Introduction to Non-Linear Optics and its Applications
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Lecture - 44
Self Phase Modulation (Contd.), Frequency Shift

So, welcome students to the next class of Introduction to Non-linear Optics and its Application. So, in the previous class we started Self Phase Modulation and try to find out the corresponding equation for self phase modulation, and from this equation we have a solution, and when we solve that thing we find that amplitude is not going to change for self phase modulation, only thing that will change is the corresponding phase. That means, if I launch an electric field in a medium where Kerr nonlinearity is there, what happen that it phase will going to modulate by itself and this phenomena is called the self phase modulation. So, today we will going to extend this phenomena or the discussion of this phenomena and we find out the corresponding frequency shift.

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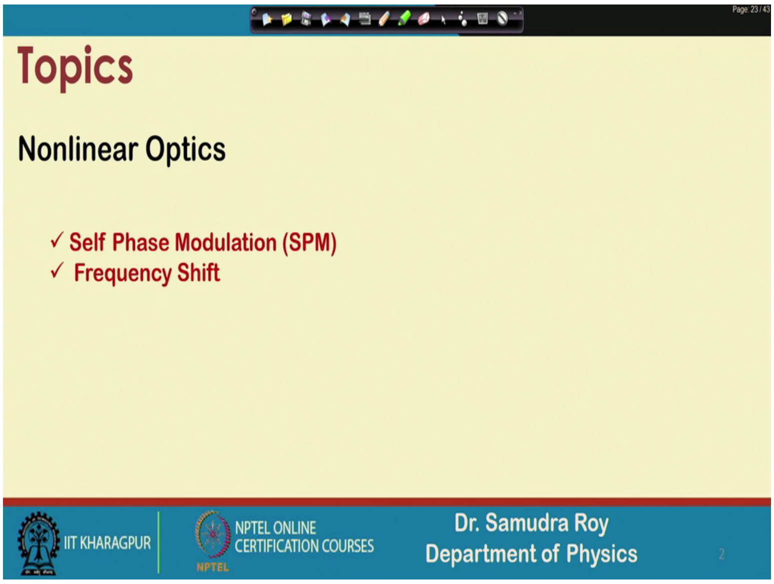

Since the phase is changing. So, what happened the some frequency shifting will be there and how this frequency shifting will take place because of this Kerr effect we will going to discuss in todays class.Well let us go back to our old equation.

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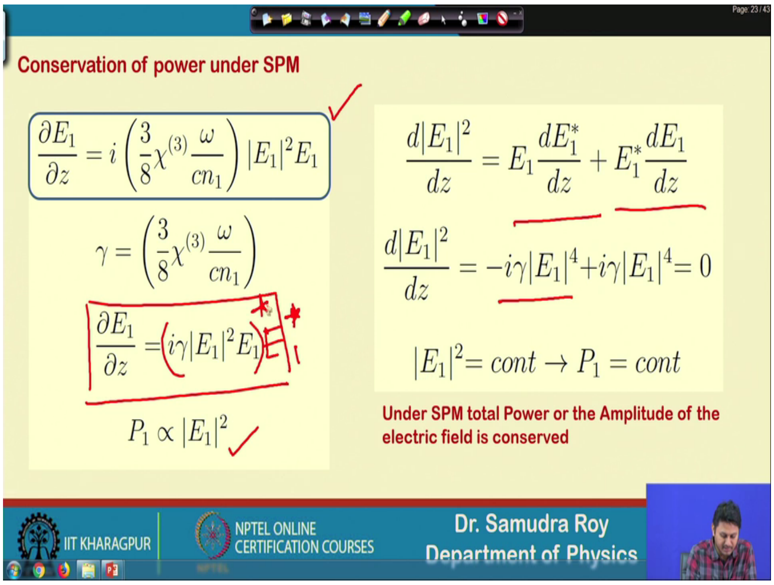

So, we have the expression of self phase modulation this d E 1 d z is i 3 divided by 8 chi 3 omega divided by c n 1. If I write this 3 8 chi 3 omega divided by c a n 1 then we have a straight forward if I define this as a gamma, we have a straight forward expression which is del E 1 till del z is I gamma mod of E 1 square E 1 this is the expression that we have, this is the differential equation for self phase modulation..

Well once we have the self phase modulation equation in our hand the next thing is to show that the power will remain conserved. So, E 1 will going to evolve over the distance, but is there any change in power and that will going to; so here we can we can find that in this expression everything is related to E 1 so; that means, there is no power exchange kind of term is may not be there, but in order to ensure this things let us find out how the power will going to evolve. So, power is clearly proportional to mod of E 1 square, power P 1 is related to the power of electric field defined by E 1.

So, if I try to find out the derivative of mod of E 1 square. So, d d z of mod of E 1 square is E 1 d 1 star z plus E 1 star d 1 z. So, E 1 star d z I can find from this equation. So, what should be the del E 1 star d z? It should be simply the complex conjugate of this quantity and the complex conjugate means there should be negative sign minus i gamma mod of E 1 square not going to change E 1 become E 1 star. Now this things will be multiplied by E 1. So, eventually we will have minus of i gamma mod of E 1 to the power 4, because this term is now will be multiplied by another E 1 star entire thing will be star this thing will be star and then it will be multiplied by.

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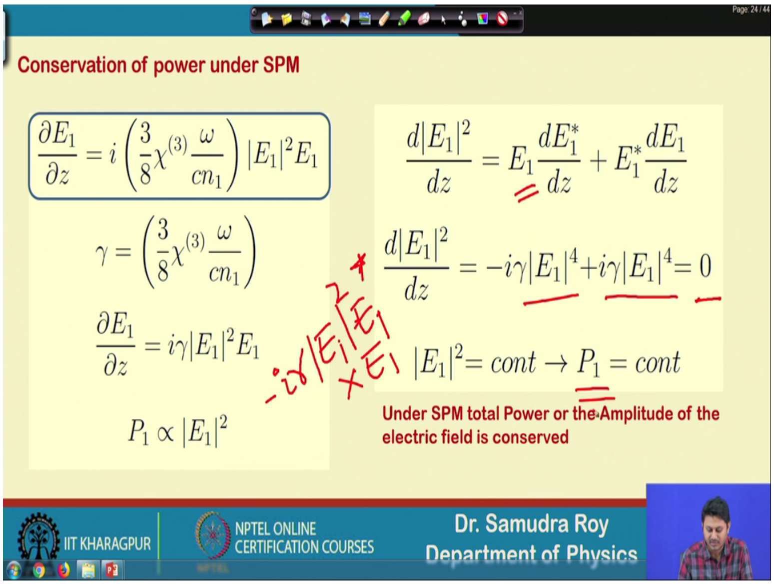

So, we will have a minus of i gamma mod of E 1 square, E 1 star and then another E 1 will be multiplied with this things. So, one we have another E 1 multiplication. So, we will have E 1 to the power of 4, mod of E 1 to the power 4. In the similar way the next term will be simply plus i gamma E 1 to the power mod of E 1 to the power 4 if I add these 2 things we will have 0 so; that means, E 1 is constant, so P 1 is constant.

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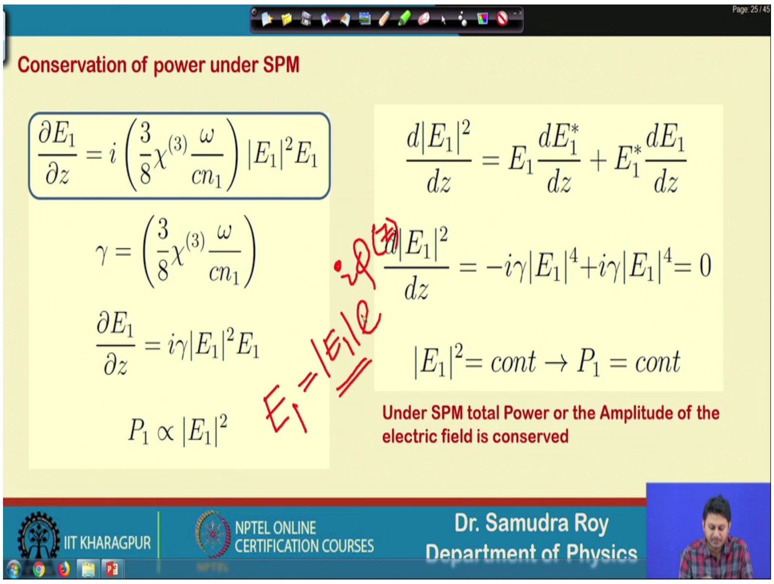

So, there is no change of power and E 1 which is a complex amplitude E 1 which is complex amplitude and can be defined as mod of E 1 plus some phase. So, this differential equations suggest that this amplitude part will not be a function of z.

So, only the phase term will be function of z. So, only the phase will going to change in the previous class we basically find out the same thing, that the phase is going to change and that is why it is called the self phase modulation.

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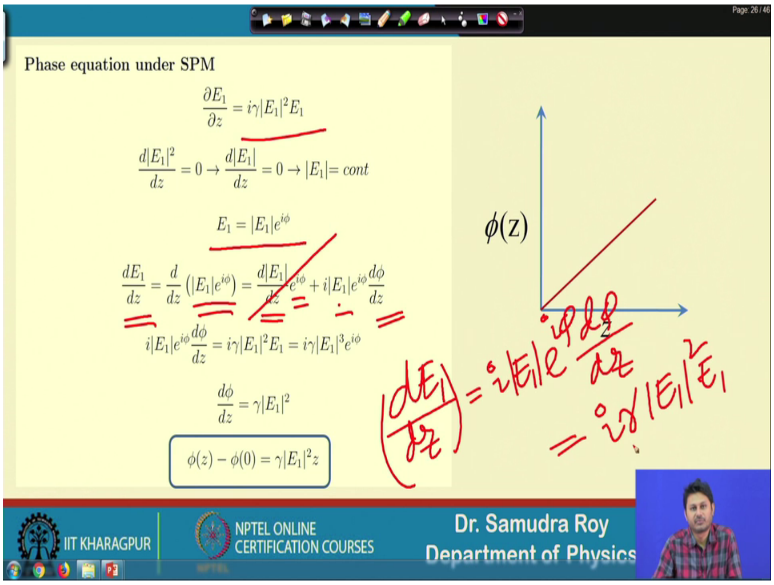

Well amplitude is now conserved, once amplitude is conserved as I mentioned in the previous slide that E 1 can be represented as mod of E 1 E to the power i phi. So, now, if I represent this E into amplitude and phase term and then if I make a derivative over that then I can write this derivative as E 1 E to the power i phi. If I divide this derivative we will have d E 1 d z e to the power i phi and i E i e to the power i phi d phi d z.

But we know that d i d z E i mod of E i is not a function it is a constant or not a function of z. So, these term will be 0, since this term will be 0 we will have d of E I d z is simply equal to i of mod of E I e to the power i phi and then d phi dz, di dz is again according to our equation is equal to i of gamma of mod of E I square E 1. So, we will have an differential equation of phi from this what we are having is a differential equation of phi. So, differential equation of phi will be simply this. So, phi is changing.

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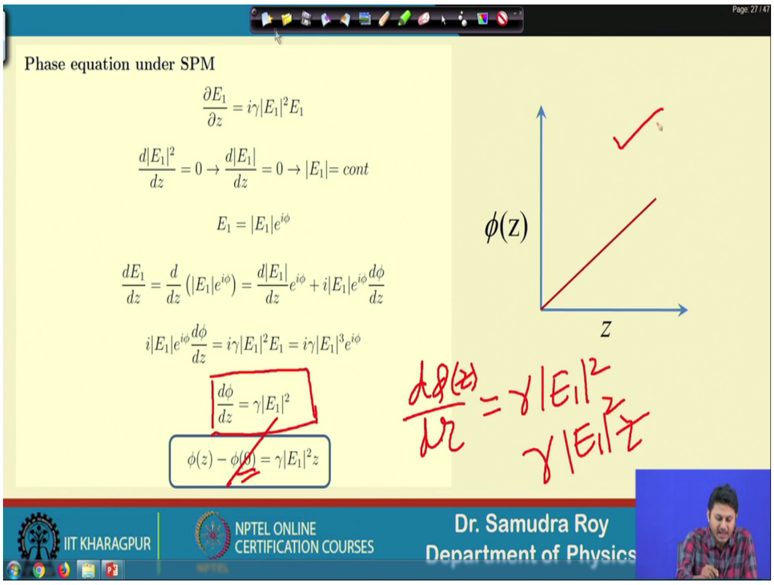

So, how the phi is changing is the thing that we are trying to find out, and we find that d phi finally, we find that d phi dz phi supposed to be a function of z is simply equals to gamma E 1 square. Well if I integrate these things because E 1 is not a function of z so we can readily integrate this if I integrate this things we will find that phi which is the phase of the electric field amplitude, will now linearly changing and it will be linear function of z and solution is simply gamma mod of E 1 square Z. So, if I integrate we can put this phi 0 so, that this is a different frequency, reference phase, that at z equal to 0 if is there any phase we can write phi 0.

But if I say this is equal to 0. So, phi z is simply function of z and you will change linearly as soon here in this figure. So, we have a expression of E and phi both, and we find that for self phase modulation the amplitude is not changing, but the phase is changing and the change of phase is linear in nature.

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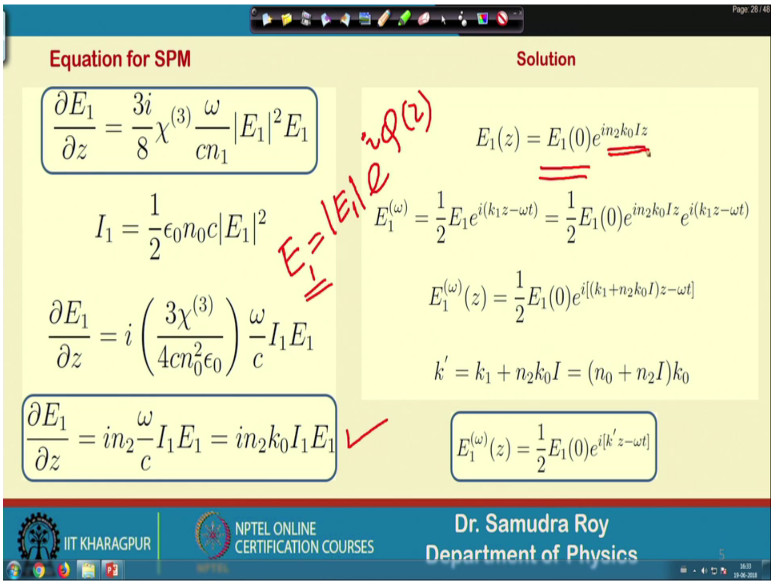

Well this thing we have already this solution we have already figured out in our previous class, once again to remind this things that how the phase is changing, in order to understand this phenomena in a different way, we again show these things. So, this is the equation these is the total equation of the electric field and this total equation of the electric field should contain both the equation of phi because this is the complex equation.

E is now having 2 part, one is mod of E if I write this E 1 mod of E 1 this the amplitude part and one is the phase part. So, one way is to do this problem by putting amplitude and phase part and separate it out and then find out the solution of E 1 and phi that we have done in the previous way or you can directly find out E 1 and put it here, and once you find out E 1 and put this things you will find that E 1 is now having E 1 0 the amplitude part, and the phase part to the solution part directly. So, the previous calculation we separately find out the solution of phi by solving this 2 equation.

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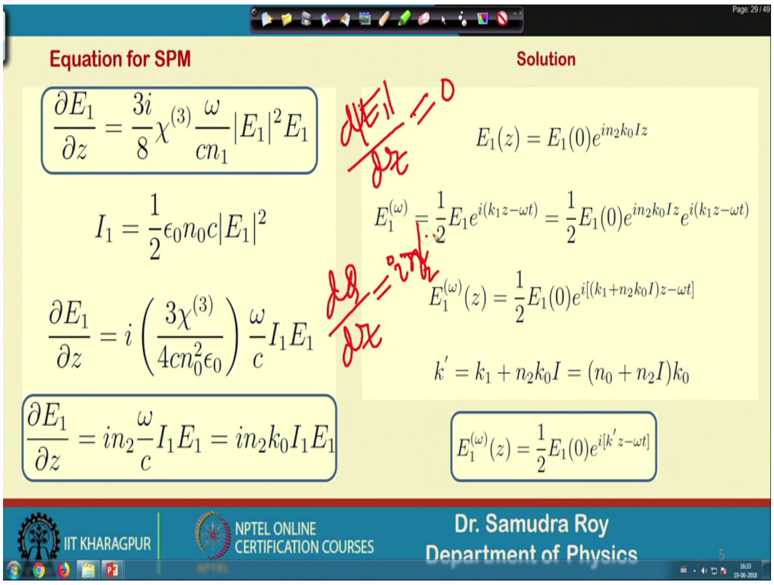

One equation is simply d 1 dz is equal to 0, that was one equation this is the amplitude equation and phase equation was d phi d z is equal to i into i in terms of gamma it was just gamma. So, let me go back and check if only gamma mod of E 1 square because inside the gamma all these n 2 and all these things were captured, but if I write this term into these things together, then we will have a solution in this solution suggest that that there is a the field will going to change, because the propagation constant k prime is now changed because of this non-linear effect.

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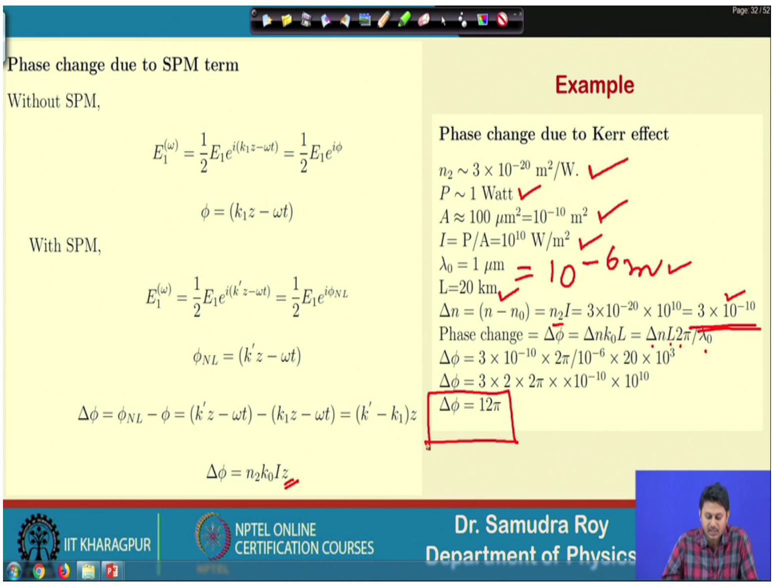

So, this solution we have already done, so just once again show that you can do that in this way also. Now the phase change can also be figure out and since we have a solution in our hand for phase we can directly do that in terms of gamma, but here the phase change one can calculate because E 1 omega is half of E 1 e to the power of i k z omega, which in terms of phase I can write as half of E 1 e to the power i phi. Phi is k 1 z omega t with SPM this is without SPM the way one can write the total electric field. With SPM one can again write same electric field, but only change is here is k prime. Because k will be k 1 was the propagation constant without any kind of non-linearity, and k is the propagation constant of the system with nonlinearity.

So, if I write k 1 z minus omega t as phi non-linear because this is coming because of this non-linear effect, then phi non-linear is this quantity. So, phi was the phase without any non-linearity and phi non-linearity is the phase with nonlinearity. Now if I try to find out what is the difference between these two so we will simply find that the difference between these 2 phase is simply n 2 k 0 and density in to z. Now all these the facility to find this form of phi 2 is we can we can readily find out for a given intensity what should be the change of the phase.

So, now n 2 k 0 i z all this value one can obtain and here is a example that what should be the phase shift non-linear phase shift if these values are given. So, the value that are given is into which is 3 into 10 to the power minus 20 meter per watt. The value of P is given as 1 watt, A is given at 100 micro millimeter square previously if you remember in one calculation we consider 100 millimeter square now it is micro millimeter square it is very small. So, 10 to the power minus 10 meter square the area. So, intensity with this area and power comes up to be 10 to the power 10 watt per meter square. The operating wave length is given as 1 micrometer which is 10 to the power this quantity is 10 to the power of minus 6 of meter.

Length here it is a z. So, over the distance this phase will going to increase the change of phase will be going to increase. So, we say this is 20 around 20 kilo meter. So, del time the change of refractive index for this if we calculate it is simply n 2 i. So, n 2 i is now 3 into 10 to the power 10, now what should be the phase change? The phase change will be delta n into k or simply n 2 k 0 I into z. So, delta n multiplied by k into 2 phi divided by lambda 0 because k 0 is 2 phi by lambda 0. So, delta n is 3 into 10 to the power minus 10 why it is delta n? Because delta n is nothing, but n 2 i.

So, just if I replace n 2 I to delta n. So, delta n l k 0. So, k 0 is 2 phi by lambda 0. So, delta phi simply becomes 3 into 10 to the power minus 10 which is this value multiplied by 2 phi divided by 10 to the power 6 the value of 2 phi by lambda and then length 20 kilo meter 20 into 10 to the power 3. So, if I calculate all these things then the final expression that we have is delta phi, the change of phase is around 12 phi, which is a good amount. That means, if pulse is launched due to this non-linear refractive index what happened that the phase will going to modify, and phase will going to change and the phase will change around 12 phi after a distance of 20 kilometer.

So, if I go with the non-linear phi with 20 kilometer the phase will change around 12 phi for this given set of parameters.

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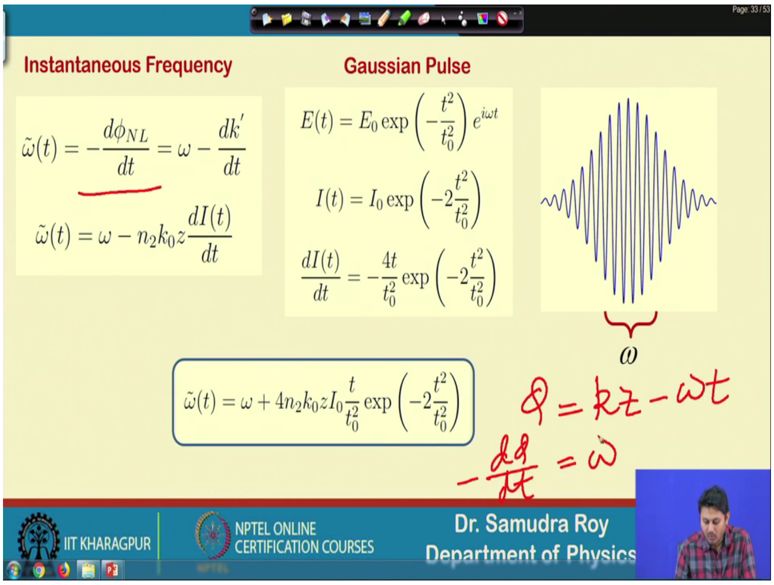

Now another important thing and that is the instantaneous frequency. So, instantaneous frequency is a non-linear phase divided by the time with a negative sign. So, this is the expression of the instantaneous frequency; since the non-linear phase phi non-linear is containing the intensity. So, now, it is important to know that this intensity is function of t so; that means so the value of instantaneous frequency can be a function of time. So, normally what happen let me let me right here. So, that so normally phase phi is given as k Z minus omega t. So, if I write d phi d t with a negative sign in the right hand side we simply have omega. This is the frequency of a system this is the frequency of a plane wave that is moving. Now our expression is slightly changed

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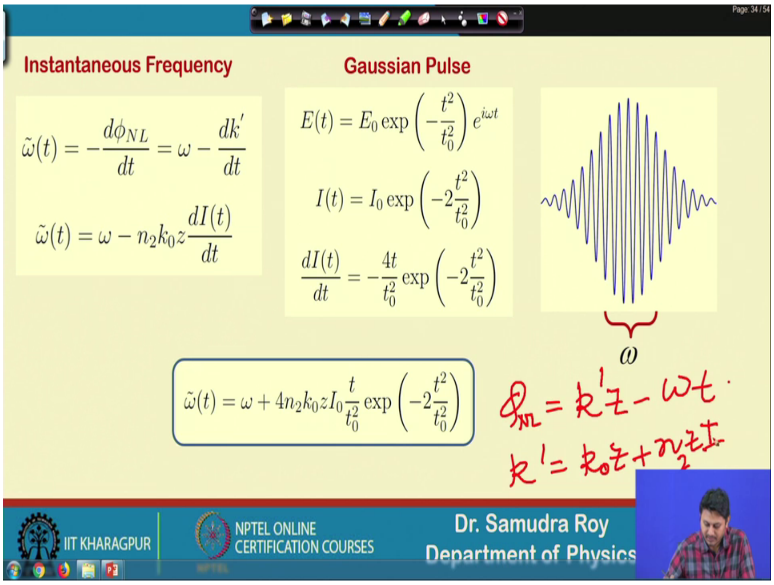

Because we now have non-linear phase and in non-linear phase what extra term we have? We have instead of k we have k prime Z minus omega t, what is k prime? K prime is k 0 Z plus n 2 of Z of n 2 Z and then I it is something like this.

So, what is k prime we already defined in the previous slide. So, let us check what is k prime. So, k prime is simply k 1 n 2 k 0 I. So, k 1 is the propagation constant corresponding to the refractive index n 1. So, now, what happen because of this k prime if I now put this entire equation.

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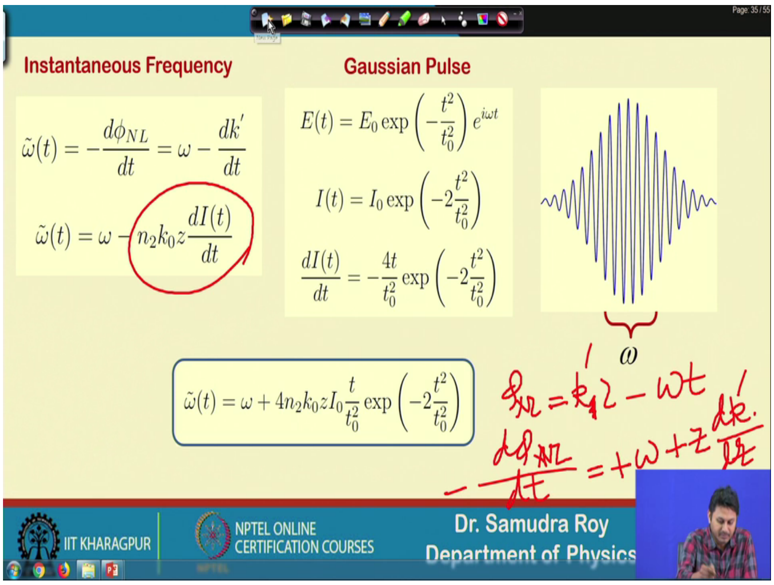

So, phi non-linear again is k prime Z minus omega t. So, when I make a derivative of these things to find out the instantaneous frequency non-linear dt.

So, we will have minus of this things is plus of omega derivative of this quantity plus Z multiplied by dk 1 k prime sorry k prime and dz. So, d k prime dz is this quantity so that means, the intensity is involved here and we know that intensity can be a function of time. If intensity is a function of time and if it is gaussian in nature then we can have the instantaneous frequency in our hand. So, this is expression of a plane wave moving with the frequency omega and this is a gaussian field. So, now, we have an electric field with gaussian distributions so this is a distribution of a gaussianic electric field with frequency component omega, if I plot the real part of this things we will have a this kind of structure.

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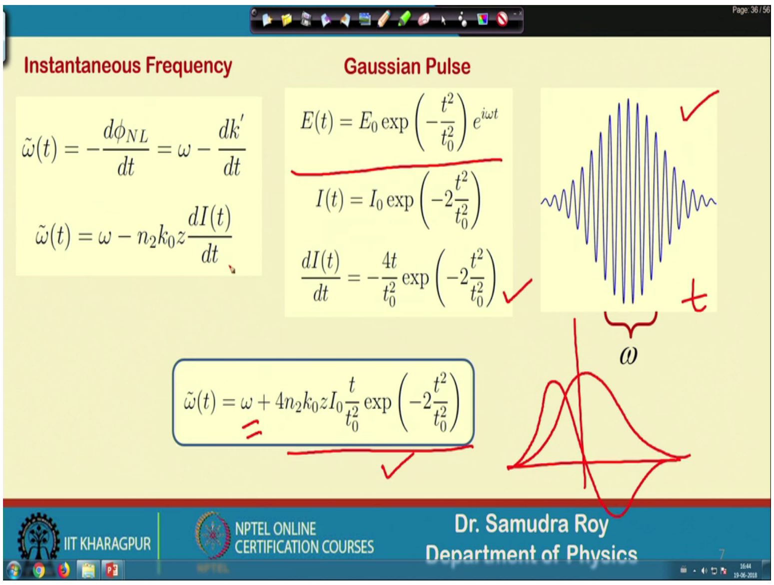

Intensity will be mod of square of these things proportional to so I 0 of this so distribution of the intensity will be something like this, it will be again a Gaussian. And now if I make a time derivative of these intensity we will have a term like this, which is the derivative of a gaussian function. So, we know that if I make a derivative of gaussian function I have plot that it will be something like this. So, instantaneous frequency is now omega 0 plus this quantity so; that means, if this is a distribution over time, see if I go over the time the distribution of the frequency which is uniform here, will not remain uniform because of this additional term and this is why because in our system we have a refractive index, which is now function of time also.

Because the intensity is a function of the refractive indices the function of intensity and now intensity is a function of time.

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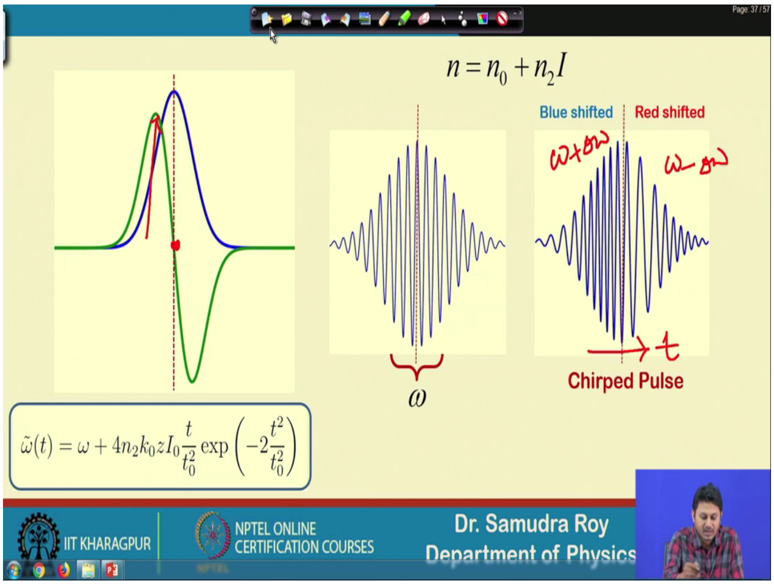

Well if I now use these things, if I now use this expression as I mentioned; then we can plot this together and if I plot this together we find the instantaneous frequency is now having change this quantity is changing. So, that the instantaneous frequency has to change and this is some sort of modulation that is over the frequency omega 0. So, omega 0 is the frequency shown in here and what happen that in the left side this quantity will add up, and then at the centre it will not going to change and then it goes to the negative direction.

So, if I look this things qualitatively, then we find that this is the uniform distribution of the frequency because of this got Gaussian distribution of the intensity, what happen the instantaneous frequency is now function of time. So, if I go this things it is a function of time. So, first part of the intensity first part of the frequency is now will be chirped or confined more. So, here the frequency is increased so we will have something omega plus delta omega, which is the addition of this quantity; and then what happened here we have some frequency reduction. So, some frequency will reduce this part because we have a negative portion in this region. So, one portion of the pulse will be blue shifted and another portion of the pulse will be right shifted, because of these typical distribution of intensity or the derivative of the intensity rather.

So, this kind of pulse is called a chirped pulse. So, one very important consequence is that when the light is propagating through a medium, and if the medium is non-linear in nature and Kerr effect is there. So, what happen that is phase will going to modulate, not only that the frequency component is also going to change. Some different frequency component will appear because of this distribution of intensity or the derivative of this intensity.

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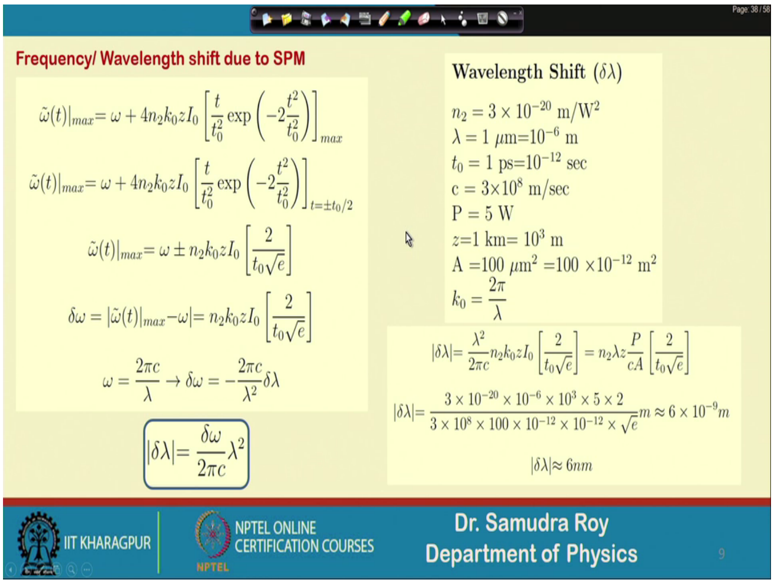

Well if I now try to find out what is the frequency change of that we can readily find out this I can give you as an homework, but all the problems are done here, but I ask you to do that by your own.

We will discuss this anyway in this class, but I will not going to discuss in detail, I just give you the overview of the problem and I ask the student to please do this problem by your own. So, the problem is what should be the maximum change of the intensity. So, if I go back to this figure then I say that here the frequency is going to going to be changed. So, here what happened that I change the frequency this is the; so we have a frequency here omega.

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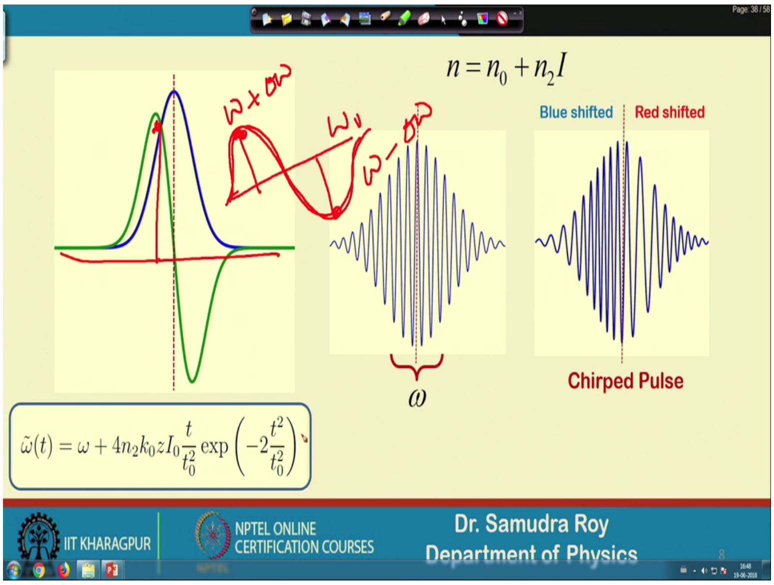

So, on top of that we have a modulation. So, some portion of the frequency will be added up, some portion of the frequency will be subtracted. So, you can see that this is a gradual change all together. So, there should be some value for which we have the maximum change of frequency omega plus delta omega this is maximum change.

And that is a maximum deviation or maximum reduction of the intensity also in this particular point. So, the question is if the distribution is Gaussian what should be the maximum change of the frequency, what should be the maximum change of the frequency. Well the result is already shown here in this slide, if you look this slide carefully I calculate these maximum value of the instantaneous frequency. So, the maximum value of the instantaneous frequency will be simply the maxima of this term which is sitting over here, and if I calculate the maxima of this function by making the double derivative of that and making it 0 then we have the value of t at which it will be maximum.

Next thing just need to put this value of t here, and eventually you will have delta omega change maximum frequency change, which is having a formula like this. Once we have this form the next question will be asked that what should be the corresponding frequency corresponding wavelength shift. Because wavelength is; that means, if I launch a particular wavelength lambda it will going to shift. So, what should be the shift of the wavelength due to this intensity distribution, tempered distribution of this intensity which is Gaussian in the form. So, these are the value that is given and if you put this value and do all this calculation you need to use this expression which is here and once you do that you will find for the given value the change of the frequency or the wavelength is 6 nanometer. So, 6 nanometer wavelength will shift because of this which is not that much but we can increase this radically if I increase the amount of power or if I reduce the corresponding area.

So, if I reduce the area the value of the non-linear effect will increase radically, and this value can change up to many nanometers, but anyway for the given values we will find that this will be around 6 nanometer. So, with this note let me finalize today's class.

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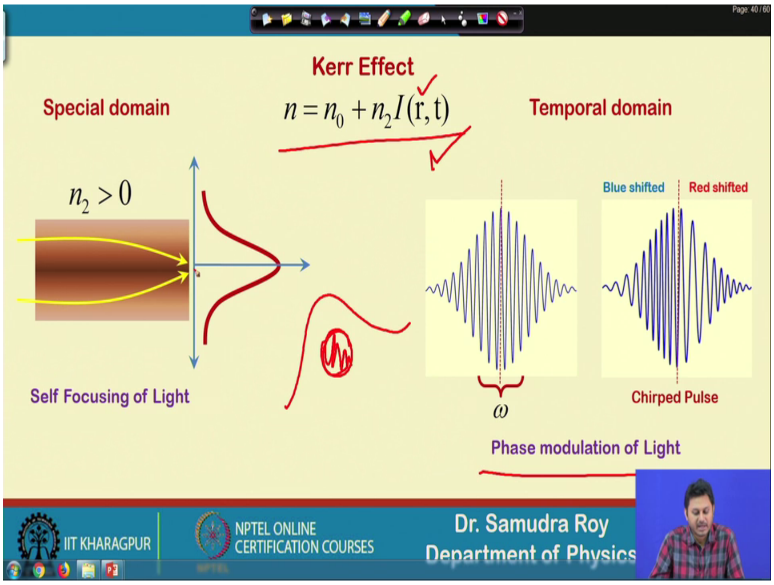

So, 2 important thing we know to this Kerr effect and that is very important this concept you should understand very clearly that this is my Kerr effect; and in the Kerr effect we have intensity in my hand and this intensity is the function of position as well as there is a temporal part of the intensity. So, when we say that the laser light is falling over some place, so it is illuminated over a space.

And this is the distribution this spatial distribution, due to the spatial distribution what happen that we have a self focusing kind of effect, but also the intensity has some distribution in temporal domain. So, when the intensity is temporal domain what happen that we have the phase modulation of light. So, light will be modulated and this 2 kind of modulation is there, in one case it will be a self focusing when you consider the intensity is distributed over space and if the intensity is distributed over time, then another phenomena will be there and this phenomena is the chirping of the pulse or the shifting of the of the frequency, this is called the non-linear shift of the frequency or the self phase modulation in other term. So, this will appear when the intensity is a function of time.

So, with this note I would like to conclude the class here, in the next class we will discuss more about other issues so.

Thank you for your attention and see you in the next class.