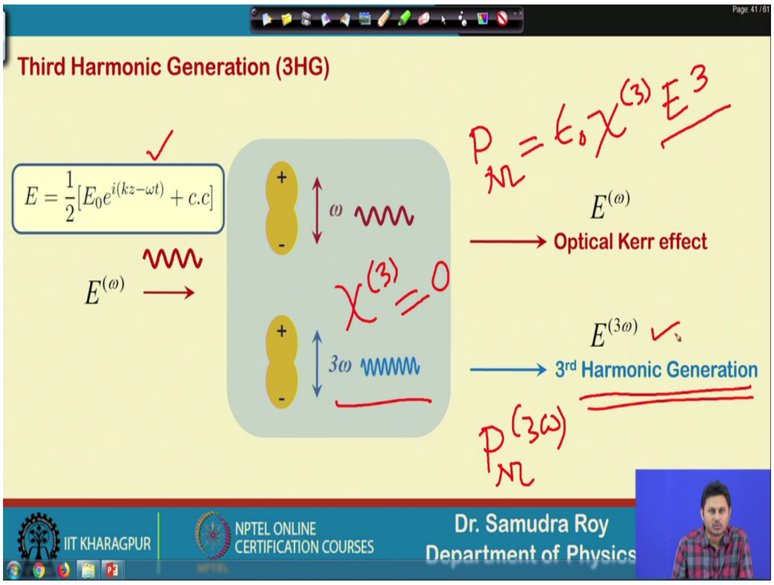

Well for third harmonic generation so, third harmonic and energy conservation. So, for third harmonic generation again go back to this schematic diagram this is a schematic diagram suggest how the third harmonic will going to generate as well as how the Kerr effect will originate. So, this is a medium where we have chi 3 equal to 0. If chi 3 equal to 0 what happened, the electric field is launched with a frequency omega, non-linear polarization is epsilon 0 chi 3 E cube due to this E cube term we have third harmonic.

So, non-linear polarization term will have frequency component 3 omega, and this 3 omega frequency component will allow the dipole or with the source term for the dipole that will going to vibrate with a frequency 3 omega. Once the dipole will vibrate with a frequency 3 omega, it will start generating a electric field of 3 omega and that is basically the origination of the third harmonic generation.

So, this is a total picture or the physical picture of the third harmonic, where the electric field of omega is launched and we will have electric field of 3 omega.

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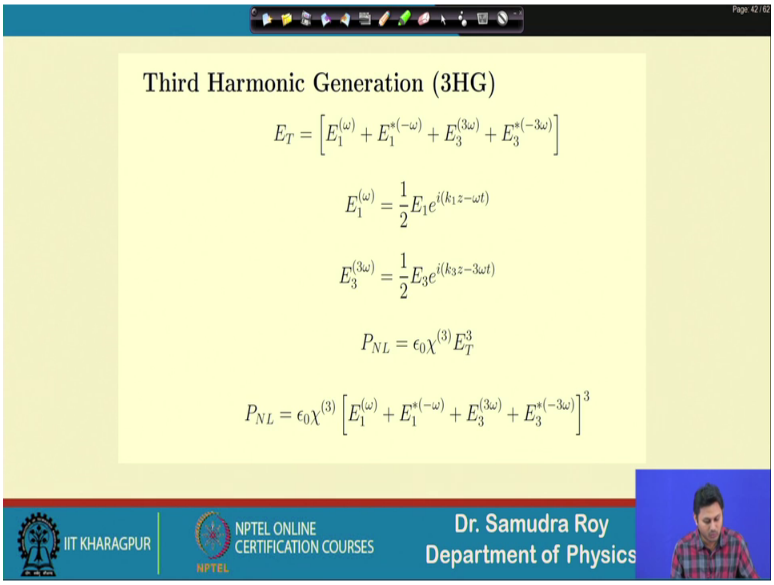

So, how these things will happen we will study in detail by using this our mother equation which is non-linear Maxwell's equation. So, exploiting non-linear Maxwell's equation we try to try to derive the evolution of the amplitude of the third harmonic wave as well as the fundamental wave.

So, here the total electric field will be the combination of the electric field so there are two electric field inside the medium already, one is E omega if I write E 1 omega another is E 3 omega. These are the 2 fields E 3 3 omega. These are the 2 fields that will be inside the system one is E 1 omega and another is E 3 of 3 omega, 3 omega means it will be vibrate at 3 omega frequency, omega means it will vibrate at omega frequency.

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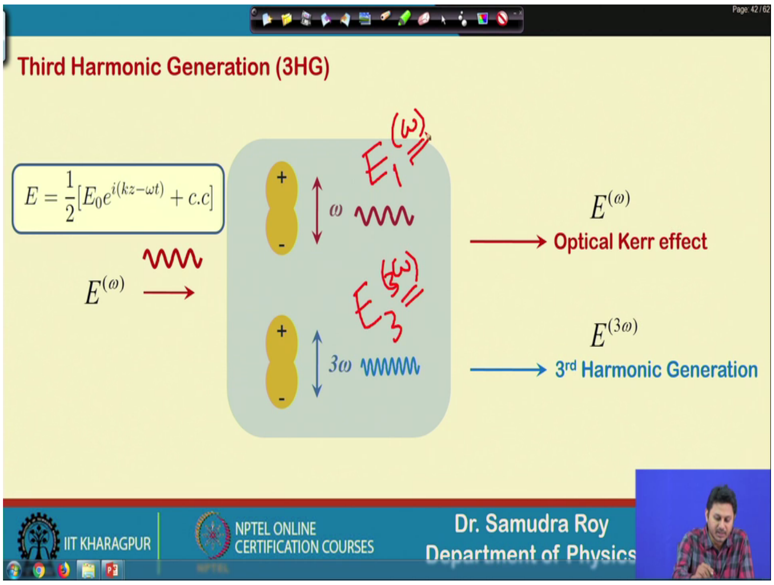

Well if I now write the total field it will be the combination of their complex conjugate. So, that this total field become a real field so E 1 omega E 1 omega E 1 of omega star, E 3 of 3 omega E 3 of 3 omega star 4 fields are there. So, P non-linear will be total field whole cube electric field component E omega; electric field component E omega can be represented as half E 1 e to the power i k 1 z omega t and half E 3 e to the power i k 3 omega minus 3 omega t for electric field with 3 omega frequency. P non-linear will be total field cubes so total field is this. So, cube of that; that means, again we will have several frequency component in P non-linear.

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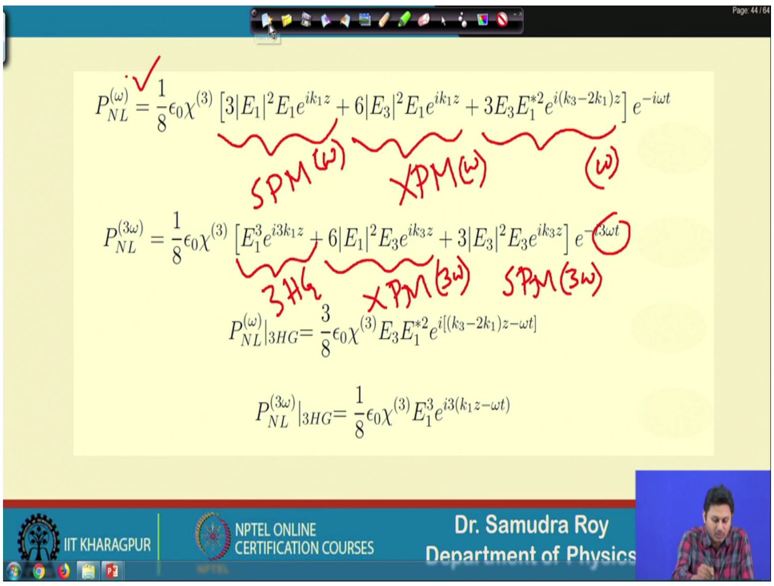

This is not the first time we are using this equation. So, we will see that there are many frequencies many frequency components are there, and we can identify this frequency components identify the terms. For example, this is the self-phase modulation term that we have been calculating for; in the last class we are calculating this term this is the cross phase modulation, this is the self-phase modulation term SPM, this is the cross phase modulation term and finally, we will have a term which is related to the third harmonic generation. This is not a third harmonic generation term because its frequency component is omega, but this term will be for a term related to third harmonic with omega field.

In the similar way for P non-linear 3 omega we have 3 terms, this is the self-phase modulation this is the self-phase modulation related omega field, cross phase modulation related to omega field, third harmonic generation term due to the omega field. This is not a third harmonic generation term critically, but this is a term which corresponds to third harmonic generation.

Here we have a third harmonic generation term, because here the frequency is 3 omega. So, 3 third harmonic generation term, this term is for cross phase modulation term with frequency 3 omega for frequency 3 omega and this is self-phase modulation term for frequency 3 omega. So, 2 fields are there individually for 2 fields we have self-phase modulation cross phase modulation and 3 omega term and third harmonic generation term. And for other 3 omega frequency we have also third harmonic generation cross phase modulation and self-phase modulation for that particular field.

So, once we identify the field, then as I mentioned we need to only consider since we are dealing with third harmonic generation we need to only consider the third harmonic terms third harmonic generation terms, inside this P non-linear. So, in P non-linear omega this field is related to third harmonic. So, I write this field here this part of this part and for P non-linear 3 omega, we only write the third harmonic term which is this one and I write this here. So, 2 term we identified one is for third harmonic generation with corresponding to omega and another is third harmonic generation corresponding to 3 omega. So, we identify these 2 terms for P non-linear, and extract these 2 term from this total terms.

This degeneracy factor again you need to consider very carefully that how this 2 fields are interacting based on that this degeneracy term will be there. If 2 distinct fields are there then we will have 3 if 3 distinct fields are there we have 6, if one distinct fields are there then we have one for example, this is related to one distinct field and this is E 1. Every field and their complex conjugate are considered as a distinct field.

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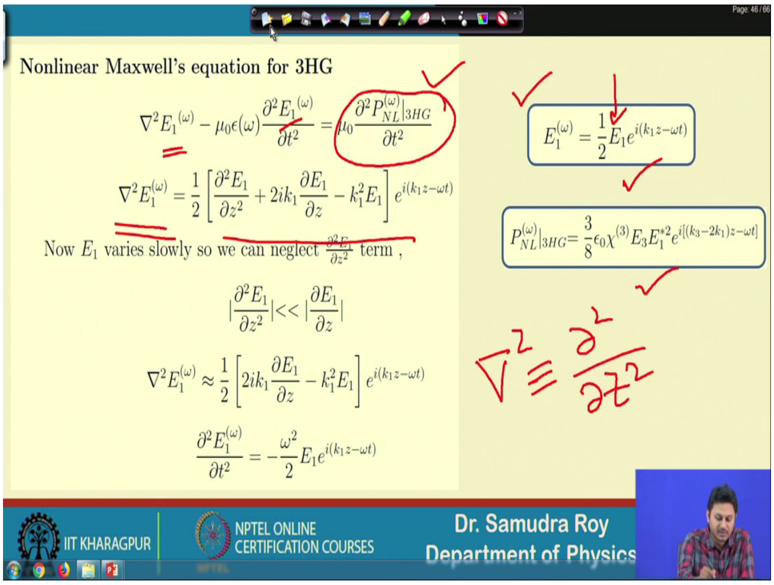

Well next we need to solve the Maxwell's equation and in order to solve the Maxwell's equation here the source term is third harmonic so; that means, E 1 how this E 1 is going to evolve we need to find out.

And in order to find out the evolution of the E 1, we need to write the source term for p from P non-linear, which containing omega term and that is related to the third harmonic. So, E 1 is given like this, where E 1 is amplitude of the fundamental field and P for P non-linear the 3 omega the 3 third harmonic generation term we extracted it is E 3 E 1 star square e to the power of i, a phase term is associated is there k 3 minus 2 k 1 z minus omega t, it has to be a frequency component omega and next what we will do, we just write this try to find out what is the value of this del square E 1 omega. So, del square E 1 omega is simply this term.

Because we are using this thing almost I mean for every class we solve this Maxwell's equation for a given E 1 see if E 1 so the double derivative corresponding to this thing is this, where this delta square is 1 dimension it is equivalent to del 2 to del Z square because x y components are not there. So, once is it is there. So, we can solve we can make a derivative of this term. And once we make a derivative we will have double derivative of E 1 single derivative of E 1 with respect to Z with a 2 i k multiplication and then k 1 square E 1. Again we use the slowly varying approximation so, this term will not going to affect, it is varying slowly. So, this is a much smaller than this. So, once this is much smaller than this term will be approximated.

And if I approximate this will be this quantity and the double derivative with respect to t of E 1 is simply minus omega square E 1 e to the power i k 1 z omega. So, left hand side we almost calculate we calculate this term and also we calculate this term, and we know that eventually this term will cancel out with this one. Because these 2 things are same with this multiplication, when I multiply this quantity here this 2 term will be same and eventually cancel out the next thing is to find out what should be the source term here, how this source term will come.

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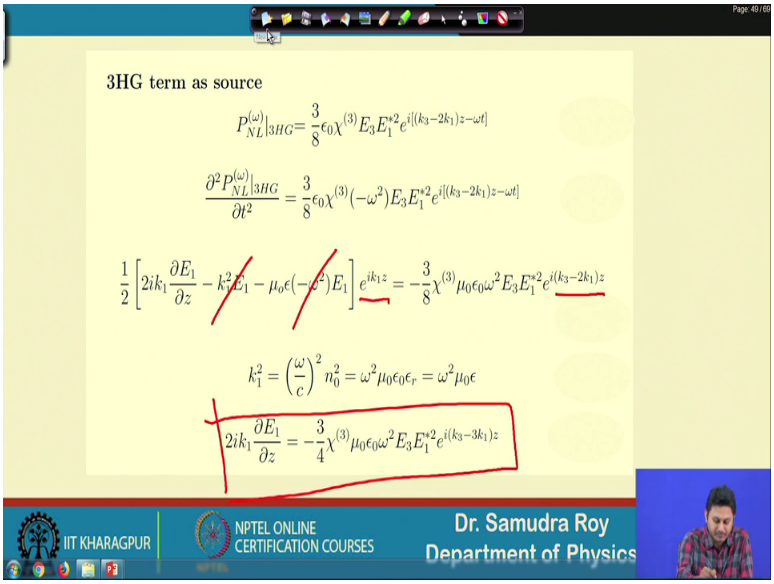

Well third harmonic term as a source as I mentioned, if I use this then we will find this is my third harmonic source term, if I make a double derivative with respect to t. So, only t is sitting here. So, we will have a minus omega square term only this.

So, once we have a minus omega term, then we have all our terms in the left hand side and right hand side if I put left hand side and right hand side together, then we find that this time and this term will cancel out and we will these leads to an expression like this.

So, in the right hand side we have E 3 E 1 star square e to the power i k 3 2 k 1 z and here we have e to the power i k 1 z. Here I should make one note that in when you calculate the self-phase modulation in the previous class, we find that in the left hand side and right hand side it is automatically phase matched. So, the phase matched term E to the power i k z minus omega t was appearing for both the cases both the sides so it was simply cancel out.

But here we find that here we have e to the power i k 3 minus 2 k 1 z and in the left hand side e to the power i k 1 z so there is no way they can cancel out each other. So, we need to take care of this term also. So, one phase term will be there exactly like the second harmonic generation, one phase term will be sitting here.

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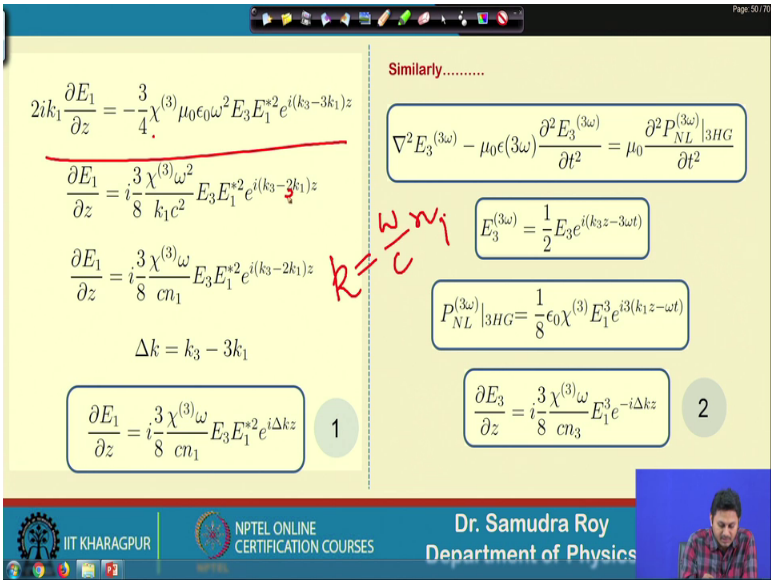

So we are almost in the; so from this now we have equation like this. So, this equation can again simplify and once we simplify the equation I write 2 k i.

So, this 2 will be 8 here because there is a 4 sitting in the numerator denominator. So, 4 into 2 will be 8, this i this minus 1 is there. So, one it should be I this k will be sitting here and if I use the value of k, k is again omega divided by c n 1. So, if I write this k as omega divided by c n 1. So, omega divided by c term will cancel out and one omega divided by c will be there and one n 1 will be there and E to the power k c k 3 divided by 3. So, here it should be. So, here it should be 3 omega it is written wrongly, because once 3 omega it should be simply 3 omega and it should be simply 3 omega.

3 omega 3 k 1 and 3 k 1 not 3 omega this 3 k 1 3 k 1, well it is written by mistake it is in 2 k 1. So, delta k will be k 3 minus 3 k 1, this is the this is the phase mismatch and if I write this phase mismatch in terms of delta k my total equation become something like delta E 1 z it is the evolution of the E 1 is i 3 by 8 chi 3 omega divided by c n 1 E 3 E 1 star E 1 star square e to the power i k z delta k z, where delta k is k 3 minus 2 k 1.

Well once we have the evolution of delta E 1 delta z, in the similar way exactly in the similar way you can find out the evolution of delta E 3 delta z; that means, this equation and you will need to follow exactly the same procedure that we have we have done for d E 1 dz. This is rather more important that is why I again put this as a as a homework and for this homework you need to use this equation, which is the non-linear Maxwell's equation your starting point. Then you need to use E 3, 3 E 3 at 3 omega this form and put this here, when you put this here you need to make a derivative of this quantity where E 3 is a function of z and k 3 is there. So, again you will have a 3 term with E 3 and k 3 and then you need to make a derivative of this with respect to time we need to make a second order derivative.

After doing that the source term is sitting here so this source term for third harmonic generation you can pick up from your P non-linear third harmonic with 3 omega should be simply E 1 e to the power i 3 k 1 z omega t; that means, it will have a frequency 3 omega, because 3 is sitting here and if we if you calculate these things carefully, then you will ended up with its this expression, which is equation 2 and which is the evolution equation for E 3. So, now, we have a equation evolution equation of E 1 and also we have evolution equation of E 3.

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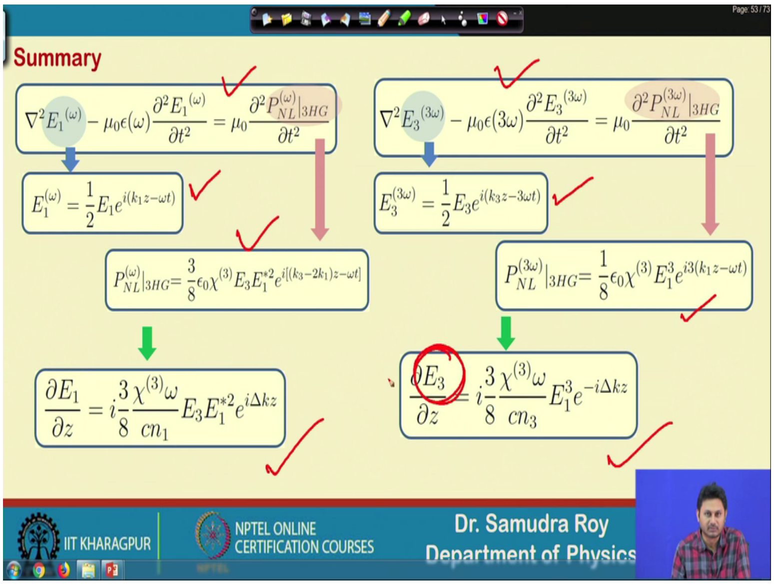

So, now, 2 equations are in our hand these 2 equations again coupled equation and this 2 coupled equations are now need to be solved. So, before solving let me give you a summary so, how you calculate these 2 things. So, you start with non-linear Maxwell's equation and then both the cases you find out what is your E 1 omega and what is your E 3 3 omega that is the fundamental wave and this is the third harmonic wave.

You put this things here and find out what is your left hand side. In the right hand side you have a source term and this source term depending on the problem you need to choose the source term, if you calculate the self phase modulation you need to find out the P non-linear for self phase modulation, which we have done in the previous class. If you try to calculate the third harmonic generation you need to pick up the P non-linear term for third harmonic generation and P non-linear for third harmonic generation is this.

You calculate this derivative for this given term, and put this things again to the Maxwell's equation. Solve this Maxwell's equation and this equation will leads to an expression of E 1. In the similar way you can do here also, you calculate if you write your E 3 this form, then you calculate the P non-linear term for 3 omega this is for third harmonic generation. So, you just pickup only the pick only the third harmonic generation term in P non-linear you have this time in your hand.

Once you have this term you combine this and put it here in the Maxwell's equation and you will have another expression which is similar to this, this is give the coupled equation and these 2 coupled equations gives you the third harmonic. So, E 3 is the amplitude of the third harmonic terms. So, we need to mind it you need to find out E 3 by solving these 2 equation together.

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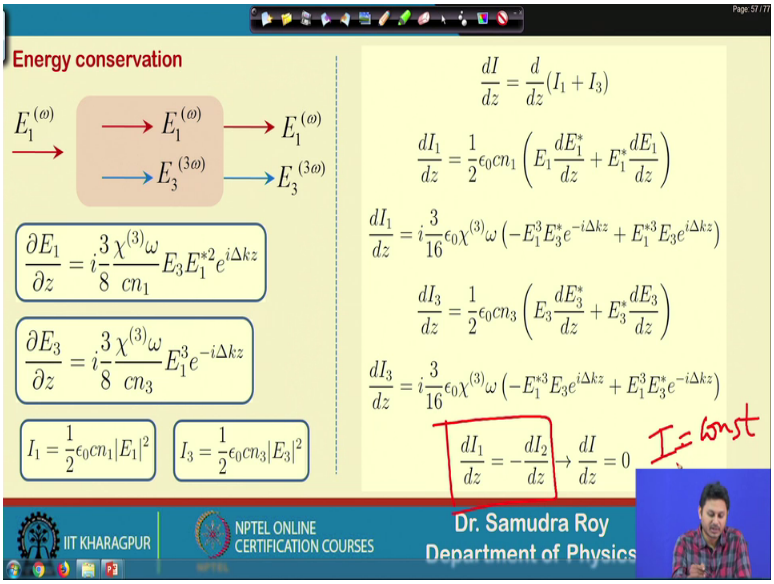

Well once we have these 2 equation finally, we need to check that whether the energy conservation is valid or not. Let us understand once again what is going on, I launch E 1 as a frequency omega here in the system, inside the system E 1 is there which is a fundamental wave and also E 3 is there which is third harmonic term. Now E 1 and E 3 you will going to evolve because they are the function of z.

So, we should have a differential equation of the amplitude of E 1 and E 3 which we have, and these are the equation of E 1 and E 3 differential equation of E 1 E 3 which coupled to each other. Under phase matching condition you can omit this term, but without phase matching this term will be there as usual and now if I write the intensity of this 2 things then intensity of the fundamental wave E 1 can be represented in this way which is the equation of that, and intensity of the third harmonic generation will again written in this form of half epsilon 0 c n 3 mode of E 3 square, several time we are using this expressions, so by that time you are familiar with this representation.

Now the next thing to find out whether the total intensity or the total energy is conserved under these or not because for this coupled equation you can readily understand that third harmonic will going to generate because of the pump here of the fundamental wave. So, the fundamental wave basically gives the energy to the third harmonic to evolve so that means, there is some energy exchange between third harmonic and the fundamental wave; exactly the way second harmonic was generated here we are doing the same thing.

So, energy is coming from the fundamental wave to the third harmonic; when the energy is coming from the fundamental wave to the third harmonic then we need to find out that these 2 equations are consistent with the energy conservation or not whatever the 2 equation we derived. So in order to find that what we do, that we find di dz this is the total intensity. So, total intensity is the derivative d z of I 1 plus I 2 because I 1 and I 2 is a total intensity which is equal to I.

So, I 1 dz again can be represented in terms of E 1 because I 1 is this if I make a derivative of this half of epsilon 0 c n 1 will be here this is a constant term and the derivative of this things will be simply this. Now d E 1 star dz we know that this quantity because this is the expression that we have in our hand, see if I put then what happened that I need to take this common with I. So, if I take this common we will have 3 divided by 16 epsilon 0 chi 3 omega in the outside and in the inside we will have E 3 cube E 3 star e to the power i delta k z plus E 3 I cube E 3 to the power i delta k z these 2 term.

In the similar way if I do for I 3 we will have the same term only a negative sign will be here. So, this term will be sitting here with the negative term and this term will be sitting here with the positive term, please do that and check it by yourself. So, once we have this equation together then we can find one expression that d I 1 d z is equal to minus of d I 2 d z. So, if I add these 2 thing we will have d z 1 I 1 plus I 3 which is equal to d I d z it is equal to 0 ; that means, the total intensity will remain conserved or the total energy will remain conserved.

So, in the third harmonic generation process whatever the expression we derived is consistent with the energy conservation. So, this equation is also known as the Manley Rowe relation which is also a similar form for second harmonic generation only thing is here one mistake we have done again this should be I 3 not I 2 because I am making a notation here for I 3. So, it should be I 3 I 1 I 3 anyway. So, exactly like the third harmonic second harmonic generation for third harmonic generation we find that, the coupled equation for fundamental wave and third harmonic wave are consistent with energy conservation; that means, the total energy will remain conserved. So, it is called the Manley Rowe Manley Rowe relation.

From this Manley Rowe relation we can also find that the number of photon that is transforming from the fundamental to the third harmonic is also conserved. So, this conservation on energy actually confirm that whatever the equation we derived is correct. So, with this note let me conclude today's class. So, thank you for your attention and see you in the next class where we discuss more about the cross phase modulation and other phenomena related to the third order effect.

So, thank you for your attention and see you in the next class.