Introduction to Non-Linear Optics and its Applications
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Lecture – 42
Optical Kerr Effect and Self-Focusing, Symmetry in Third Order Susceptibility

So, welcome student to the next class of a Introduction to Non-linear Optics and its Application. So, we have already started the chi 3 effect which is the higher order effect for Centro symmetric system for which chi 2 is equal to 0; that means, for a centro symmetric system, if I want to find out the non-linear effect then chi 3 is the first non-linear effect that we will take place because chi 2 is 0 here. So, let us see what we have in our lecture today.

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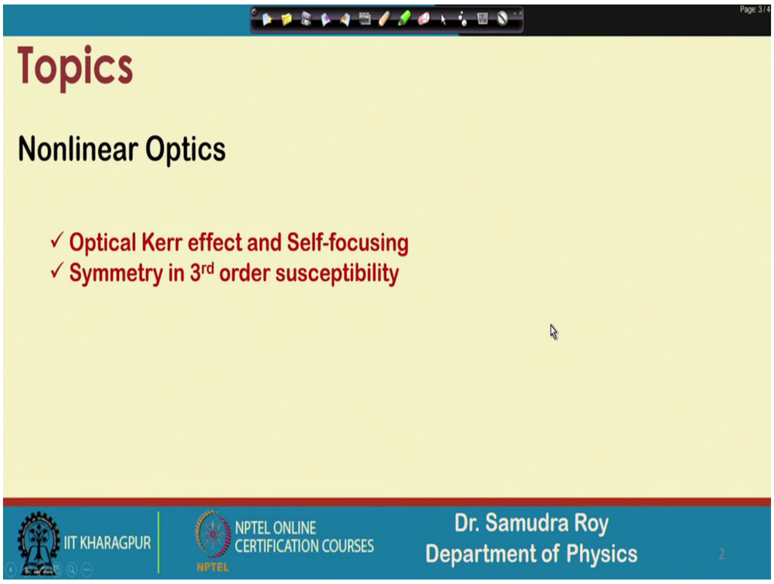

So, today we will going to discuss two important things, one is optical Kerr effect and self focusing and then symmetry in third order susceptibility. So, optical Kerr effect we have already discussed in our previous class, but the detail calculation we will do in today's class. And then try to find out what is the meaning of self-focusing, this is the consequence of a Kerr nonlinearity.

And then susceptibility here also we will have many components like the susceptibility for second order case we define that d i j k. Here also we will find that there will be many numbers or many elements and due to the symmetry there are many elements that we will going to be vanished. So, we will find out what is the procedure to find out the different co efficient of a chi 3 material define linear coefficient. So, let us start with this optical Kerr effect.

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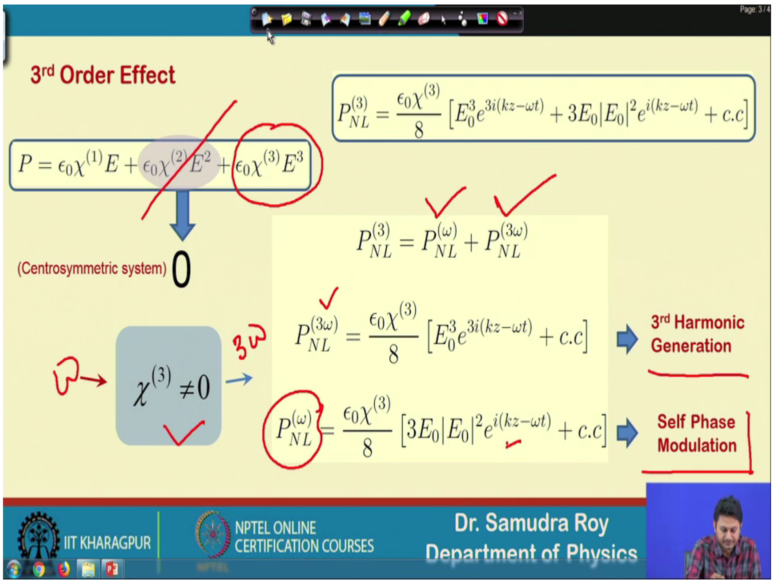

So, this is basically a third order effect as I mentioned for centro symmetric system the chi 2 is 0 as shown here. So, this term will not be there, so it will be vanished because chi 2 is equal to 0. So, chi 3 is the first higher order effect that will give some kind of non-linearity. So, in the system chi 3 is not equal to 0 as mentioned here and we will have something in the input and we will have some output. So, our aim is to find out what kind of output one can expect. So, non-linear polarization here will contain two different terms.

One term will be having a frequency component omega which is the fundamental frequency, and another component will have a frequency component 3 omega. So, this omega component basically leads to self-phase modulation this one. And other component which is having a 3 mega frequency will leads to third harmonic generation. It is quite obvious that when the non-linear polarization will vibrate with a 3 omega frequency, it will going to generate a electric field with 3 omega frequency.

So, if I launch omega frequency here then at the output we can expect 3 omega frequencies. So, like the second harmonic generation here we will have third harmonic generation and the procedure will be exactly same we will see in the future classes, but the phase matching condition need to be satisfied in order to have this kind of things. On the other hand, self-phase modulation is a process this one, where the phase matching condition is automatically satisfied. Because here the electric field will vibrate with the same frequency omega and it will contain the same vector k.

So, that is why this term will not be there, so exponential term the phase term will not going to appear here, but because of the presence of this term we will have something in refractive index. So, refractive index will going to modify and because of the modification of the refractive index; what extra thing we will going to get that will we will discuss .

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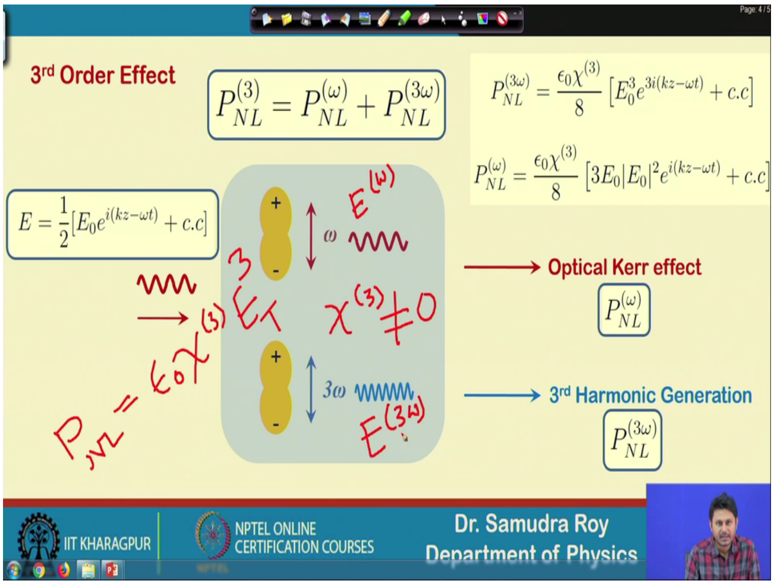

Once again third order effect; the physics of third order effect is important, that if I launch an electric field E which is defined as half E 0 e to the power i k z minus omega t this is a standard way to define an electric field having frequency component omega, and the propagation constant k this is a planar representation of an electric field.

These electric field when insert into the system where chi 3 is not equal to 0. We will have some kind of frequency mixing, because P non-linear in general will be epsilon, chi 3 and E total cube this is the form of polarization non-linear polarization E T is nothing, but the total electric field. So, total electric field will also contain an electric field E omega and also some electric field E 2 omega; E 3 omega. So, this these 2 frequency will going to mix up and will generate something we will see that.

But even if I launch an electric field E then non-linear polarization can be written in 2 terms. First one is this which will going to vibrate at 3 omega frequency as mentioned earlier, and second one is this which will going to vibrate at frequency omega. First one will give 2 optical Kerr effect this is the source term for that and P non-linear 3 omega will give third harmonic generation, because it will it will leads to a vibration of dipole of 3 omega and because of that we will have a electric field of 3 omega, these things we have already discussed in the previous class.

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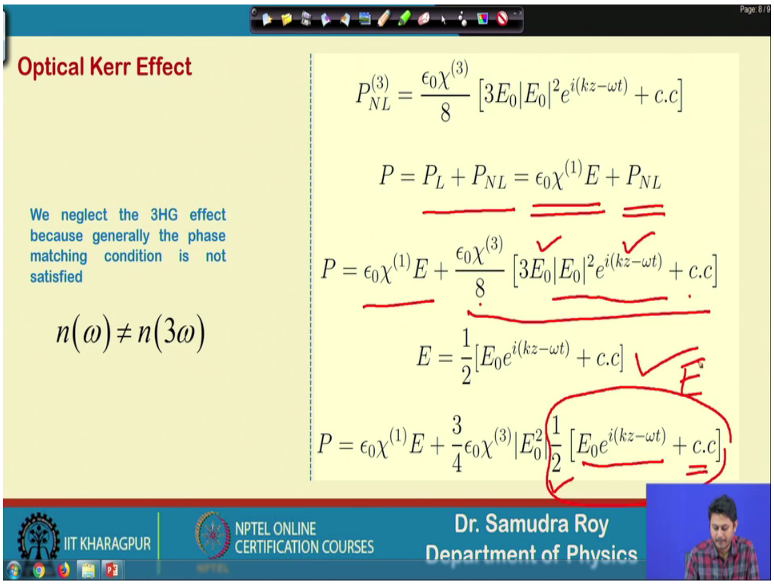

So, today we will see few more things, so P non-linear 3 is a nson-linear polarization here. It will contain 2 term; P linear and P non-linear and we are only considering here we are only considering, the self-phase modulation term with frequency omega. Why we are considering? Because P non-linear if I go to the previous slide, if you look carefully P non-linear basically the combination of P linear P non-linear omega and P non-linear 3 omega. P non-linear 3 omega give raised to third harmonic generation, but in order to, in order to have third harmonic generation 1 important criteria is the phase matching should be there.

But normally we do not have the phase matching condition, so that is why we will not going to take this term for the time being; we will just discard this term. So, P non-linear 3 means it is a third order effect is nothing, but the P non-linear omega; that means, only this term I will take care, this term will contain a frequency component omega mind it and leads to the optical Kerr effect as shown here. So, if I take only these term so what will be my total polarization? My total polarization will be P linear plus P non-linear.

P linear is epsilon 0 chi 1 E as usual where E is a totally electric field and with that we should have P non-linear as a addition, because we are considering the total polarization. Now if I write this things; so, P linear term is epsilon 0 chi i E plus I just replace this P non-linear term here and I will have these things in our hand. Now my total electric field here or whatever the electric field I launched is simply E equal to half E 0 e to the power i k z minus omega t plus the complex conjugate to make it real.

So, this is my total electric field, so now, if I look very carefully this term here we have E 0 here we have E to the power of i k z minus omega t and here we have 1 half term and complex conjugate of these things. So, if I take E to the E 0 square common from that then this term will simply become E 0 E to the power i k z plus complex conjugate with a half. So, half E 0 e to the power i k z omega t plus complex conjugate this entire term is nothing, but the electric field E so, now if I write my total polarization in this form.

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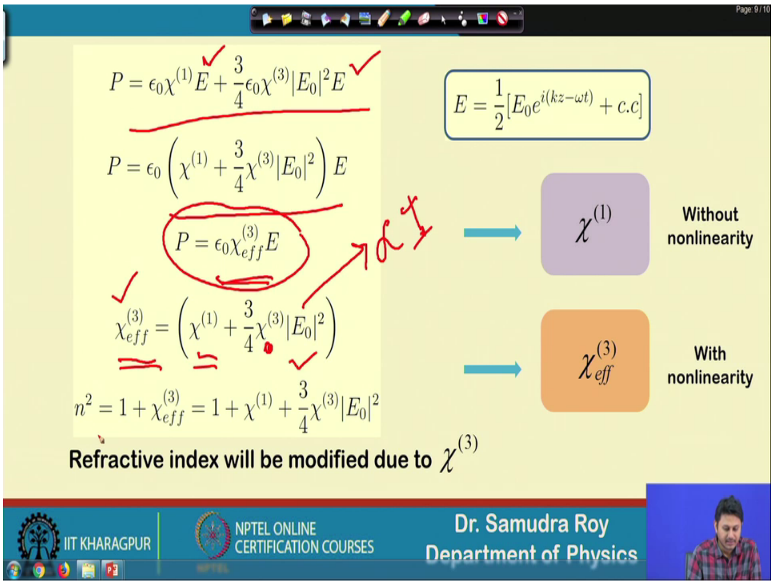

I can have an expression something like this; where we have E here and also 1 E here taking E common I can write epsilon 0 epsilon 1 plus 3 by 4 chi 3 mod of E square into E and eventually will it leads to epsilon 0 chi effective 3 E. So, I actually try to write this P in a standard form, so this is a standard form and when I write this in standard form; we find that the susceptibility term is now modified. So, modified susceptibility chi effective is now written as fundamental susceptibility or chi i chi 1 plus 3 by 4 third order susceptibility mod of E 0 square.

This mod of E 0 square by the way is directly proportional to the intensity; that means, we have some term, inside the susceptibility which contains the intensity in it. So, that mean now because of the presence of third order non-linearity here this term, I can modify the total susceptibility and this total susceptibility can be modified with this intensity term. So, finally if I write this things in terms of refractive index which is a more physical term then refractive index square n square is 1 plus susceptibility we know.

Now, in place of susceptibility we write the effective susceptibility, so 1 plus chi effective. Chi effective again can be represented in terms of this; so now total refractive index is 1 plus chi i plus 3 by 4 epsilon 3 by 4 chi 3 E 0 square mod of E 0 square. So, this additional term is now appearing inside the refractive index so; that means, refractive index is modified due to the presence of third order nonlinearity or the chi 3 effect.

Now, what is the meaning of that? Here in the right hand side we schematically try to show that if my system has only chi 1. So, there is no refractive index changed here so refractive index will remain unchanged. If my system has chi 3 within it so; that means, if the nonlinearity is there; if I consider the these nonlinearity, then what happened my refractive index will modify. This effect is eventually called the Kerr effect, Kerr effect is a effect where we can modify the refractive index by the launching light with the combination of third order nonlinearity and the intensity of the light. Because mod of E 0 square is sitting here, we will try to understand these things in a more general way.

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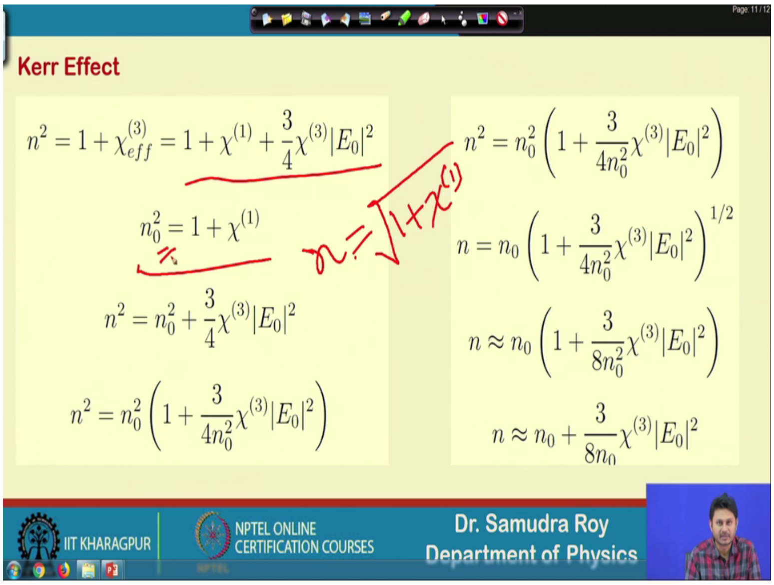

So, Kerr effect as I mentioned if I write the refractive index in terms of susceptibility effective susceptibility, it will be simply this. Now refractive index without any nonlinearity can be represented as n 0 square where n 0 is the refractive index without any kind of nonlinearity. Which is simply written 1 plus chi i we know because refractive index refractive index is root over of 1 plus chi i?

So, refractive index square is square of that; the 0 suggest that I am talking about a refractive index which is non-linear in nature. So, the higher order effect is not included if that is the case then I can write this total refractive index in this particular form n square is equal to n 0 square because this term is now n 0 plus 3 by 4 susceptibility into mod of E square.

So, in the previous calculation we can see that how this term is coming; so we are just continuing this calculation only we replace this 1 plus susceptibility term to n square so that the entire equation can come in terms of refractive index n. Well if I take n square common then it is 1 plus 3 divided by 4 n square susceptibility third order susceptibility mod of E square. And then I can make some kind of approximation; n is now replaced by this by taking a half to the power half term and once we have to the power half then we can expand with a binomial series.

Because this term normally is very small chi 3 is very small we already shown in the last class it is of the order of 10 to the power of minus 20. So, it is very small term since it is a very small term what happened? That I can approximate it is a first order. And if I approximate these thing into the first order it will be simply 3 divided by 8 n 0 square chi 3 mod of E square. So, my total n is now n 0 plus something.

n 0 is a refractive index without any non-linearity, so because of the presence of non-linearity the refractive index is now modified. And we can see that n 0 is a refractive index with without any non-linearity and top of on top of that we can add something, and this addition basically gives the total refractive index .

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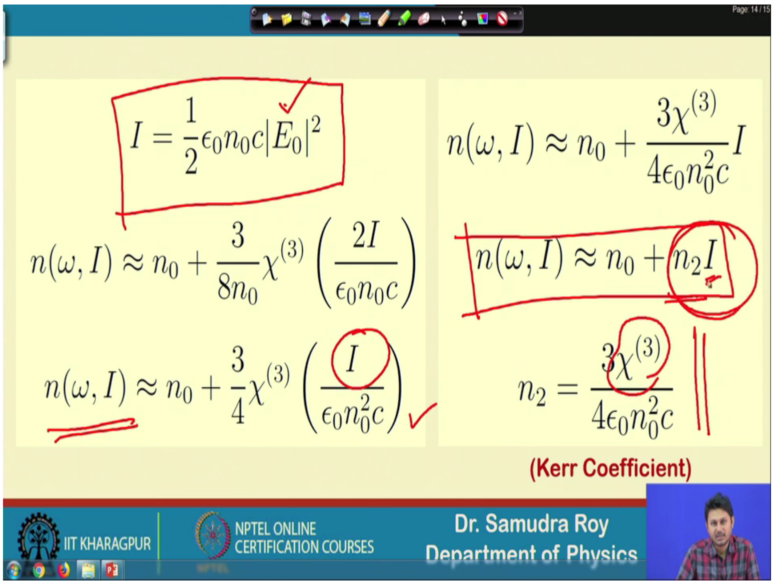

Now, the total refractive index can be represented in more convincing form which is normally given in the literature, and we try to do that. Because as mentioned E mod of E 0 square is there and we can readily change this E 0 mod of E 0 square to intensity; so if I do we just need to use this relation which we are using almost every class that how the mod of field square is related to the corresponding intensity is half epsilon 0; n 0 c mod of field amplitude square.

So, if I replace this E 0 square into the equation then refractive index can be represented as 3 by 8 chi 3 n 0. And now here I replace this mod of E 0 square to 2 I divided by epsilon 0 n 0 divided by c. And then we can write these things a more simpler form and once I have this entire equation. Now here I you can see that if I write refractive index n as a function of frequency and intensity. Normally the refractive index is a function of frequency, but now we find that because of the third order effect refractive index is also function of intensity because intensity is now sitting here explicitly. So, here if I replace these things I can readily see that these thing is nothing, but n 2.

We have already calculated n 2 in our previous class you please check that you will find you will have the same value which we have for n 2. And if I write this n 2 then this is basically the Kerr coefficient and my refractive index can be represented in a more simple and a convincing form which is this. So, total refractive index is the refractive index without non-linearity plus n 2 into I, where n 2 is a Kerr coefficient and this Kerr coefficient basically give us the amount of non-linearity and I is a intensity of the material intensity of the field.

So, there are two properties involved here in order to find out what is my refractive index and non-linearity these is basically the term that is modifying. And this term is now containing two things 1 is n 2 which is a material property because it is related to the suspect third order susceptibility. So, the third order susceptibility will going to change material to material that is one issue second thing is that it also depend on the intensity. If the intensity is very high also we can change or increase our non-linear part of the refractive index. So, there are two way we can increase the non-linear effect in refractive index or the Kerr effect.

One is by choosing suitable material for which chi 3 is very high or we can increase the intensity we launch an electric field and we increase the intensity, and because of the increment of the intensity we can increase this refractive index.

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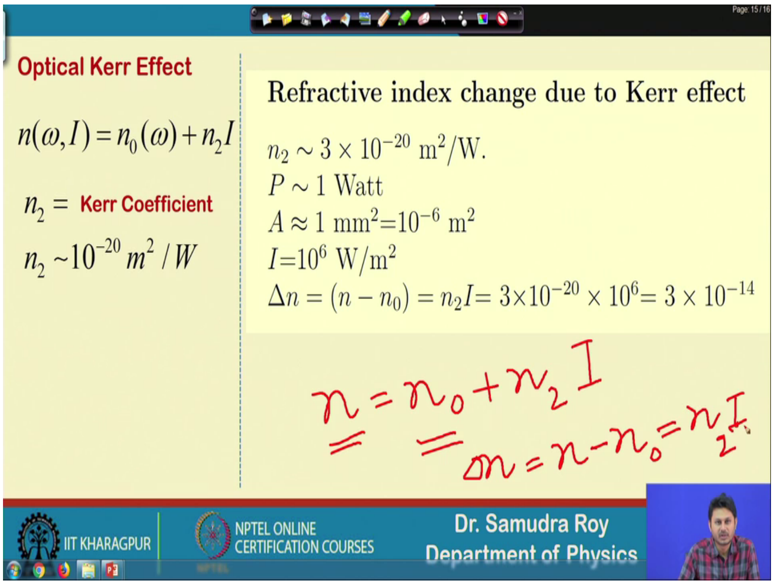

Well, let us try to find out what is the rough; that is a rough estimation that; how the refractive index will going to change we are talking about the equation. So, let me write the equation here in this place that n; n is equal to n 0 plus n 2 I, this is the refractive index without nonlinearity and this is the refractive index with nonlinearity. So, the increment of the refractive index is nothing, but delta n is equal to n minus n 0 which is nothing, but n 2 I.

So, if I calculate n 2 I then we can readily understand that what is the amount of change of refractive index we are talking about. So, in order to do we just try to calculate it with some given values. So, n 2 is order of 10 to the power minus 20 meter square per watt in for example, in silica. So, it is the more correct value is 3 into 10 to the power minus 20 meter square per watt which is n 2. Let us consider a power of 1 watt and area of 1 millimetre square. So, normally when we use a very tiny web guide or fibre then the area is the area of for example, this is a structure of web guide say planar web guide is a planar web guide and we launch the light here in this place.

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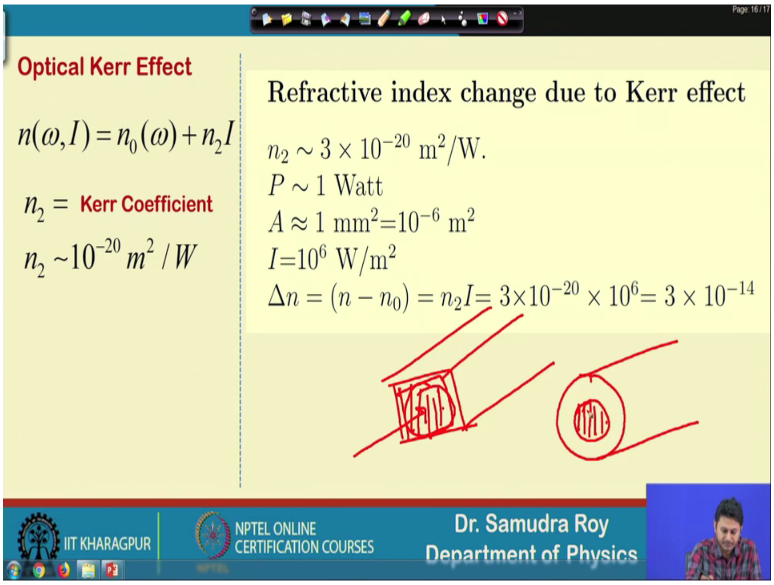

So, what happened that light will be confined inside this web guide so this region is normally the area I am talking about. For fibre this is a cylindrical kind of structure and we have a core here and if I launch the light will be confined inside this core. So, the area of the core is typically the area I am talking about, so the area where the light is confined.

So, here we take a arbitrary value 1 millimetre square which is 10 to the power minus 6 metre square. And if I try to find out the intensity with that because power is given and intensity is also amplitude is also given the area is also given. So, then we can find out the intensity and this intensity is now comes out to be 10 to the power 6 watt per metre square.

Now, n 2 is given and intensity is given if I write this n 2 minus I which is 3 into 10 to the power minus 20 multiplied by 10 to the power 6, it will be nearly 3 into 10 to the power minus 14 very, very less value very, very less amount. So, the increment of the refractive index here because n 2 is positive normally n 2 is positive. So, that is why intensity and the refractive index change will always be positive.

So, the refractive index will increase with a very small value, so increment is very, very small; why the increment is small? Because the effective area I am talking about is very large and also the power is small. In order to increase this what we need to do? We need to put huge amount of power with a very tiny space or very tiny region. So, that the effective area become quite small and the power become quite high; so that the intensity will be very high. So, that I can increase this quantity right now it is 10 to the power 6, if I increase this into 10 to the power 10 or 20, then we have some significant value here which basically gives the change of refractive index.

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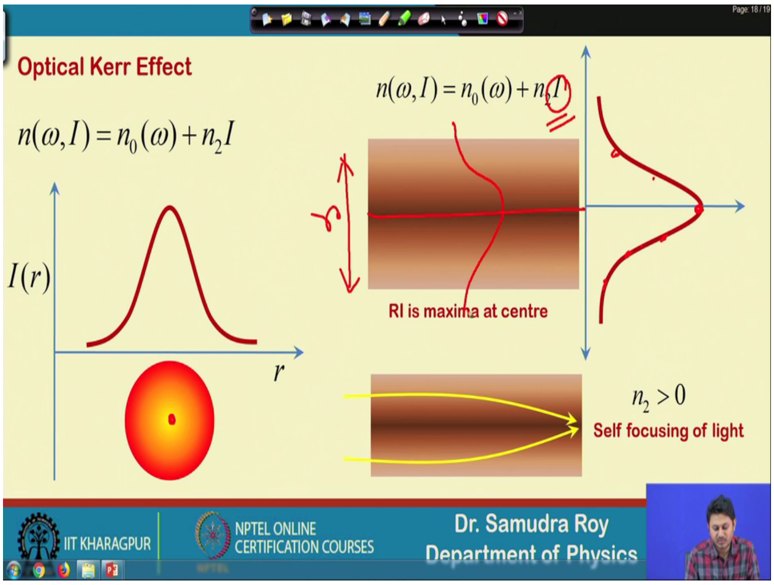

Well after having this idea, so let us see what is the consequence of this change of refractive index? The consequence is this is the distribution of the intensity; so, if the intensity is distributed in this way so this is the distribution of intensity in space. So, in the middle part we have a very high intensity and gradually the intensity will go down so; that means, it should have a this kind of a Gaussian kind of distribution, so now, if this intensity distribution is like that.

So, what happened that the refractive index in this region this if this is my r, because the refractive index because the intensity is changing like this what happened the refractive index will be very high at the middle, because I is maxima at this point and gradually the value of I is reducing because it is a distribution over space. So, the refractive index is gradually reducing.

And as result what we have? We have a graded kind of structure because of the presence of the non-linear refractive index; we have a graded kind of structure. So, the refractive index variation will be something like this. So, refractive index will be maxima at the core or the central part and gradually decreases in both the side; it will be the replica of the intensity distribution because my intensity distribution is something like this. So, as a result we have this kind of distribution, since the refractive index is distributed in this way. If the light is propagating inside this kind of refractive index what happened that the light will confined or gradually bend to its centre.

Because of the positive refractive index this kind of phenomena will happen and we call this phenomenon as self-focusing of light. So, reflective index is maxima here so, gradually refractive index is reducing. So, light will going to bend and some sort of lensing effect will be there. So, light will converge to a point so this is called the self-focusing; so, this phenomena is happening because the refractive index is now function of r and r is now, function of I.

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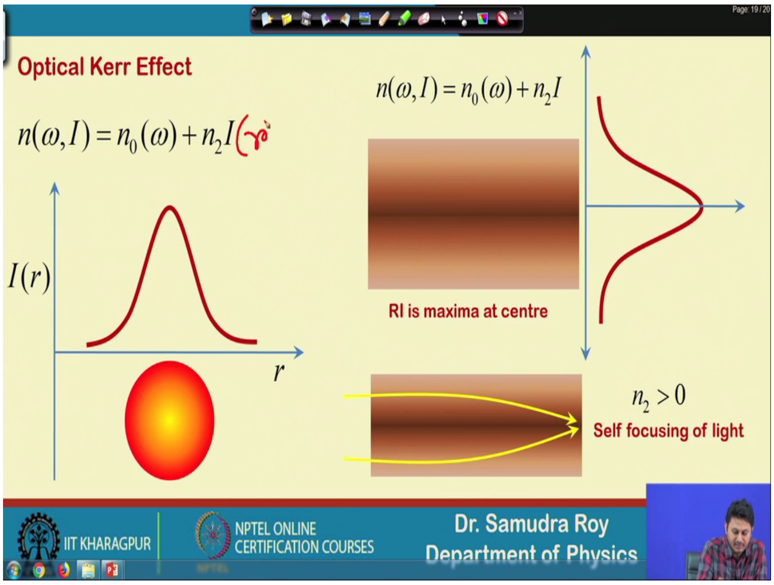

And I is now function intensity is now function of r because it is distributed over space. So, refractive index will now function of r also and it will distribute like this so that the light will confined maxima the maxima of the refractive index in the middle, so light will confined inside the system; well this is the consequence of the Kerr effect.

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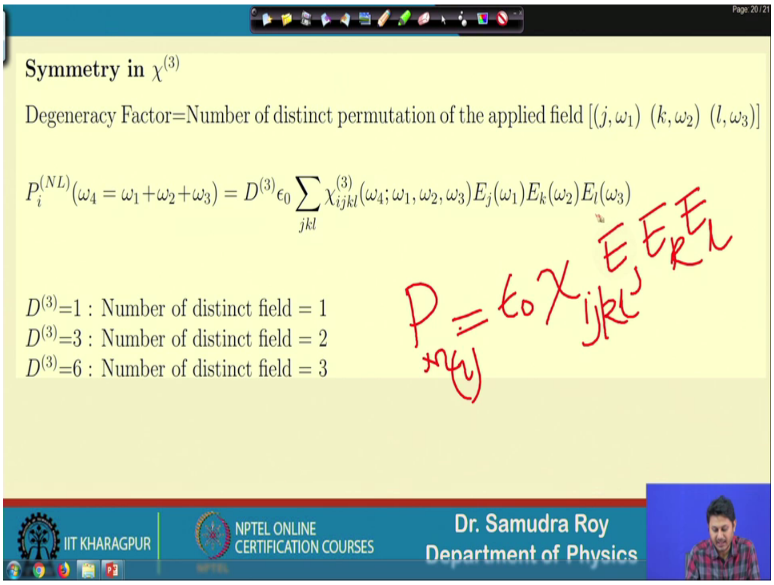

So, now we will quickly study the symmetry of this chi 3; so before going to the symmetry we try to understand few things 1 is the if my non-linear polarisation. So, non-linear polarization is written in this form. So, P non-linear is epsilon 0 chi i j k l this is the ith component. So, it will be E j E k and E l; so there will be 3 electric field component i j k l this is the standard way to represent the non-linear polarization terms of this i j k. So, now if I write this things here j k l then there will be a degeneracy factor.

And the degeneracy factor is that if there are two same fields and one different field. So, number of distinct field is two; then we have 3 term here if there are one different field. So, we will have a degeneracy term 1 here, and if there is 3 different field all the fields are different we have 6 term as a degeneracy factor.

So, we will discuss this issue, in the next class today we do not have that much of time. So, we will start from this place and try to understand what is the degeneracy factor? Because it is important to understand in the previous in the next few classes, where we study the self-phase modulation and cross phase modulation some terms will appear and this term is nothing, but this degeneracy factor. So, with this note let me conclude here, so see you in the next class.

Thank you for your attention.