Introduction to Non-Linear Optics and its Applications
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Lecture – 43
Symmetry in Third Order Susceptibility (Contd.), Self Phase Modulation (SPM)

So, welcome student to the next class of Introduction to Non-linear Optics and its Applications. In the previous class we discuss about the Kerr effect and then we started something related to the symmetry in susceptibility third order susceptibility.

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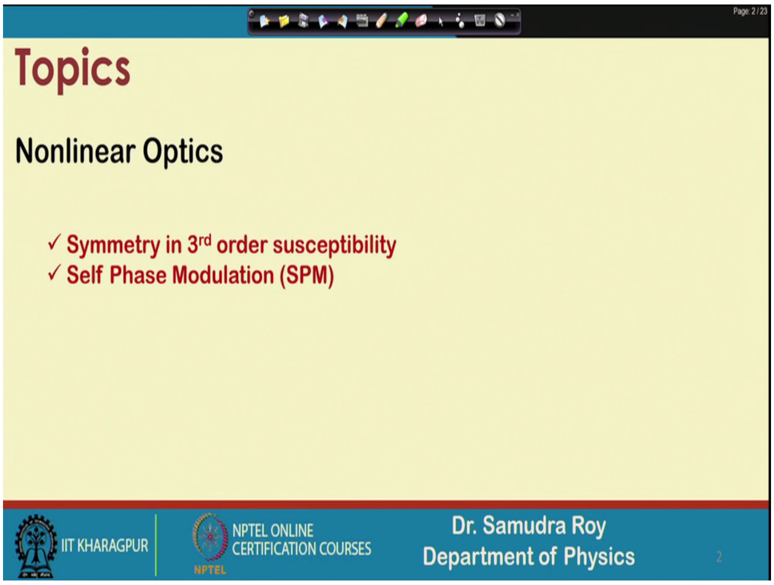

So, today we have lecture number 43 and let us see what we have in today's lecture. We have the symmetry in third order susceptibility and then study the self phase modulation.

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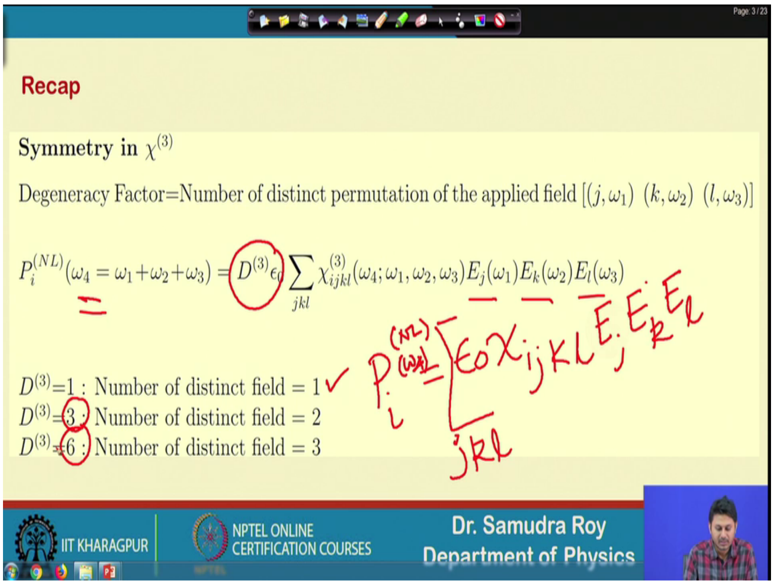

So, symmetry in the third order susceptibility we started this very quickly we mentioned in the last class that, if I want to find out the degeneracy factor.

Because when the non-linear polarization is written, it is in the component form it is written as ith component of non-linear polarization if I write in non-linear here is epsilon 0 chi i j k l and then E j E k and E l. Now when I write E j E k E l and i j k here these are the repetitive indices. So, we know that this repetitive indices means there should be a summation sign over here. So, we should have a summation sign over j k and l.

Now, the question is if we have these kind of formation that the if this has a frequency component omega 1 omega 2 and omega 3 and then if these 3 things are multiplied together this things j k and l are distinguished, then what happen then we will have a frequency component omega 1 plus omega 2 plus omega 3 and with this we write this frequency component as omega 4.

So; that means, p non-linear here omega 4 will be the multiplication of this terms E j E k E l both having frequency component 1 2 and 3. Now if any of these 2 are same or 3 of them are same or none of these things are same, there should be 3 different conditions and for 3 different conditions what we have a degeneracy factor. So, one factor will appear here, and this degeneracy will come this degeneracy factor will come because these things may same.

If they are same then we will have one distinct field and we have D equal to 1. So, this is 1. If two of them are same and one is different then we have a value 3 here and if none of them are same. So, 3 are totally distinct field, then we will have a degeneracy factor 6 here.

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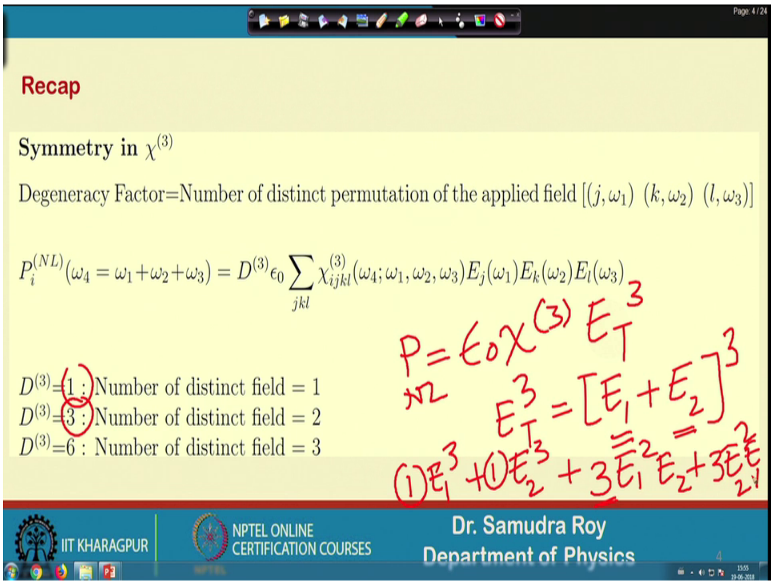

So, this term will appear several times when we do the calculation and why they are coming we can understand very easily, because this term if I write my total polarization in scalar form the non-linear polarization rather in the scalar form, which is very effective as I mentioned several time it should be E total whole cube.

Now, if E total contain 2 different fields say E 1 and E 2. So, this cube is nothing, but the cube of this. So, eventually we have a plus b whole cube. So, when we will we have a plus b whole cube. So, we will have one term like E 1 cube one term like E 2 cube and the combination of these 2 like 3 E 1 square E 2, and 3 E 2 square E 3.

Since we have only 2 distinct field, we have in number 3 here and in number 1 here as a degeneracy factor, because if I expand it should be E 1 cube, which has a degeneracy factor 1 plus E 2 cube which are the degeneracy factor 1 here, and then we have E 1 square E 2 two fields are involved among which we have 2 identical fields.

So; that means, its follows this. So, we have 3 term here and another case we have 3 E 2 square E 1. This is also possible and here also we need to write 3 because 2 distinct field and one field is different.

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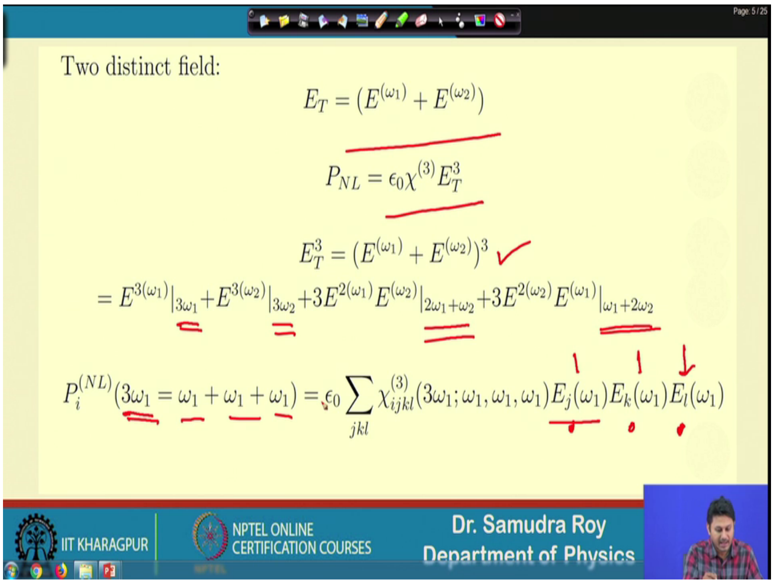

So, in order to understand these things let us put some kind of example the same example I showed just I show. So, here this is the case, so total electric field is this E plus E 2 where the frequency component omega 1 and omega 2 are there.

So, 2 different frequency component, so P NL is nothing, but E T whole cube E T is this plus this. So, I need to make a cube of that. So, when I make a cube of that what should be the frequency component that is important, so in first case we have E E cube of omega 1. So, when we have E cube of omega 1 the frequency component written is written here, we have 3 omega 1 frequency, E 3 of omega 2 we have what frequency component we have 3 omega 2 frequency component.

So, total frequency component here is 3 omega 2 3 of E 2 omega 1 and then E omega 2. So, what will be the frequency component? The frequency component will be 2 omega 1 plus omega 2 and in the similar way here the frequency component will be omega 1 plus 2 omega 2.

Now, if I want to find out my frequency component this is a specific structure and for the specific structure we have 3 fields sitting here, and from this 3 fields I now want to find out my non-linear polarization with a particular frequency, you can see there are different frequency that can appear in P non-linear.

So, P non-linear can contain 3 omega frequency it can contain 3 omega 2 frequency, it can contain 2 omega 1 omega 2 frequency component and omega 1 2 omega 2 frequency component for this particular structure. So, therefore different frequency components are there which can appear in P non-linear, but we now only need to find out the frequency component 3 omega 1.

So, how we can find 3 omega 1 frequency by adding omega 1 omega 1 and omega 1? So, when I add 3 omega 1 together then only I can have 3 omega 1 frequency. So, this we can only have when we have multiplication of this first 3 things and once we have a multiplication of this first 3 things. So, these 3 fields have to be the same field because they have the same frequency components, and we have a degeneracy factor here and this degeneracy factor have to have value 1 because they are all same. What should be the frequency component of 3 omega 2?

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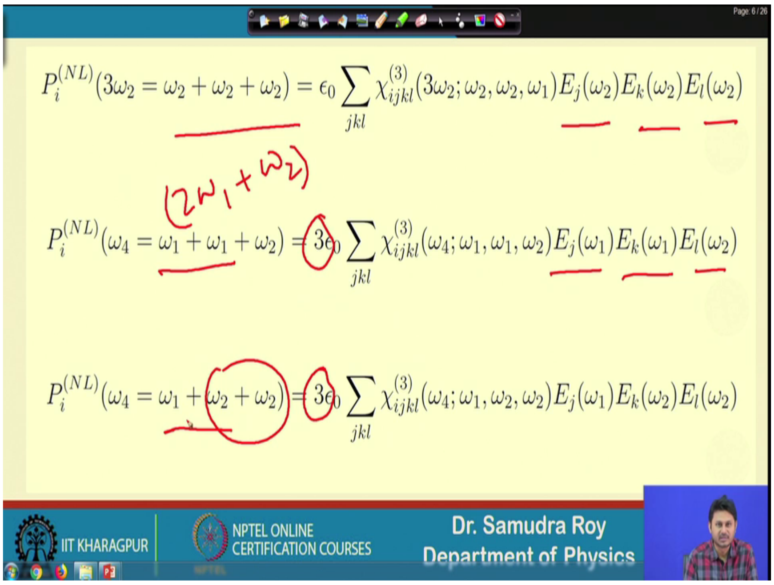

So, 3 omega 2 frequency component one can have for this particular case by omega 2 plus omega 2 plus omega 2, again the degeneracy factor has to be 1 here because all this frequency components should be same. What should be the frequency component of 2 omega 1 plus omega 2? This is the frequency component that should have in P non-linear in that case what happened I need to add 2 same frequency omega 1 omega 2 and one different frequency component; that means, other field will be multiplied and that is why we have a term 3 here.

In the similar way we have a degeneracy factor 3 here, because again we have 2 omega and 1 omega 1. So, 2 distinct field and one different field; that means, a degeneracy factor 3 will be multiplied here if I want to calculate that non-linear polarization of frequency omega 4. So, in this way we will calculate this. So, please remember this because we will going to use this in some future classes.

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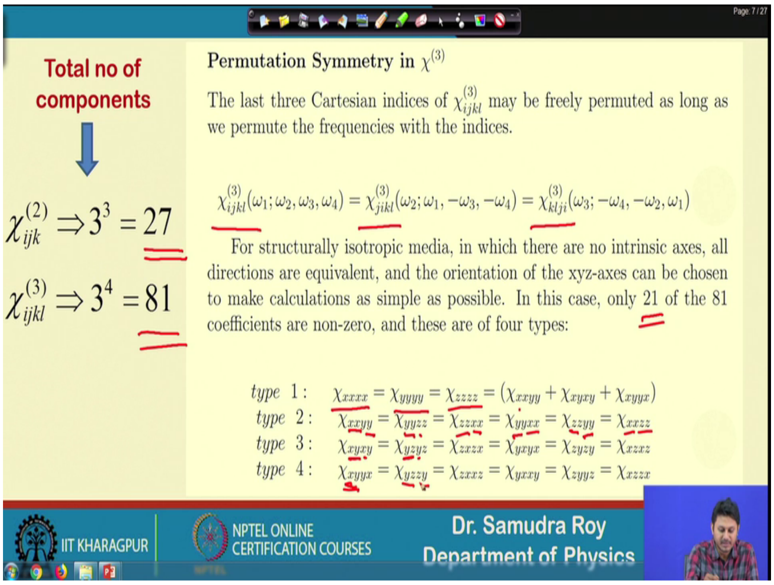

So now some other permutation symmetry of chi 3 so permutation symmetry we have already seen when we calculate the second order susceptibility, because of the Neumann symmetry and other symmetries we find there are many components that will going to be vanished and there are many components which are equal to each other, and eventually we find that there are few components, which are distinguished other components are same. Here in this case also we will have a similar thing. So, here we have a comparisons so total number of component of chi 2 is 3 to the power 3 which is 27 that we know. So, there should be 27 different components, but we find that among this 27 not are not all are different there are many same components are there.

So, we can reduce the total matrix, here also for chi 3 we have 81 different components, but it should have a permutation symmetry. So, because of this permutation symmetry what happened that susceptibility i j k l should be equal to susceptibility j i k l, k j k l j i and so on and also for a specific isotropic system, the susceptibility terms this different terms the 81 different terms can be reduced to only 21 different terms because they have some internal symmetry here.

So, this internal symmetry suggest that x x x x, y y y y and z z z z they are same if all these components are same then they are equal and these things will be again equal to x x y x, x y y x, y x y and x y y x there can be 4 different types of this. So, one type is 2 components x x and y y as sitting side by side like y y z z and z z x x and then again x x y y, x x z z, y y and x x z z.

So, these all these components are same again there are some components x y x y, y z y z these are again of type 2 they are same. Finally, we have a mirror kind of things. So, x y y x, y z z y these components are again same to each other. So now if I try to find out so this these are 21 different elements that one can reduce for isotropic system other components are not there so, how this 21 components are there you can see 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21.

So, these 21 components are there, but they are also related to each other. So, eventually we will have only 3 this this and this and this this 4 different and again we have one equation to in our hand so, that we can have only 3 components that are different. So, among 81 components we have only 3 components that are difference. So, this kind of symmetry can be possible in chi 3.

So, you just remember that it looks that it should have 81 different components, but the thing is that because of the application of the symmetry or the material symmetry we can reduce this radically and eventually have 1 to 2 components which are non-vanishing or they are distinguished.

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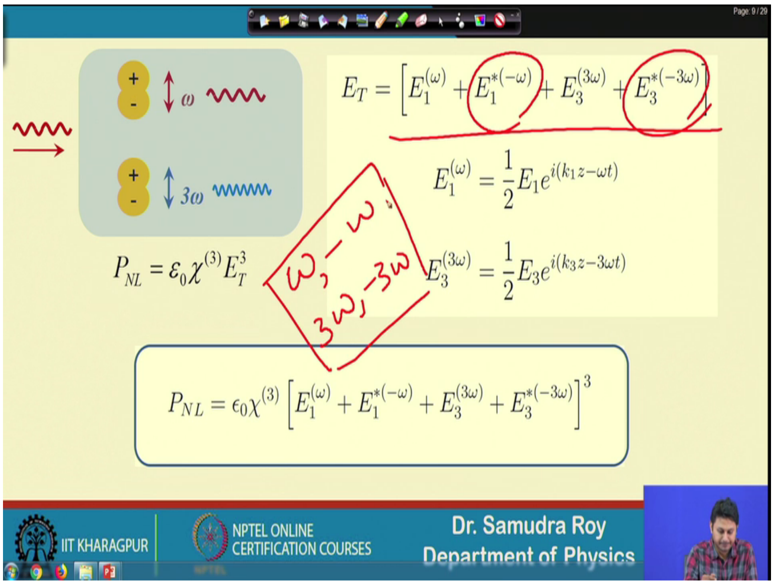

Well after having this knowledge of susceptibility, symmetry of the susceptibility, let us again go back to our original problem.

That is if I launch an electric field omega then in the system, 2 frequency components will generate of 3 omega and omega and if it is generated, then the total electric field inside the material is now represented in terms of omega 1 and minus omega 1 and 3 omega and minus 3 omega like this. So, it is omega minus omega 3 omega minus omega, so complex conjugates are added here.

So, eventually we have 3 4 different frequency component, we can consider this complex conjugate components are also different frequency. So, there are 4 different frequency component omega, minus omega, 3 omega and minus 3 omega. So, inside the system let us consider 3 different 4 different frequency components and now they are combined with themselves and find different frequency component of P NL. So, the challenge here is to find out the different P NL value with different frequency component.

So, now the P NL can be represented as susceptibility epsilon 0 into third order susceptibility multiplied by E 1, E 1 star, E 3, E 3 star. E 1 have frequency component omega it has frequency component minus omega it is 3 omega and it is minus 3 omega and cube of that thing.

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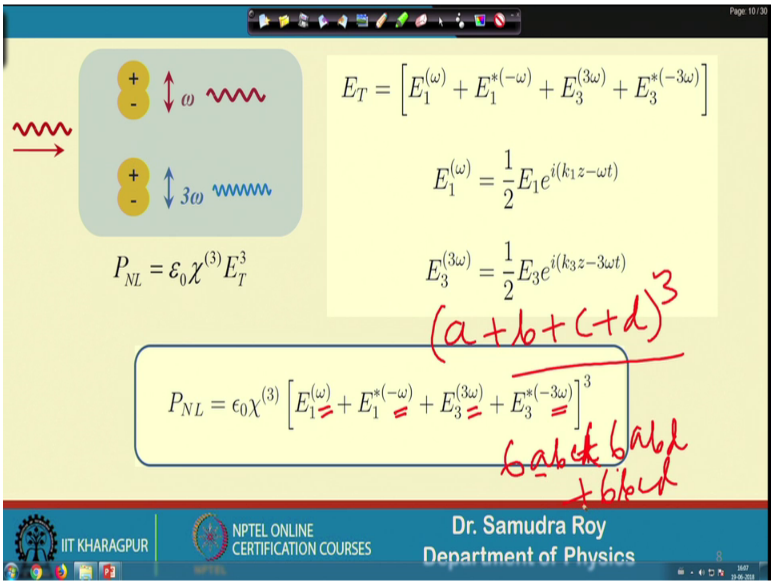

Once we have cube of that thing eventually what we are doing, we are solving the problem a plus b plus c plus d whole cube of that. So, how many terms will be there you can imagine. So, these amount of terms are there, so these amount of different frequency component will be there, a cube b cube c cube d cube will be there for one frequency component will be 3 omega, another frequency component will be minus 3 omega.

Another frequency component will be 9 omega and then minus 9 omega and so, on there will be other term also. If I if you calculate this a plus b plus c plus d whole cube, you will have a cube plus b cube plus c cube plus d cube plus 3 of a square b plus 3 of a square c plus 3 of a square d plus 3 of b square a 3 of b square c 3 of c square d and so, on. So, all the all the combinations will be there and also one combination will be there and that is 6 of a b c d something like this 6 of abc and then plus 6 of a b d, plus 6 of b c d this kind of components are there.

So, abc is a 3 different components that is way 6 are there a b d are different components that is why 6 are there. So, for all different components we have 6 degeneracy term are there which we have already discussed in the previous slide.

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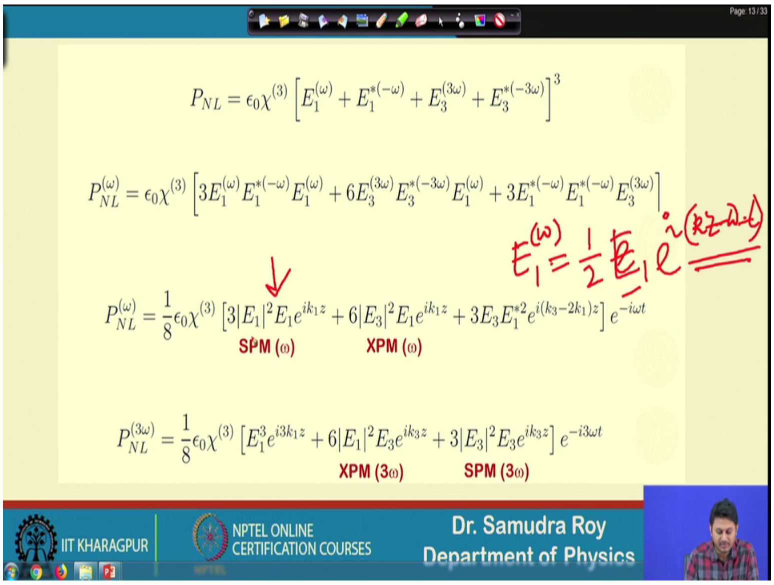

So, as I mentioned there are different components will be there, but our goal is to find out only few components and this few components are omega, how the omega components are there and 3 omega components. So, if I make a cube of this quantity.

So, how can get how we can get the omega components? So, omega components the first way to generate the to get omega component is E omega multiplied by the E omega star. So, there will be omega 1 omega minus omega. So, 0 and then multiplied by E 1 omega. So, one term will be here, so that we have omega component also we have E 3, E 3 star which gives you 0 omega component and then multiplied by E 1 omega we will have the omega component. What we need to do that whatever is written here in the bracket you just add and check whether we you are having a omega or not.

So, if you have omega then these are the omega components so; that means, the total contribution of this this terms is omega. So, this term will have a frequency component omega. Again one can have omega by E 1 star, E 1 star which is minus of 2 omega multiplied by E 3 with 3 omega. So, if you add minus of 2 omega minus of 2 omega which is minus of omega minus of omega it is minus of 2 omega plus 3 omega again you will have omega.

Interesting thing is this again the degeneracy factor you can see we have have to have a degeneracy factor 3 here, because E 1 E 1 star is different, but E 1 is same. So, how many distinct fields are there only 2 distinct fields are there E 1 star and E 1. By the way by the way here E 1 star and E 1 should be considered 2 distinct fields.

Here we how many distinct fields E 3 E 3 star and E 1, 3 distinct fields are there since 3 distinct fields combined to generate omega we will have a degeneracy factor 6 here because according to our rule if 3 fields are different. So, we will have a degeneracy factor 6 here again E 1 E 1 are same, E 1 star E 1 star are same E 3 is different. So, we will have degeneracy factor 3.

So, now if I recollect all this term we will have few terms here, and this is this. The first term is mod of E 1 square E 1 with a 3 multiplication, and E to the power of i k 1 z because k 1 z will be the phase term, this term is related to self phase modulation. Because we know that E 1 is half of e to the power E 1 omega is half of E 1 e to the power of i kz minus omega t minus omega t. So, if I use this E 1 here then what happen this exponential term will have we will have this exponential term and this amplitude term and we write this in this way.

So if I write this in this way, so exponential term will come here so entire term will have a frequency component omega. So, e to the power i omega t will appear all the times so it I can take it as common and then we will have this term, which is due to self phase modulation this term which is related to the cross phase modulation and finally, other term which is related to the third harmonic generation term we will discuss later.

Well in the similar way if you try to find out what are the 3 omega components, then I can I can give this as a simple home work for you. Then you will find that there are terms which you will gather if E 1 cube which is the third harmonic generation term and cross phase modulation term and self phase modulation term.

What is cross phase modulation, what is self phase modulation let us try to understand very briefly, here we have a self phase modulation because all the terms are related to E E 3. Here we have a cross phase modulation term because one term is related to mod of E 1 square and another term is E 3. E 3 is a natural term because we are talking about 3 omega frequency.

Here also we have a cross phase modulation because E 3 cube is there and natural frequency term E 1 is sitting over here and also E 1 mod of E 1 and E 1 is sitting here.

So, this is the same terms so that is why it is called the self phase modulation and cross phase modulation. In self phase modulation what happened the pulse phase will modulated by its own and for cross phase modulation; that means, the pulse will get modulated the phase of a pulse is get modulated by other other field.

And here the other field is the E 3 field that is why it is called the cross phase modulation well.

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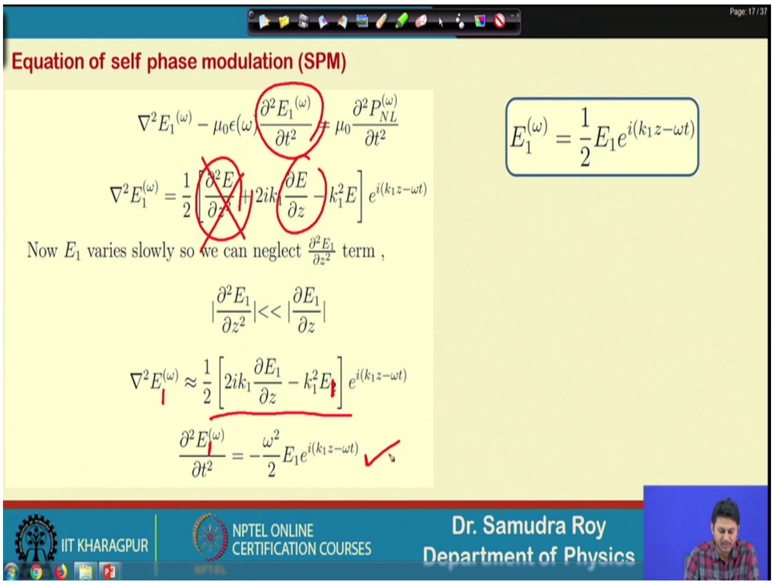

Once we have all this term, now it is time to find out the equation for self phase modulation and in order to find out the equation for self phase modulation, we just need to solve the non-linear Maxwell's equation that we have been solving for last few classes.

So, this is the standard form of Maxwell's non-linear Maxwell's equation, we need do not need to explain this every time because I believe by the time you are very much aware of this equation, because this is the mother equation and from that we need to derive the evolution of the fields, the field amplitude rather total electric field if this. So, once we have the total electric field from, now it is time to have this value and this value related to E 1.

So, if I do the grad square of E 1 is simply this quantity; double derivative E single derivate of E and the derivate of exponential of term, which gives me E 1 k 1 square with

E to the power ikz minus omega t phase sitting over here. Now we use the slowly very approximation he is suggest that this term will going to very very fast compared to this. So, we can neglect this term and if I neglect by equation will now confined with these terms, also we have the derivative of E 1. So, once we have the derivative of E 1. So, if I do this things. So, here in the equation I just miss E 1. So, you should put it E 1 you had (Refer Time: 24:53) it should be E 1 and here it is typing mistake. So, here should be E 1 anyway.

So, if I do this derivative with respect to this t, then what happened that we will have minus omega square term outside. So, minus omega square term outside and rest of the term will be simply E 1 E to the power i k 1 z minus omega t. So, we can evaluate this thing quite easily because you have been doing that for our last few classes, but interesting thing is the source term. Interesting thing is this source term this is the source term. If you if you remember you will find that this source term is containing many things here.

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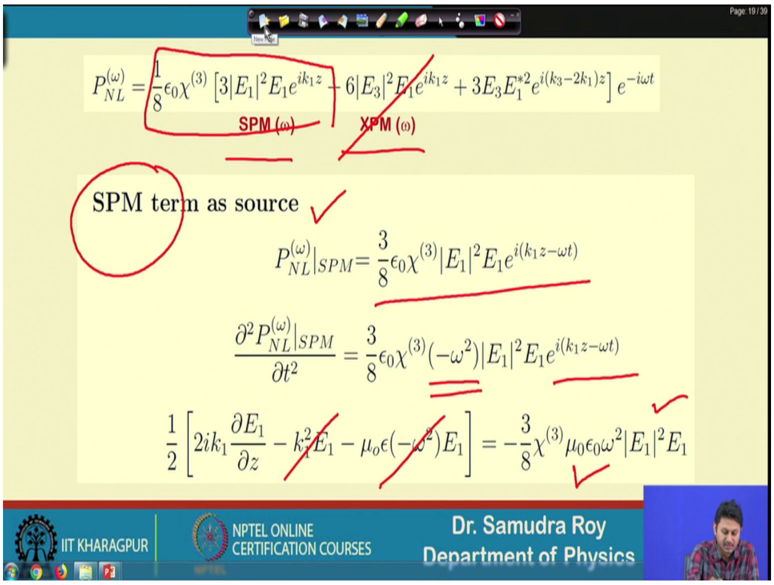

So, if I consider P NL omega then we find that it is containing many terms here. So, we need to take care of these issues. So, now, we will have a source term containing the self phase modulation cross phase modulation term, but we are now only looking for the self phase modulation effect. So, I am only taking the self phase modulation as a source term.

So; that means, only consider this term, we are now not considering this effect for the time being we will do that in the later cases. So, if I consider this as a source term, the derivative of this quantities simply will be 3 8 3 by 8 epsilon 0 chi 3 and then minus of omega square mod of E 1 square E 1 and this quantity.

Now, if I plan this things to our original equation, this equation will be simply this. So, we know that this term and this term cancel out eventually because k we know that this is always this 2 terms are always there and they will going to cancel out each other, and now this source term is slightly different. One interesting thing here is note that this a naturally phase matching thing.

So, in the right hand side we have E to the power i k 1 z minus omega t in the right hand side also we have E to the power i k 1 z omega t. So, this phase thing will not be there. So, it is a naturally phase match term. So, self phase modulation will naturally happen whether it is a phase matching or not.

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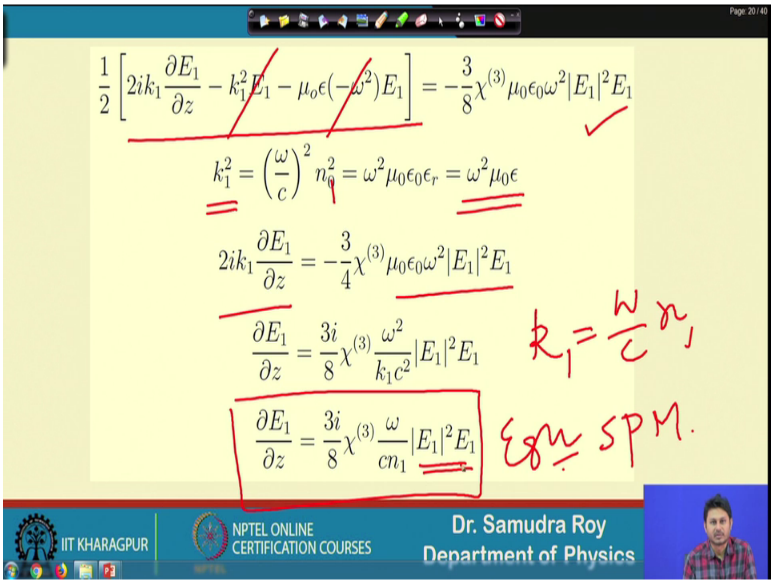

So, we have this term in our hand again this thing and this thing as I mention cancel out because k 1 if we calculate it will be omega square mu 0 epsilon, and k 1 if this term is omega square mu 0 epsilon this negative negative term will be positive and this term this term will cancel out and right hand some terms will be as usual. So, then we will have 2 i k and this things in the right hand side and then if I put this 2 k i in the denominator and just put the value of k 1, k 1 is omega by c into n n 1 here I put n 0 it should be n 1 because I am my notation it is now n 1 anyway.

So, it will be simply this quantity. So, this is the equation for self-phase modulation SPM. So, this equation suggest that a field amplitude E 1 will evolve and in the right hand side all the term are E 1, so that means, that field will going to evolve and this evolution will not depends on any other field.

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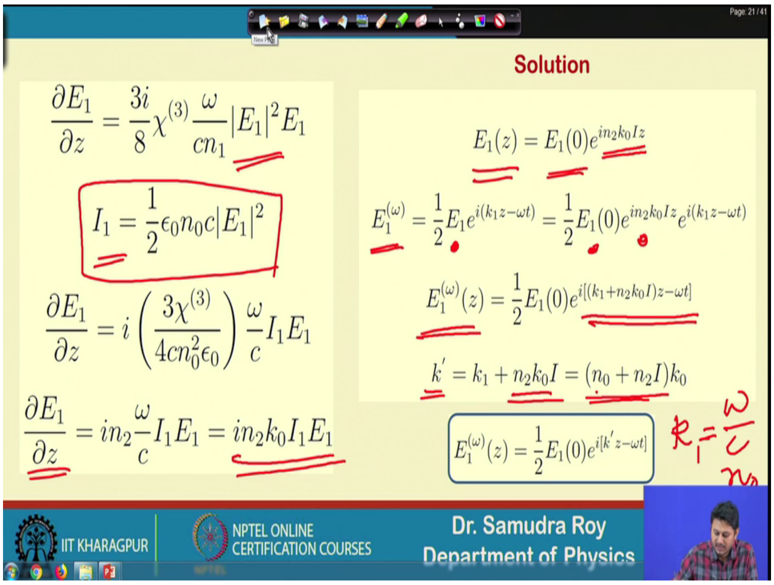

So, E 1 will evolve by itself if I want to write this things if I want to write this things in terms of intensity we can always write this. So, mod of E 1 square I can write in terms of intensity, and if I replace this intensity with this our famous equation then eventually we will have del E 1 digit is i in 2 k 0 I 1 E 1.

So, my equation become quite simplified and this simplified thing should have some solution. And if I find this solution it will be simply E 1 z is equal to E 1 0 e to the power of I into k 0 i z said. So, we have a modulation in phase. We will see that in the next class also, but here if I want to find out the total electric field E 1 omega which is half of E 1 E to the power i k 1 z minus omega t, then this E 1 which is the amplitude of this quantity can be now replaced by this solution. So, I write E 1 0 and then the corresponding phase so that means, amplitude of these things will not going to change only the phase will going to evolve with z here you can see that when z is equal to 0 E 1 0 is E 1 z is equal to E 1 0.

So, gradually the phase will going to evolve and if I now write this total phase of the total field E 1 z, then I find that these things is now changing these total electric field the phase term is changing and in this phase we have a propagation constant. Now my non-linear propagation constant keep k prime is k 1 plus n 2 k 0 I. So, n 2 k 0 I can be written in this because k 1 is simply omega by c into n 0 so omega by c is k 0 so k 0 if I take common. So, n 0 plus n 2 I multiplied by k 0. So, this refractive index change multiplied by k 0 is our new phase. So, this is our solution this is our new phase.

So, what I find that only the phase of this things will going to change, when I calculate the entire thing we find that only the phase of electric field is going to change because of this Kerr effect, which is called the self phase modulation. So, today we will like to conclude our class so today we learn about the self phase modulation and for self phase modulation we find that, if a electric field is launch in a system which has a (Refer Time: 31:30) non-linearity what happen that it will going to evolve and because of the evolution of the phase the propagation constant k is now changed, previously it is k right now it is k plus some term and this extra term is coming because of this non-linear refractive index change. So, with that note let me conclude here. So, thank you for your attention, see you the next class and we will discuss the self phase modulation again in the next class in detail.

Thank you.